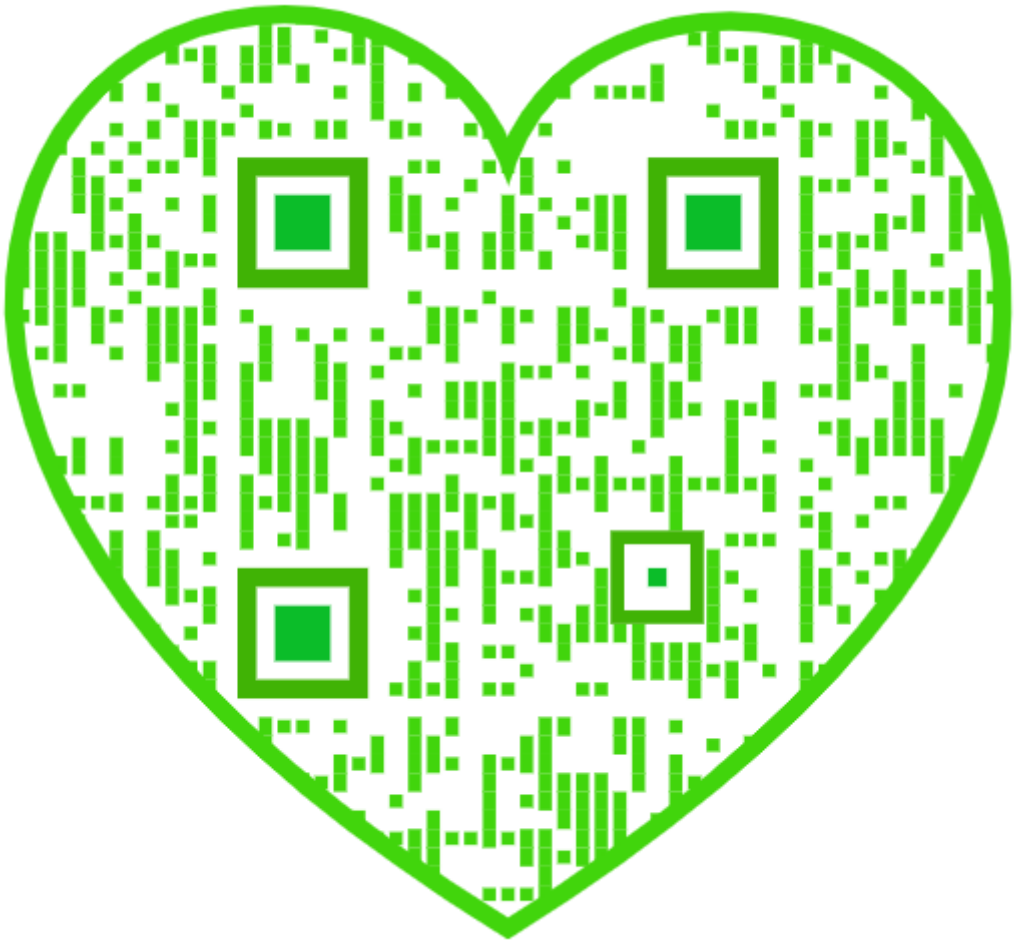

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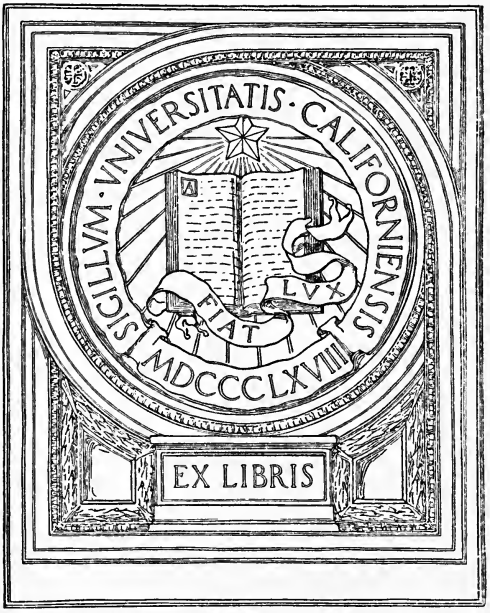

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ELECTRICAL MACHINE DESIGN

THE DESIGN AND SPECIFICATION OF DIRECT AND ALTERNATING CURRENT MACHINERY

BY

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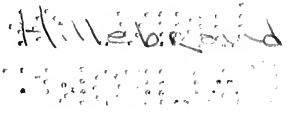

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PREFACE

The following work was compiled as a course of lectures on Electrical Machine Design delivered at McGill University. Since the design of electrical machinery is as much an art as a science no list of formulæ or collection of data is sufficient to enable one to become a successful designer. There is a certain amount of data, however, sifted from the mass of material on the subject, which every designer finds convenient to compile for ready reference. This work contains data that the author found necessary to tabulate during several years of experience as a designer of electrical apparatus.

A study of design is of the utmost importance to all students, because only by such a study can a knowledge of the limitations of machines be acquired. The machines discussed are those which have become more or less standard, namely, direct-current generators and motors, alternating current generators, synchronous motors, polyphase induction motors, and transformers; other apparatus seldom offers an electrical problem that is not discussed under one or more of the above headings.

The principle followed throughout the work is to build up the design for the given rating by the use of a few fundamental formulæ and design constants, the meaning and limits of which are discussed thoroughly, and the same procedure has been followed for the several pieces of apparatus.

The author wishes to acknowledge his indebtedness to Mr. B. A. Behrend, under whom he learned the first principles of electrical design and whose influence will be seen throughout the work; to the engineers of the Allis-Chalmers-Bullock Company of Montreal, Canada, and particularly to Mr. Bradley T. McCormick, Mr. G. P. Cole and Mr. H. F. Eilers; to Mr. A. McNaughton of McGill University for criticism of the arrangement of the work and to Mr. A. M. S. Boyd for assistance in the proof-reading.

McGill University,
September 2, 1912.

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ELECTRICAL MACHINE DESIGN

CHAPTER I

MAGNETIC INDUCTION

1. Lines of Force.—A magnetic field is represented by lines of force. These are continuous lines whose direction at any point is that of the force acting on a north pole placed at the point, therefore, as shown in Fig.1, lines of force always leave a north pole and enter a south pole.

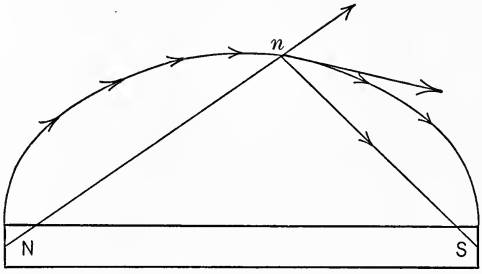


FIG. 1.—Direction of lines of force.

2. Direction of an Electric Current.—P and Q, Fig. 2, are two conductors carrying current. The current is going down in conductor P and coming up in conductor Q. If the direction of

FIG. 2.—Direction of an electric current.

the current be represented by an arrow, then in conductor P the tail of the arrow will be seen and this is represented by a cross; in conductor Q the point of the arrow will be seen and this is represented by a point or dot.

3. Magnetic Field Surrounding a Conductor Which is Carrying Current.—In conductor P, Fig. 3, an electric current is passing downward. It has been found by experiment that in such a case the conductor is surrounded by a whirl of magnetic lines in the direction shown. This direction can be found by the following rule: "If a corkscrew be screwed into the conductor in the direction of the current then the head of the corkscrew will travel in the direction of the lines of force."

4. Magneto Induction.—Faraday's experiments showed that when the magnetic flux threading a coil changes, an e.m.f. is generated in the coil, and that this e.m.f. is proportional to the rate of change of flux in the coil.

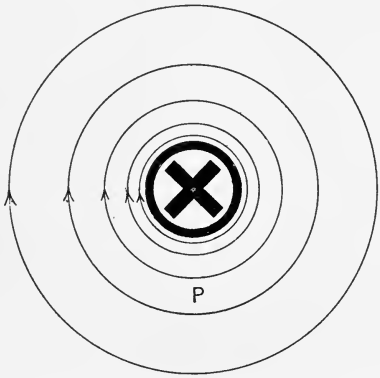


FIG. 3.—Magnetic field surrounding a conductor.

The unit of e.m.f. is so chosen that one unit of e.m.f. is generated in a coil of one turn when the rate of change of flux in the coil is one line per second. This is called the c.g.s. unit; the practical unit, called the volt, is 10^8 c.g.s. units.

5. Direction of the Generated E.M.F.—N and S, Fig. 4, are the north and south poles of a magnet, ϕ is the total number of lines of force passing from the north to the south pole, A is a coil of one turn.

When the coil A is moved from position 1, where the number of lines threading the coil is ϕ, to position 2, where the number of lines threading the coil is zero, in a time of t seconds, then the average e.m.f. generated in the coil $= \frac{\phi}{t} 10^{-8}$ volts; or at any instant the e.m.f. $= \frac{d\phi}{dt} 10^{-8}$ volts.

The quantity $\frac{d\phi}{dt}$ is the rate at which conductor xy, Fig. 4, is cutting lines of force, so that the voltage generated by a conductor which is cutting lines of force is equal to

(the lines cut per second) $\times 10^{-8}$.

The direction of this e.m.f. is found by *Fleming's Rule* which states that "If the thumb, forefinger and middle finger of the right hand are all set perpendicular to one another so as to represent three co-ordinates in space, the thumb pointed in the direction of motion of the conductor relative to the magnetic

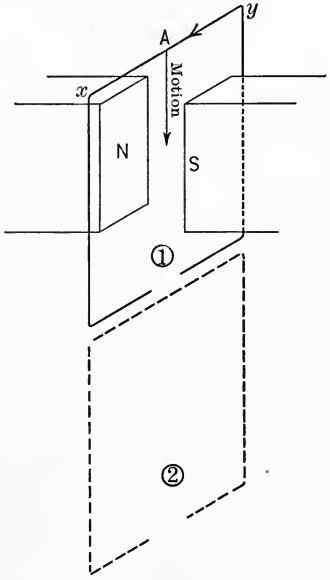


FIG. 4.—Direction of the generated e.m.f.

field, and the forefinger in the direction of the lines of force, then the middle finger will point in the direction in which the generated e.m.f. tends to send the current of electricity."

The direction of the e.m.f. in Fig. 4 is found by Fleming's three-finger rule, and the current due to this e.m.f. is in such a direction as to tend to maintain the flux threading the coil, or, as stated by the very general law known as *Lenz's Law*, "the generated e.m.f. always tends to send a current in such a direction as to oppose the change of flux which produces it." The complete

statement of the e.m.f. equation is, therefore, that the e.m.f. at any instant $= -\frac{d\phi}{dt}10^{-8}$ volts.

6. Magnetomotive Force.—In Fig. 5 the current I passes through the T turns of the coil C which is wound on an iron core, and a magnetic flux ϕ is set up in the magnetic circuit. This flux is found to depend on the number of ampere-turns TI, and, corresponding to Ohm's law for the electric circuit, there is a law for the magnetic circuit—namely, m.m.f. $= \phi R$, where
m.m.f. is the magnetomotive force and depends on TI,
ϕ is the flux threading the magnetic circuit,
R is the reluctance of the magnetic circuit.

The most convenient unit of m.m.f. would have been the ampere-turn, but in order to conform to the definition of potential as used in hydraulic and electric circuits, another unit has to be adopted.

The difference of potential in centimeters between two points in a hydraulic circuit is the work done in ergs in moving unit mass of water from one point to the other, and the difference of magnetic potential (m.m.f.) between two points in a magnetic circuit is the work done in ergs in moving a unit pole from one point to the other.

Let a unit pole, which has 4π lines of force, be moved through the electro-magnet from A to B in a time of t seconds, then an e.m.f. $E = -T \times \frac{4\pi}{t}$ c.g.s. units will be generated between a and b. In order to maintain the current I constant against this e.m.f. an amount of work $= EI$ ergs per second must be done; so that the m.m.f. between A and B

$$= EIt \text{ ergs},$$
$$= \frac{4\pi T}{t} It \text{ ergs},$$
$$= 4\pi TI \text{ ergs when } I \text{ is in c.g.s. units},$$
$$= \frac{4\pi}{10} TI \text{ ergs when } I \text{ is in amperes},$$

therefore the unit of m.m.f. is not the ampere-turn, but is equal to $\frac{4\pi}{10}$ ampere-turns.

It must be understood that this m.m.f. is what might be called the generated m.m.f., thus in the electro-magnet shown

in Fig. 5 the generated m.m.f. is equal to $\frac{4\pi}{10} TI$, but the effective m.m.f. between A and B is equal to this generated m.m.f. minus the m.m.f. necessary to send the magnetic flux round the iron part of the magnetic circuit. Take for example the extreme case shown in Fig. 6, where the core is bent round to form a complete annular ring. The generated m.m.f. between the points A and $B = \frac{4\pi}{10} TI$, but A is the same point as B so that there can be no difference of magnetic potential between them, therefore all the generated m.m.f. is used up in sending the

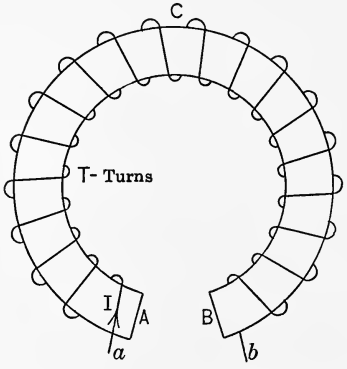


FIG. 5.—Magnetic circuit.

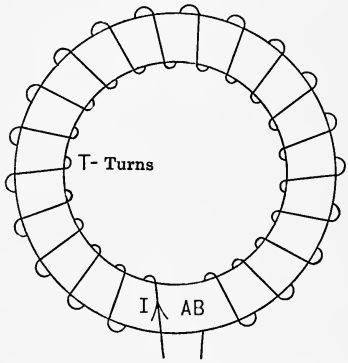


FIG. 6.—Closed magnetic circuit.

flux ϕ through the ring itself. By means of such a ring the magnetic materials used in electrical machinery are tested; the cross-section S of the ring and also the mean length l are known, and for a given generated m.m.f. the flux ϕ can be measured. Then since

$$\text{m.m.f.} = \phi R$$

therefore $\frac{4\pi}{10} TI = \phi \frac{kl}{S} = Bkl$, from which k can be found.

B is the flux density or number of lines per square centimeter,
k is the specific reluctance and $=1$ for air,
l is in centimeters.

For an air path, B, the flux density in lines per square centimeter $= \frac{4\pi}{10} \frac{TI}{l}$, where l is in centimeters. When inch units are

used, so that l is in inches, and the flux density is in lines per square inch, then

$$B, \text{ the flux density in lines per square inch} = 3.2\frac{TI}{l}. \qquad (1)$$

For materials like iron, k, the specific reluctance, is much less than 1, and the value of k varies with the flux density. For practical work the value of k is never plotted; it is more convenient to use curves of the type shown in Fig. 42, page 47, which curves are determined by testing rings of the particular material and plotting B in lines per square inch against $\frac{TI}{l}$, where l is in inches.

CHAPTER II

ARMATURE WINDING

7. Definition of Armature Winding.—In Fig. 7 the armature A of a generator is revolving in the magnetic field NS in the direction of the arrow. The directions of the e.m.fs. which are generated in the conductors of the armature are found by the three-finger rule and shown in the usual way by crosses and dots. The principal purpose of the armature winding is to connect the armature conductors together in such a way that a desired resultant e.m.f. can be maintained between two points which are connected to an external circuit. The conductors and their interconnections taken together form the winding.

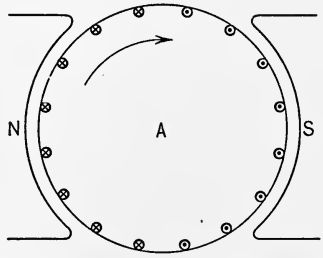


FIG. 7.—Direction of current in a D.-C. generator.

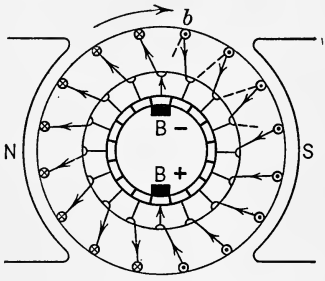


FIG. 8.—Two-pole simplex Gramme winding.

8. Gramme Ring Winding.—This type of winding, which is shown diagrammatically in Fig. 8, was one of the first to be used. Although the winding is now practically obsolete it is mentioned because of its simplicity, and because it shows more clearly than does any other type of winding the meaning of the different terms used in the system of nomenclature.[1]

The two-pole winding shown in Fig. 8 is the simplest type of Gramme winding; it has only two paths between the + and the − brush and is called the *simplex winding* to distinguish it from

[1] The system of nomenclature adopted in this chapter is that of Parshall and Hobart.

the other two fundamental Gramme windings, shown in Figs. 9 and 10. Inspection of these latter figures shows that in each of these cases there are four paths between the + and the − brush, or twice as many as in the case of the simplex winding; for this reason they are called *duplex windings*. There is, however, an essential difference between the two duplex windings and to distinguish between them it is necessary to define the term re-entrancy.

9. Re-entrancy.—If the winding shown in Fig. 8 be followed round the machine starting at any point *b*, it will be found that the winding returns to the starting-point, or is re-entrant, and that before it becomes re-entrant every conductor has been taken in once and only once; such a winding is called a *singly re-entrant winding*.

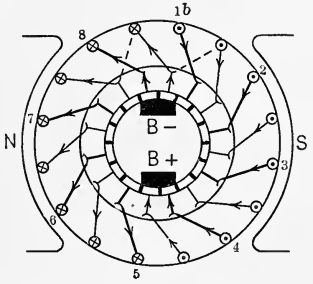


Fig. 9.—Doubly re-entrant duplex winding.

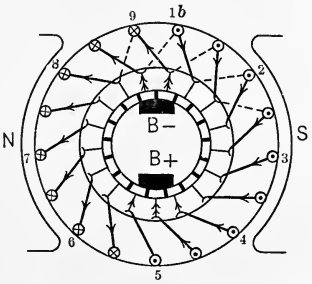


Fig. 10.—Singly re-entrant duplex winding.

If the winding shown in Fig. 9 be followed round the machine starting at any point *b*, it will be found to be re-entrant when only half of the conductors have been taken; in fact the winding is simply two singly re-entrant windings put on the same core, and is called a *doubly re-entrant duplex winding*.

If, on the other hand, the winding shown in Fig. 10 be followed round the machine starting at any point *b*, it will be found that it does not become re-entrant until every conductor has been taken in once and only once; it is therefore a *singly re-entrant duplex winding*.

It is evidently possible to carry this process of increasing the number of paths through the winding much further so as to get multiplex multiply re-entrant, multiplex singly re-entrant, and many other combinations; but such windings are rarely to be found in modern machines, in fact even duplex windings are

seldom used except for large-current low-voltage machines. In such machines the large current entering the brush is divided up and passes through the several paths, so that during commutation the current which is being commutated is only half of what it would have been had a simplex winding been used. This is shown at the positive brush, Fig. 10, where it will be seen that only the current in coil 5, or half of the total current, is being commutated at that instant.

In the case of a duplex winding the brush must be wide enough to cover two segments in order to collect current from all four paths.

10. Objections to the Gramme Winding.—In the case of small machines it is difficult to find space below the core for the return part of the winding without making the diameter of the machine unnecessarily large; while in the case of large machines, where the winding is made of heavy strip copper, it is difficult to remove and replace damaged coils.

The number of coils and commutator segments is twice that required for a machine of the same voltage and with the other type of winding, namely the drum winding.

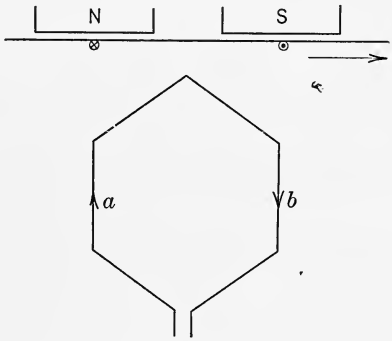


FIG. 11.—Coil for drum winding.

In many cases the active part of a coil is only a small portion of the total coil since the side and return connections do not cut lines of force.

Since the whole winding lies close to the iron of the core the coefficient of self-induction of the coils is large and the machine on that account is liable to spark.

11. Drum Winding.—This winding was developed to overcome the objections to the Gramme winding. In the simplest

case two conductors are joined together to form a coil of the shape shown in Fig. 11. This coil is placed on the machine in such a way that when one side *a* of the coil is under a north pole, the other side *b* is under the adjacent south pole, therefore both sides of the coil are active and the e.m.fs. generated in the two sides act in the same direction round the coil. Since each coil consists of at least two conductors, the total number of conductors for a drum winding must be even.

Fig. 12 shows a two-pole simplex singly re-entrant drum winding with 16 conductors. It might seem that conductors

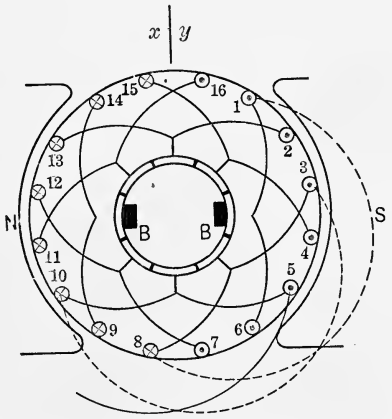


FIG. 12.—Two-pole simplex singly re-entrant drum winding.

which are exactly opposite to one another should be connected in series to form a coil, so that conductor 1 should be connected to conductor 9 and then back to conductor 2, but a few trials will show that in order to get a singly re-entrant winding it is necessary to make the even conductors the returns for the odd. Starting then at conductor 1 the winding goes to the nearest even conductor to that which is exactly opposite, namely conductor 8, then returning back to the south pole the next odd conductor is number 3, which again is connected to conductor 10, and so on; so that the complete winding can be represented by the following table:

$$1^{-}8_3^{-}10_5^{-}12_7^{-}14_9^{-}16_11^{-}2_13^{-}4_15^{-}6_1,$$

which shows clearly that the winding is re-entrant, and also that every conductor has been taken in once and only once.

The connections of this winding to the commutator are shown in Fig. 12.

Fig. 13 is another method adopted to show the connections of the same winding and is obtained by splitting Fig. 12 at xy and opening it out on to a plane. This gives what is called the developed winding and shows clearly the shape of the coil which is used. If Fig. 13 be cut out and bent around a drum it will give the best possible representation of a drum winding.

Inspection of Figs. 12 and 13 shows that, just as in the case of the simplex Gramme winding, there are two paths between the + and the − brush.

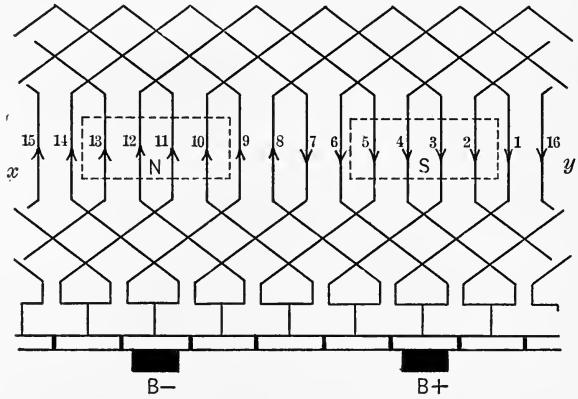


FIG. 13.—Developed two-pole simplex singly re-entrant drum winding.

12. The E.M.F. Equation.—If

ϕ_a is the flux per pole which is cut by the armature conductors,
p is the number of poles,
p_1 is the number of paths through the armature,
Z is the total number of face conductors on the armature surface,
r.p.m. is the speed of the armature in revolutions per minute,
then one conductor cuts $\phi_a p$ lines per revolution
or,

$$\phi_a p \frac{\text{r.p.m.}}{60} \text{ lines in one second.}$$

Since the number of conductors between a + and a − brush $= \frac{Z}{p_1}$

the e.m.f. between the terminals in volts

$$= \frac{Z}{p_1} \phi_a p \frac{\text{r.p.m.}}{60} 10^{-8} \qquad (2)$$

The number of poles and paths is determined by the designer of the machine.

13. Multipolar Machines.—One of the easiest ways by which to get several paths through the winding is to build multipolar

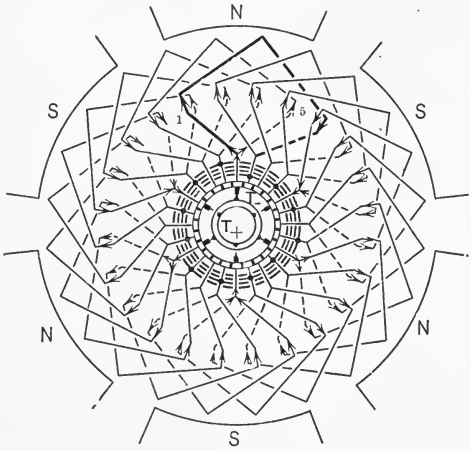


FIG. 14.—Six-pole simplex singly re-entrant multiple drum winding.

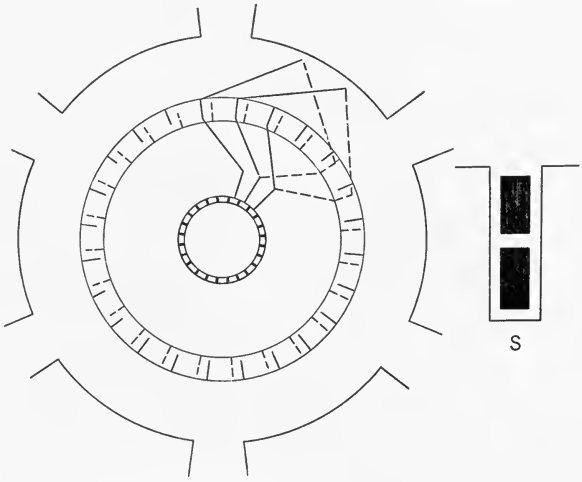


FIG. 15.—Part of a six-pole simplex singly re-entrant multiple drum winding.

machines, thus Fig. 14 shows a six-pole drum winding which has six paths in parallel between the + and the − terminals. There are three + and three − brushes and like brushes are connected together outside of the machine at T_+ and T_-. The

diagram which is used in this case is a slight modification of the developed diagram and gets over the difficulty of splitting the winding. Fig. 14 is rather complicated and will be explained in detail.

Figure 15 shows the armature conductors, the poles and the commutator of the same machine. There are two conductors in each slot so that a section through one of the slots is as shown at *S*, Fig. 15; the winding lies in two layers and is called a *double layer winding.* In Fig. 14 there are 24 slots and 48 conductors; a conductor in the top half of a slot is an odd-numbered conductor and that in the bottom half of a slot is an even-numbered conductor, so that, since the even conductors are the returns for the odd, each coil has one side in the top of one slot and the other side in the bottom of the slot which is one pole pitch further over. The coils are all alike and are made on the same former; a few of the coils are shown in place in Fig. 15, where conductors that are in the top layer are represented by heavy lines while those in the bottom layer are represented by dotted lines.

14. Equalizer Connections.—In the winding shown in Fig. 14 there are six paths in parallel between the + and the − terminals, and it is necessary that the voltages in all six paths be equal, otherwise circulating currents will flow through the machine. In Fig. 16, for example, is shown a case where, due to wear in the bearings, the armature is not central with the poles, and therefore the flux density in the air gap under poles *A* is greater than that in the air gap under poles *B*, so that the conductors under poles *A* will have higher voltages generated in them than those that are under poles *B*, and the voltage between brushes *c* and *d* will be greater than that between brushes *f* and *g*. Since *c* and *f* are connected together, as also are *d* and *g*, a circulating current will flow from *c* to *f*, through the winding to *g*, then to *d* and back through the winding to *c*, as shown diagrammatically at *C*, Fig. 16.

Since the circulating currents pass through the brushes, some of the brushes will have to carry more current than they were designed for, and sparking will result. To prevent the circulating current from having any large value it is necessary to prevent unequal flux distribution in the air gaps under the poles. This can be done by setting up the machine carefully, taking off the outside connections T_+ and T_- between the brushes, and adjusting the thickness of the air gaps under the different poles until

the voltages between + and − brushes are all equal, the machine being fully excited and running at full speed and no load.

It is impossible to eliminate entirely this circulating current, but its effect must be minimized. This is done by providing a low resistance path of copper between the points c and f and also between d and g, inside of the brushes, so that the circulating current will pass around this low-resistance path rather than through the brushes. It must be understood that the equalizer connections, as these low-resistance paths are called, do not eliminate the circulating current, but merely prevent it

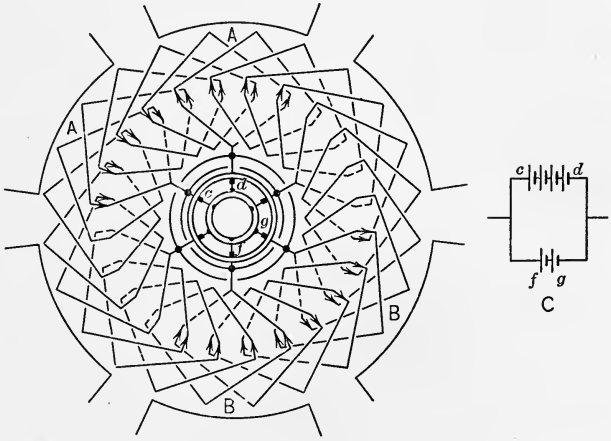


FIG. 16.—Machine with unequal air-gaps to show the action of equalizer connections.

from passing through the brushes. When the armature in Fig. 16 revolves into another position a different set of conductors have to be supplied with equalizers, and in order that the machine may be properly equalized in all positions of the armature, all points which ought to be at the same potential at any instant must be connected together.

Figure 14 is the complete diagram showing all the windings and also the equalizers. It is not necessary to equalize all the coils, because when, as in Fig. 14, the brush is on a coil which is not directly connected to an equalizer, there is still a path of lower resistance than that through the brushes, namely round one turn of the winding and then through the equalizer connection. It is usual in practice to put in about 30 per cent. of the maximum possible number of equalizer connections.

The developed diagram corresponding to that of Fig. 14 is shown in Fig. 17; the only change that has been made is that the equalizer connections have been put at the back of the machine where they can be easily got at for repair. When these connections are placed behind the commutator it is impossible to get at them for repair without disconnecting the commutator from the armature winding.

When an armature is supplied with equalizer connections each brush will carry its proper share of the total current, because at

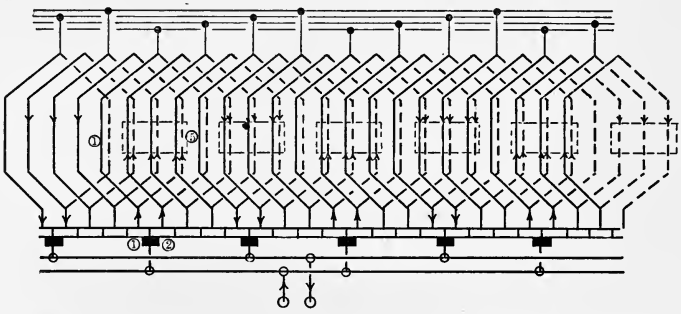


FIG. 17.—Six-pole simplex singly re-entrant drum winding.

one side the brushes are all connected together through the terminal connections and at the other side through the equalizer connections, so that the voltage drop across each brush is the same, and since the brushes are all made of the same material they must have the same current density.

15. Short Pitch Windings.—In Fig. 14 the two sides of each coil are exactly one pole pitch apart; such a winding is said to be *full pitch.* In Fig. 18 is shown a winding in which the two sides of each coil are less than one pole pitch apart; such a winding is said to be *short pitch.* Fig. 19 shows the developed diagram for this short-pitch winding at the instant when the coils in the neutral zone are short-circuited. It will be seen that the effective width of the neutral zone has been reduced by the angle α; this disadvantage, however, is compensated for by the fact that the conductors of the coils which are short-circuited at any instant are not in the same slot. This, as shown in Art. 67 page 82, lessens the mutual induction between the short-circuited coils and tends to improve commutation. Shortening the pitch by more than one slot decreases the neutral zone but does not

further decrease the mutual induction, so that there is no advantage in shortening the coil pitch more than one slot, but rather the reverse. Since there are seldom less than 12 slots per pole the effect of the shortening of the pitch on the generated voltage and on the armature reaction can be neglected.

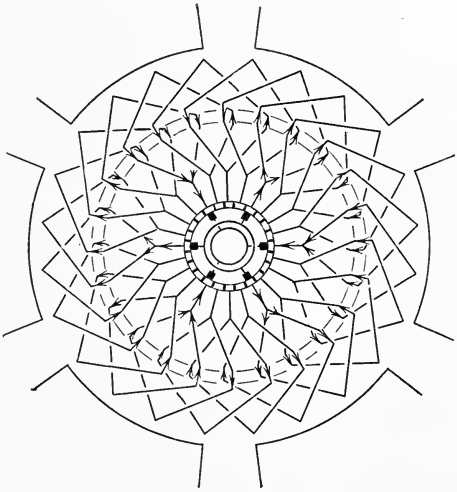


FIG. 18.—Six-pole simplex singly re-entrant short-pitch multiple drum winding.

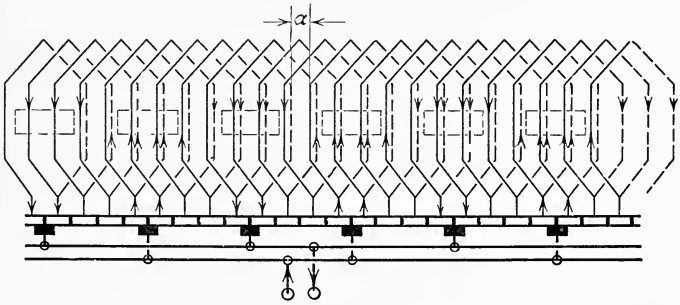


FIG. 19.—Corresponding developed drum winding.

16. Multiple Windings.—The windings shown in Figs. 14 and 18 have the same number of circuits through the armature as there are poles. Had they been duplex windings they would have had twice as many circuits. When the winding has a number of circuits which is a multiple of the number of poles

it is called a *multiple* winding to distinguish it from that described in the next article which is called a *series* winding.

Since windings that are not simplex and singly re-entrant are very rare, these two terms are generally left out, so that, unless it is actually stated to the contrary, all windings are assumed to be simplex singly re-entrant, and the windings shown in Figs. 14 and 18 would be called six-pole multiple drum windings.

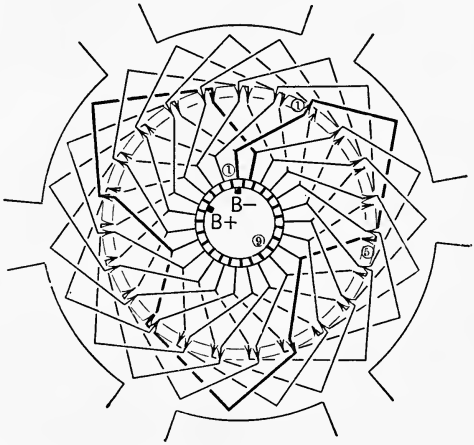


FIG. 20.—Six-pole series progressive drum winding.

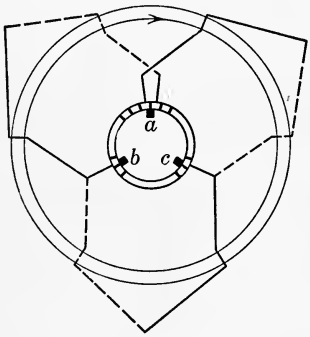


FIG. 21.—Small part of the above winding.

17. Series Windings.—It is possible to wind multipolar machines so that there are only two paths through the armature winding. Such windings are called *series* windings and an example of one is shown in Fig. 20 for a six-pole machine; in Fig. 21 a small portion of the winding is shown to make the complete diagram clearer.

If the winding be followed through, starting from the − brush, it will be seen that there are only two circuits through the armature, and that only two brushes are required. At the instant shown in Fig. 21 the − brush is short-circuiting two commutator segments and in so doing short circuits three coils. Since the points *a*, *b* and *c* are all at the same potential it is possible to put − brushes at each of these points, so that the current will not be collected from one set of brushes but from three; this will allow the use of a commutator of 1/3 of the length of that required when only one set of positive and one set of negative brushes are used. A machine with a series winding will therefore have in most cases the same number of brush sets as there are poles.

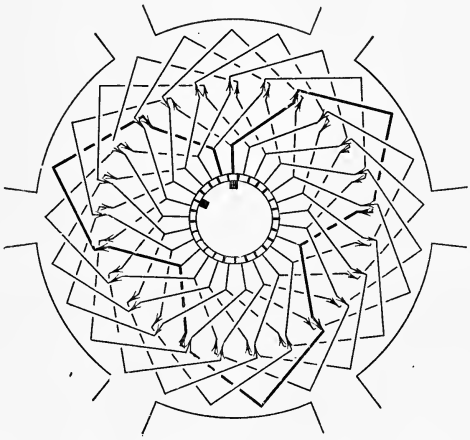


Fig. 22.—Six-pole series retrogressive drum winding.

It may be seen from Fig. 21 that the number of commutator segments must not be a multiple of the number of poles otherwise the winding would close in one turn round the machine; to be singly re-entrant the winding must progress by one commutator segment, as shown in Fig. 20, or retrogress by one commutator segment, as shown in Fig. 22, each time it passes once round the armature, the condition for this is that S, the number of commutator segments, $= k\frac{p}{2} \pm 1$, where k is a whole number and p the number of poles. When the − sign is used the winding is progressive, as in Fig. 20, where the number of commutator

segments is 23, and when the + sign is used the winding is retrogressive, as in Fig. 22, where the number of commutator segments is 25.

The paragraph in Art. 14, page 15, on the division of the total current of the machine, holds to a certain extent for the series winding also; the one side of all the brushes are connected together through the short-circuited coils, as shown in Fig. 21, while the other side of the same brushes are connected together through the terminal connections. It is important to notice, however, that since the number of commutator segments S is not a multiple of $\frac{p}{2}$, the number of pairs of poles, and since the brushes of like polarity are spaced two pole pitches apart, a kind of *selective commutation* will take place, thus when, as shown in Fig. 21, brush *a* is short-circuiting a set of three coils in series, brush *b* has just begun to short-circuit an entirely different set of three coils in series and brush *c* has nearly finished short-circuiting still another set of three coils in series.

The series winding has the great advantage that equalizers are not required, since, as shown in Fig. 21, the winding is already equalized by the coils themselves, so that there can be no circulating current passing through the brushes; further there can

FIG. 23.—Coils with several turns.

be no circulating current in the machine due to such causes as unequal air gaps because each circuit of the winding is made up of conductors in series from under all the poles. On account of this fact, and also because of the property that only two sets of brushes are required, the series winding is used for D.-C. railway motors, because in a four-pole railway motor the two brushes can be set 90 degrees apart so as to have both sets of brushes above the commutator, where they can be easily inspected from the car.

18. Lap and Wave Windings.—From the appearance of the individual coils of the two windings, Figs. 14 and 20, the former is sometimes called a lap winding and the latter a wave winding. It must be understood, however, that each of the coils shown in these diagrams may consist of more than one turn of wire, thus Fig. 23 shows both a lap and a wave coil with several turns per coil, so that the terms *lap* and *wave* apply only to the connections to the commutator and not to the shape of the coil itself; the terms *multiple* and *series* are more generally used.

19. Shop Instructions.—It would be a mistake to send winding diagrams such as those described in this chapter into the shop and expect the men in the winding room to connect up a machine properly from the information given there; the instructions must be given in much simpler form. For a winding such as that shown in Fig. 17 the shop instructions would read: "Put the coil in slots 1 and 5 and the commutator connections in segments 1 and 2," where the position of segment 1 relative to that of slot 1 is fixed by the shape of the end of the coil. All the coils are made on formers and are exactly alike, so that, having the first coil in place, the workman can go straight ahead and put in the other coils in a similar manner.

In the case of a winding such as that shown in Fig. 20, the instructions would read: "Put the coil in slots 1 and 5 and the commutator connections in segments 1 and 9," where the position of segment 1 relative to that of slot 1 is again fixed by the shape of the coil.

20. Duplex Multipolar Windings.—The multipolar windings discussed so far have all been simplex and singly re-entrant. It is evident that both multiple and series windings can be made duplex if necessary; such windings can easily be drawn from the information already given in this chapter and no special discussion of them is necessary.

21. Windings with Several Coil Sides in One Slot.—There are generally more coils than there are slots. Fig. 24 shows part of the winding diagram for a machine which has a multiple winding with four coil sides in each slot and two turns per coil, and Fig. 25 shows part of the winding diagram for a series wound machine also with four coil sides in each slot and two turns per coil. A section through one slot in each case is shown in Fig. 26; there are eight conductors per slot and conductors are numbered similarly in Figs. 25 and 26. In each case there are

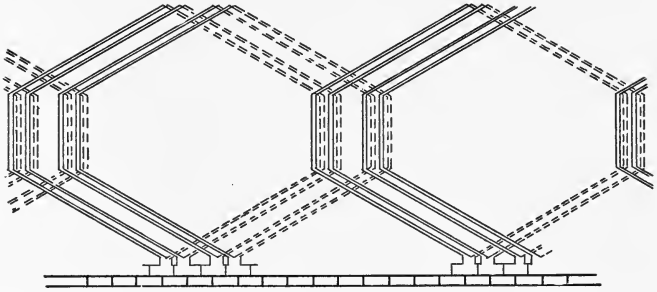


FIG. 24.—Multiple winding with four coil sides per slot and two turns per coil.

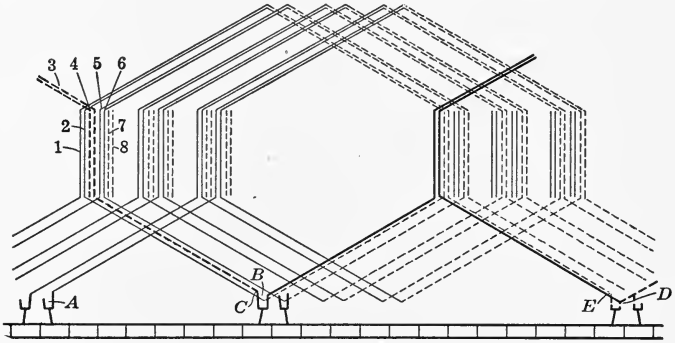


FIG 25.—Series winding with four coil sides per slot and two turns per coil.

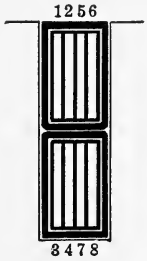


FIG. 26.—Section through one slot of the above windings.

two commutator segments to a slot since the number of commutator segments is always the same as the number of coils, in fact a coil may be defined as the winding element between two commutator segments.

22. Number of Slots and Odd Windings.—For series or wave windings the number of coils = S, the number of commutator segments, $= k\frac{p}{2} \pm 1$, therefore the number of coils must always be odd, and the number of slots should also be odd. Even when an odd number of slots is used it is not always possible to get a wave winding without some modification. Suppose, for example, that a 110-volt four-pole machine has 49 slots and two conductors per slot, then the number of coils $= 49 = 24\frac{p}{2} + 1$, which will give a satisfactory winding; when wound for 220 volts the machine requires twice the number of conductors, or four conductors per slot, so that the number of coils $= 98 = 49\frac{p}{2} \pm 0$ which will not give a wave winding. In such a case, however, the winding can be made wave by cutting out one coil so that the machine has really 97 coils instead of 98, and has also 97 commutator segments; the extra coil is put into the machine for the sake of appearance but is not connected up, its two ends are taped so as to completely insulate the coil, and it is called a *dead coil*.

When the armature is large in diameter it is built in segments as described in Art. 28, page 28. In such a case it is difficult to get an odd number of slots on the armature; indeed, it can only be done when the number of segments that make up one complete ring of the armature and also the number of slots per segment are both odd numbers; however, for reasons to be discussed under commutation, series or wave windings are seldom found in large machines.

For multiple or lap windings the number of slots must be multiple of half the number of poles if equalizers are to be used; this can be ascertained from Fig. 16, where it is seen that in the case of a six-pole machine the equalizers must each connect together three points on the armature exactly two pole pitches apart from one another. It is found, however, that small four-pole machines with multiple windings run sparklessly without equalizers, and since the only condition that limits a multiple winding without equalizers is that the number of con-

ductors be even, for such small machines the same punching is used as for the machine with the series winding, namely, a punching having an odd number of slots; the number of conductors will be even since the winding is double layer, and has therefore a multiple of two conductors per slot. By the use of the same punching for both multiple and series windings, a smaller stock of standard parts needs to be kept, quicker shipment can be made, and lower selling prices given for small motors, than if different punchings were used. In the case of large machines it is best to make the winding such that equalizers can be used, because the sparking caused by the want of equalizers becomes worse as the number of poles and as the output of the machine increases.

CHAPTER III

CONSTRUCTION OF MACHINES

The construction of electrical machinery is really a branch of mechanical engineering but it is one which requires considerable knowledge of electrical phenomena.

Figure 27 shows the type of construction that is generally adopted for D.-C. machines of outputs up to 100 h.p. at 600 r.p.m.

23. The Armature.—*M*, the armature core, is built up of laminations of sheet steel 0.014 in. thick, the thinner the laminations the lower the eddy current loss in the core, but sheets thinner than 0.014 in. are flimsy and difficult to handle.

The laminations are insulated from one another by a layer of varnish and are mounted directly on the shaft of the machine and held there by means of a key, as shown at *K*. It will be noticed that in the key-way *K* there is a small notch, called a marking notch, the object of which is to ensure that the burrs on the punchings all lie the same way; it is impossible to punch out slots and holes without burring over the edge, and unless these burrs all lie in the same direction a loose core is produced.

The laminations are punched on the outer periphery with slots *F* which carry the armature coils *G*. The type of slot shown is that which is in general use and is called the open slot. The other type which is sometimes used is closed at the top; the coils in this case have to be pushed in from the ends. The open slot has the advantage that the armature coils can be fully insulated before being put into the machine, and that the coils can be taken out, repaired, and replaced, in the case of a breakdown, more easily than if the closed type of slot had been adopted.

The armature core is divided into blocks by means of brass vent segments, shown at *P*; the object of the vent ducts so produced is to allow free circulation of air through the machine to keep it cool; they divide the core into blocks less than 3 in. thick and are approximately 3/8 in. wide; narrower ducts are

not very effective and are easily blocked up, while wider ducts do not give increased cooling effect and take up space which might be filled with iron.

The vent segments are mounted directly on the shaft, and, along with the laminations of the core, are clamped between two cast-iron end heads N. These end heads carry the coil supports L which are attached by arms shaped so as to act as fans and maintain a circulation of air through the machine.

The armature coils are held against centrifugal force by steel band wires, five sets of which are shown.

24. Poles and Yoke.—The armature revolves in the magnetic field produced by the exciting coils A which are wound on and insulated from the poles B. In Fig. 27 the poles are of circular cross-section so as to give the required area for the flux with the minimum length of mean turn of the field coil. They are made of forged steel and, attached to them by means of screws, there is a laminated pole face E made of sheet steel 0.025 in. thick, to prevent, as far as possible, eddy currents in the pole faces.

The pole face laminations are stacked up to give the necessary axial length and are held together by four rivets. In Fig. 27 it will be seen that each end lamination has a projection on it for the purpose of supporting the field coil.

The axial length of the pole in the machine shown is 3/8 in. shorter than the axial length of the armature core. This is done to enable the revolving part of the machine to oscillate axially and so prevent the journals and bearings from wearing in grooves. In order that the armature may oscillate freely it is necessary that the reluctance of the air gap does not change between the two extreme positions, the condition for which is that the armature core be longer or shorter axially than the pole face by the amount to be allowed for oscillation, which is usually 3/8 in. for motors up to 50 h. p. at 900 r.p.m., and 1/2 in. for larger machines.

The poles are attached to the yoke C by means of screws, the yoke also carries the bearing housings D which stiffen the whole machine so that the yoke need not have a section greater than that necessary to carry the flux.

The housings and yoke are clamped together by means of through bolts, and the construction must be such that the housings are capable of rotation relative to the yoke through

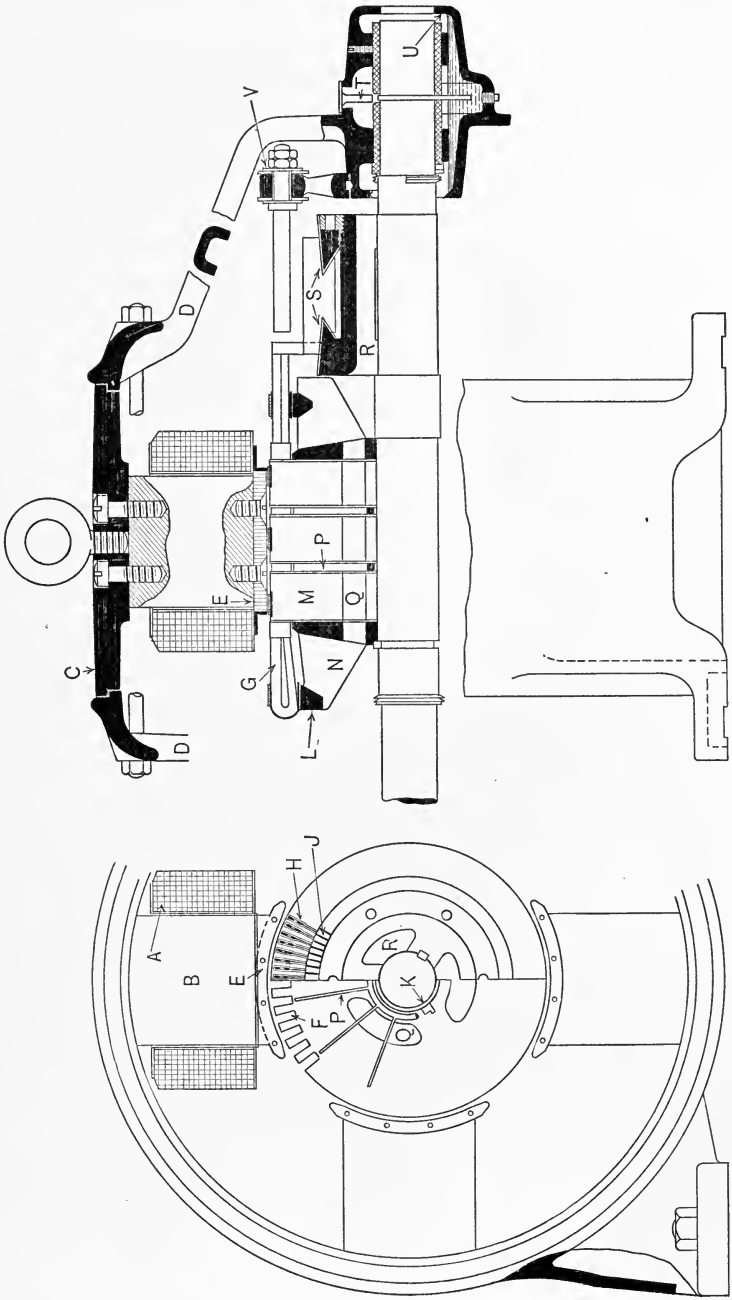


FIG. 27.—Small D.-C. motor.

90 or 180 degrees in order that the machine, usually a small motor, may be mounted on the wall or on the ceiling. This rotation of the bearings is necessary in such cases because, since the machines are lubricated by means of oil rings, the oil wells must be always below the shaft.

25. Commutator.—The commutator is built up of segments J of hard drawn copper insulated from one another by mica which varies in thickness from 0.02 to 0.06 in., depending on the diameter of the commutator and the thickness of the segment. The mica used for this purpose must be one of the soft varieties, such as amber mica, so that it will wear equally with the copper segments.

The segments of mica and copper are clamped between two cast-iron V-clamps S and insulated therefrom by cones of micanite 1/16 in. thick. The commutator shell, as the clamps and their supports are called, is provided with air passages R which help to keep the machine cool.

The commutator segments are connected to the armature winding by necks or risers H which, in all modern machines, have air spaces between them as shown, so that air will be drawn across the commutator surface and between the risers by the fanning effect of the armature. This air is very effective in cooling the commutator.

26. Bearings.—The construction of a typical bearing is shown in detail and is self-explanatory. The points of interest are: the projection T on the oil-hole cover, the object of which is to keep the oil ring from rising and resting on the bushing; the oil slingers on the shaft, which prevent the oil from creeping along the shaft and leaving the bearing dry; the bearing construction with a liner of special bearing metal, which is a snug fit in the bearing shell and which can readily be removed and replaced when worn; the method adopted for draining the oil back into the oil well. The level of the oil is shown and it will be seen that it is in contact with a large portion of the bearing shell and is therefore well cooled. The oil may be drained out when old and dirty by taking out the plug shown at the bottom. There is a small overflow at U which prevents the bearing from being filled too full.

The brushes are carried on studs which are insulated from the rocker arm V. The rocker arm is carried on a turned seat on the bearing and can be clamped in a definite position.

27. Slide Rails.—When the machine has to drive or be driven by a belt, the feet of the yoke are slotted as shown, so that it can be mounted on rails and a belt-tightening device supplied.

28. Large Machines.—For large machines the type of construction is somewhat different from that already described; Fig. 28 shows the type of construction generally adopted for large direct-connected engine units.

When the armature diameter is larger than 30 in., so that the punchings can no longer be made in one ring, the armature core is built up of segments which are carried by dovetails on the spider; the segments of alternate layers overlap one another so as to break joint and give a solid core.

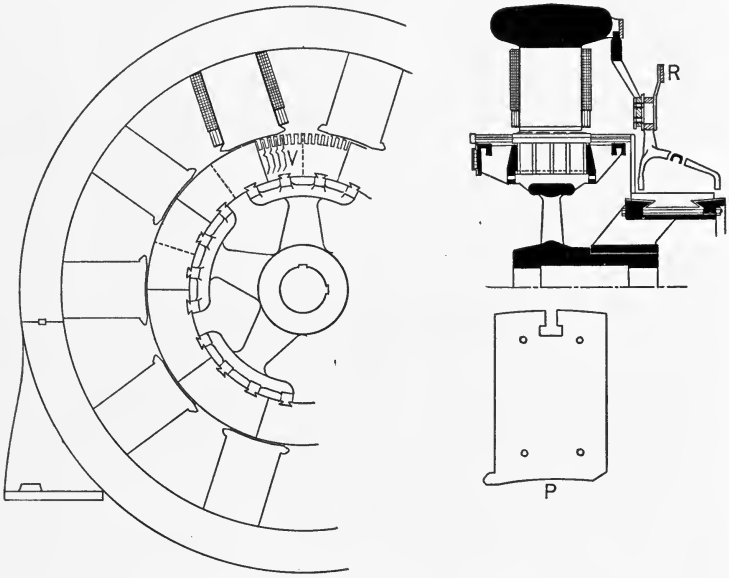


Fig. 28.—Engine type D.-C. generator.

The vent ducts in this machine are obtained by setting strips of sheet steel on edge as shown at V; these strips are carried up to support the teeth and are held in position by projections punched in the lamination adjoining the vent duct. Vent ducts are placed at the ends of the core, partly for ventilation but principally to support the teeth.

The poles are of rectangular cross-section and are built up of laminations of the shape shown at P. The laminations are

0.025 in. thick and are assembled so that the cutaway pole tips of adjacent laminations point in opposite directions. A saturated pole tip is therefore produced, which is an aid to commutation. The laminations are riveted together to form a pole which is then fastened to the yoke by screws.

The shaft, bearings, and base of such a machine are generally supplied by the engine builder. The bearings are similar to those shown in Fig. 27 except that the bushing is generally made of babbitt metal, which is cast and expanded into a cast-iron shell. One oil ring is put in for each 8-in. length of the bearing bushing.

Since the shaft is supplied by the engine builder it is necessary to support the commutator from the armature spider, and one way of doing this is shown in Fig. 28. The brush rigging must also be supported from the machine in some way and is generally carried from the yoke as shown. Brushes of like polarity are joined together by copper rings R which carry the total current of the machine to the terminals.

The yoke of a large D.-C. generator is always split so that, should the armature become damaged, the top half of the yoke can be lifted away and repairs done without removing the armature.

CHAPTER IV

INSULATION

29. Properties desired in Insulating Materials.—A good insulator for electrical machinery must have high dielectric strength and high electrical resistance, should be tough and flexible, and should not be affected by heat, vibration, or other operating conditions.

The material is generally used in sheets and its dielectric strength is measured by placing a sheet of the material 0.01 in. thick between two flat circular electrodes and gradually raising the voltage between these electrodes until the material breaks down. To get consistent results the same electrodes and the same pressure between electrodes should be used for all comparative tests; a size of 2 in. diameter, with the corners rounded to a radius of 0.2 in., and a pressure of 1.5 lb. per square inch, have been found satisfactory. The value of the dielectric strength is defined as the highest effective alternating voltage that 1 mil will withstand for 1 minute without breaking down.

The material should be tested over the range of temperature through which it may have to be used, and all the conditions of the test and of the material should be noted; for example, the dielectric strength depends largely on the amount of moisture which the material contains and is generally highest when the material has been baked and the free moisture expelled.

The flexibility is measured by the number of times the material will bend backward and forward through 90 degrees over a sharp corner without the fibers of the material breaking or the dielectric strength becoming seriously lessened. Materials which are quite flexible under ordinary conditions often become brittle when baked so as to expel moisture.

30. Materials in General Use.—For the insulation of windings the choice is limited to the following materials. Micanite, mica, varnished cloth, paper, cotton, various gums and varnishes.

Cotton tape which is generally 0.006 in. thick and 0.75 in. wide is put on coils in the way shown in Fig. 29, which is called half-lap taping. Such a layer of tape will withstand about 250

volts when dry. When the tape is impregnated with a suitable compound so as to fill up the air spaces between the fibers of the cotton such a half-lap layer will withstand about 1000 volts.

Cotton Covering.—Small wires are insulated by spinning over them a number of layers of cotton floss; the wire generally used for armature and field windings is insulated with two layers of cotton and is called double cotton covered (d.c.c.) wire.

FIG. 29.—Half-lap taping.

Single cotton-covered wire is sometimes used for field windings, and for very small wires silk is used instead of cotton because it can be put on in thinner layers. The thickness of the covering varies with the size of the wire which it covers, and its value may be found from the table on page 508. A double layer, with a total thickness for the two layers of 0.007 in., will withstand about 150 volts. When impregnated with a suitable compound it will withstand about 600 volts.

Micanite, as used for coil and commutator insulation, is made of thin flakes of mica which are stuck together with a flexible varnish. The resultant sheet is then baked while under pressure to expel any excess of varnish and is afterwards milled to a standard thickness, usually 0.01 or 0.02 in.

It is a very reliable insulator and, if carefully made, is very uniform in quality. It can be bent over a sharp corner without injury because the individual flakes of mica slide over one another. To make this possible the varnish has to be very flexible. Being easily bruised, it must be carefully handled, and when put in position on the coil, must be protected by some tougher material.

The dielectric strength of micanite is about 800 volts per mil on a 10-mil sample, and is not seriously lessened by heat up to 150° C., but long-continued exposure to a temperature greater

than 100° C. causes the sticking varnish to loose its flexibility. Micanite does not absorb moisture readily, but its dielectric strength is reduced by contact with machine oil.

Varnished Cloth.—Cloth which has been treated with varnish is sold under different trade names. Empire cloth for example, is a cambric cloth treated with linseed oil. It is very uniform in quality, has a dielectric strength of about 750 volts per mil on a 10-mil sample, and will bend over a sharp corner without cracking. It must be carefully handled so as to prevent the oil film, on which the dielectric strength largely depends, from becoming cracked or scraped.

Various Papers.—These go under different trade names, such as fish paper, rope paper, horn fiber, leatheroid, etc. When dry, they have a dielectric strength of about 250 volts per mil on a 10-mil sample. They are chosen principally for toughness and, after having been baked long enough to expel all moisture, should bend over a sharp corner without cracking. The presence of moisture in a paper greatly reduces its dielectric strength and generally increases its flexibility; all papers absorb moisture from the air. It is good practice to mould the paper to the required shape while it is damp, then bake it to expel moisture and impregnate it before it has time to absorb moisture again.

Impregnating Compound.—The compound which has been referred to is usually made with an asphaltum or a paraffin base, which is dissolved in a thinning material. It should have as little chemical action as possible on copper, iron, and insulating materials. It should be fluid when applied; must be used at temperatures below the break-down temperature of cotton, namely 120° C.; should be solid at all temperatures below 100° C., and should not contract in changing from the fluid to the solid state.

Elastic Finishing Varnish.—This is usually an air-drying varnish and is put on the outside of insulated coils. It should be oil-, water-, acid- and alkali-proof, should dry quickly, and have a hard surface when dry.

31. Thickness of Insulating Materials.—It is not generally advisable to use material which is thicker than 0.02 in., because if there is any flaw in the material that flaw generally goes through the whole thickness, whereas if several thin sheets are used the flaws will rarely overlap; thick sheets also are not so flexible as are thin sheets. For these reasons it is better to use several

layers of thin material to give the desired thickness rather than a single layer of thick material.

32. Effect of Heat and Vibration.—It is not advisable to allow the temperature of the insulating materials mentioned in Art. 30 to exceed 85° C. because, while at that temperature the dielectric strength is not much affected, long exposure to such a temperature makes the materials dry and brittle so that they readily pulverize under vibration. It must be understood that the final test of an insulating material is the way it stands up in service when subjected to wide variations in voltage and temperature and to moisture, vibration, and other operating conditions found in practice.

33. Grounds and Short-circuits.—If one of the conductors of an armature winding touch the core, the potential of the core becomes that of the winding at the point of contact, and if the frame (yoke, housings and base) of the machine be insulated from the ground, a dangerous difference of potential may be

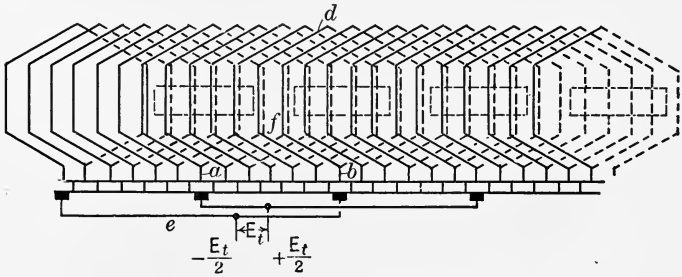


FIG. 30.—Winding with grounds.

established between the frame and the ground. For safety it is advisable to ground the frame of the machine, and then the potential of the winding at the point of contact with the core will always be the ground potential.

If the winding be grounded at two points a short-circuit is produced and a large current flows through the short-circuit, this will burn the windings before the circuit-breakers can open and put the machine out of operation.

If, for example, the winding shown in Fig. 30 becomes grounded at the point a, the difference of potential between the point b and the ground changes from $\frac{1}{2}E_t$ to E_t, but no short circuit is produced unless there exists another ground in the

winding at some point d, or in the system at some point e. If the machine were a motor, a short-circuit would open the circuit-breakers, but not before some damage had been done. If the machine were a generator, and a short-circuit took place between points a and d, the circuit-breakers would not open unless power could come over the line from some other source, such as another generator operating in parallel with the machine in question, or from motors which are driven as generators by the inertia of their load.

34. Slot Insulation and Puncture Test.—As shown in the preceding article, the insulation between the conductors and a core which is grounded may, under certain circumstances, be subjected to a difference of potential equal to the terminal voltage of the machine. Due to operating causes still greater differences of potential are liable to occur. To make sure that there is enough insulation between the conductors and the core, and that this insulation has not been damaged in handling, all new machines are subjected to a puncture test before they are shipped; that is, a high voltage is applied between the conductors and the core for 1 minute. If the insulation does not break down during this test it is assumed to be ample. The value of the puncture voltage is got from the following table which is taken from the standardization rules of the A. I. E. E.

Rated terminal voltage of circuit	Rated output	Testing voltage
Not exceeding 400 volts	Under 10 kw	1000 volts
Not exceeding 400 volts	10 kw. and over	1500 volts
400 and over, but less than 800 volts	Under 10 kw	1500 volts
400 and over, but less than 800 volts	10 kw. and over	2000 volts
800 and over, but less than 1200 volts	Any	3500 volts
1200 and over, but less than 2500 volts	Any	5000 volts
2500 and over	Any	Double normal rated voltage

35. Insulation of End Connections.—Examination of Fig. 30 will show that the voltage between two end connections which cross one another may be, as at point f, almost equal to E_t, the terminal voltage. The end connections must therefore be insulated for this voltage.

36. Surface Leakage.—If the end connections had only sufficient insulation to withstand the voltage E_t this insulation would break down during the puncture test due to what is

called surface leakage. Fig. 31 shows part of a motor winding and the insulation at the point where the winding leaves the slot. The slot insulation is sufficient to withstand the puncture test and is continued beyond the slot for a distance *ef*. When a high voltage is applied between the winding and the core the stress in the air at *b* may be sufficient to ionize it, then the air between *e* and *f* becomes a conductor, the drop of potential between *e* and *f* becomes small, and the voltage across the end-connection insulation at *f*, which equals the puncture voltage minus the

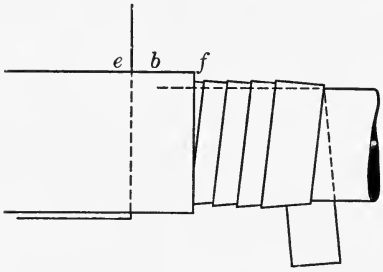


FIG. 31.—Insulation where coil leaves slot.

drop between *e* and *f*, becomes high. To prevent break-down of the end-connection insulation due to this cause the distance *ef* is made as large as possible without increasing the total length of the machine to an unreasonable extent, and the end-connection insulation is made strong enough to withstand the full puncture voltage but with a lower factor of safety than that used for the slot insulation. Suitable values of *ef*, taken from practice, are given in the following table.

Rated terminal voltage of circuit	Length
Not exceeding 800 volts	0.75 in.
800 volts and over, but less than 2500 volts	1.25 in.
2500 volts and over, but less than 5000 volts	2.0 in.
5000 volts and over, but less than 7500 volts	3.0 in.
7500 volts and over, but less than 11000 volts	4.5 in.

37. Several Coil Sides in One Slot.—In Fig. 25, page 21, is shown part of the winding diagram for a machine with four coil sides in each slot and two turns per coil, and Fig. 26 shows a section through one of the slots of the machine. This latter figure is duplicated and shown in greater detail in Fig. 34.

Since the voltage between two adjacent commutator segments seldom exceeds 20 volts and is more often of the order of 5 volts,

the amount of insulation between adjacent conductors need not be large, thus the conductors shown are insulated from one another by one layer of tape on each conductor and the group of conductors is then insulated more fully from the core. The completely insulated group of coils is shown in Fig. 32, and Fig. 33 shows the same group of coils before they are insulated.

When the individual coils are made up of a number of turns of d.c.c. round wire it is advisable to put a layer of paper between

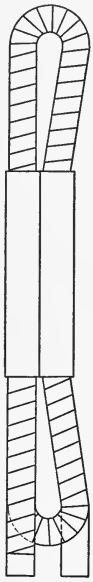


Fig. 32.

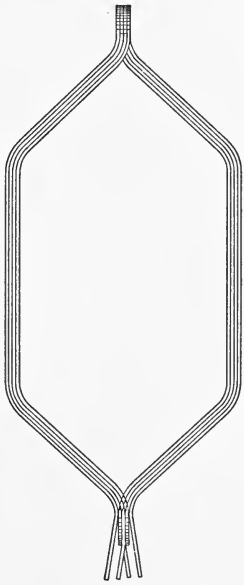


Fig. 33.

Coil for double layer winding with two turns per coil and 8 cond. per slot.

them, as shown at *a*, Fig. 35, because the cotton covering may become damaged when the coils are squeezed together to get them into the slot. The voltage between adjacent turns of the same coil is so low that the cotton covering on the conductor is ample for insulating purposes.

38. Examples of Armature Insulation.—The methods adopted in insulating coils, and the reasons for the various operations, can best be understood by the detailed description of some actual examples.

Example 1.—The insulation for the winding of a 240-volt D.-C. generator. The winding is a double-layer multiple one

with two turns per coil and four coil sides or eight conductors per slot; the conductors are of strip copper wound on edge. A section through the slot and insulation is shown in Fig. 34 and the various operations are as follows:

(*a*) After the copper has been bent to shape, tape it all over with one layer of half-lap cotton tape 0.006 in. thick. This forms the insulation between adjacent conductors in the same slot.

(*b*) Tape together the two coils that form one group with one layer of half-lap cotton tape 0.006 in. thick all round the coils. This forms the end connection insulation and also part of the slot insulation.

(*c*) Bake the coil in a vacuum tank at 100° C. so as to expel all moisture, then dip it into a tank of impregnating compound at 120° C. and leave it there long enough to become saturated with the compound.

(*d*) Put one turn of empire cloth 0.01 in. thick on the slot part of the coil and lap it over as shown at *d*. This empire cloth is $1\frac{1}{2}$ in. longer than the core so that it sticks out $\frac{3}{4}$ in. from each end.

(*e*) Put one turn of paper 0.01 in. thick on the slot part of the coil and lap it over as shown at *e*. This paper also is $1\frac{1}{2}$ in. longer than the core; it is not put on for insulating purposes but to protect the other insulation which is liable otherwise to become damaged when the coils are being placed in the slots.

(*f*) Heat the coil to 100° C. and then press the slot part to shape while hot. The heat softens the compound and the pressing forces out all excess of compound. The coil is allowed to cool while under pressure and comes out of the press with such a shape and size that it slips easily into the slot.

(*g*) Dip the ends of the coil into elastic finishing varnish.

Example 2.—The insulation for the winding of a 10 h.p. 500-volt motor with a double layer winding having five turns per coil and thirty conductors per slot. The conductors are of double cotton-covered wire.

(*a*) Put one turn of paper 0.005 in. thick round the slot part of two of the groups of conductors that form the individual coils. This paper is $1\frac{1}{2}$ in. longer than the core and forms part of the insulation between individual coils in the same slot and also part of the insulation from winding to core.

(*b*) Put one turn of empire cloth 0.01 in. thick round the three

coils that form one group and lap it over as shown at *b*. This empire cloth is $1\frac{1}{2}$ in. longer than the core.

(*c*) Put one turn of paper 0.005 in. thick on the slot part of the coil, make it also $1\frac{1}{2}$ in. longer than the core, and lap it on the top.

(*d*) Tape the ends of the group of three coils with one layer of half-lap cotton tape 0.006 in. thick and carry this tape on to the paper of the slot insulation for a distance of $\frac{1}{2}$ in. so as to seal the coil.

(*e*) Wind the machine with these coil groups putting a lining of paper 0.01 in. thick in the slot and a strip of fiber 1/16 in. thick on the top of the coils and then hold the coils down with band wires.

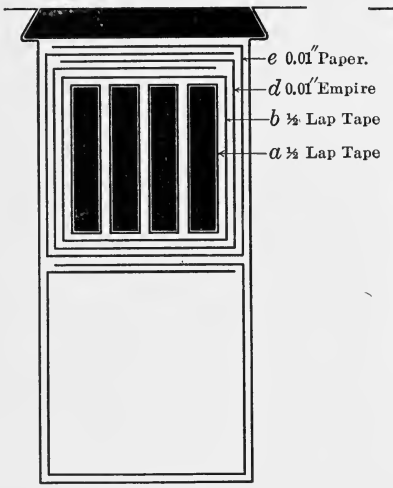


FIG. 34.

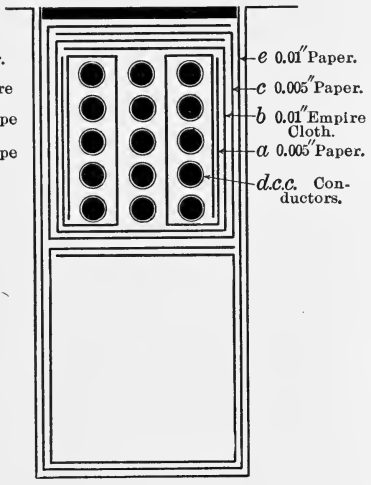


FIG. 35.

Armature slot insulation.

(*f*) Place the armature in a vacuum tank and bake it at 100° C. to expel moisture, then force impregnating compound into the tank at a pressure of 60 lb. per square inch and maintain this pressure for several hours until the winding has been thoroughly impregnated.

(*g*) Rotate the armature, while it is still hot, at a high speed so as to get rid of the excess of compound which will otherwise come out some day when the machine is carrying a heavy load.

(*h*) Paint the end connections with elastic finishing varnish taking care to get into all the corners.

39. Total Thickness and Apparent Strength of Slot Insulation.—Example 1, Fig. 34.

	Width, inches	Depth, inches	Volts
Tape on conductor	.....	.....	1,000
Tape on group of coils	0.024	0.024	1,000
Empire cloth	0.02	0.03	7,500
Paper	0.02	0.03	2,500
Total	0.064	0.084	12,000

In the above table under the heading of width is given the space taken up in the width of the slot by the different layers of insulation. The tape on the conductor has not been added because it is a variable quantity and depends on the number of conductors per slot.

Under the heading of depth is given the space taken up in the depth of half a slot by the different layers of insulation; here also the tape on the individual conductors has not been added since it varies with the number of conductors which are vertically above one another in the slot.

The apparent strength of the above insulation is 12,000 volts and the required puncture test is 1500 volts so that there is a factor of safety of 8.

Example 2, Fig. 35.

	Width, inches	Depth, inches	Volts
D. c. c. on wire	.....	.....	600
Paper on coils	0.03	0.01	1,250
Empire cloth	0.02	0.03	7,500
Paper	0.01	0.015	1,250
Paper	0.02	0.005	2,500
Total	0.08	0.06	13,100

The puncture voltage is 2000 and the factor of safety $=6.5$.

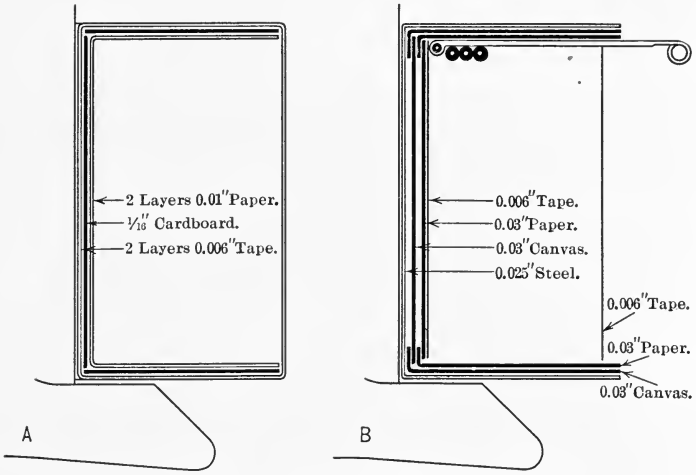


FIG. 36.—Field coil insulation.

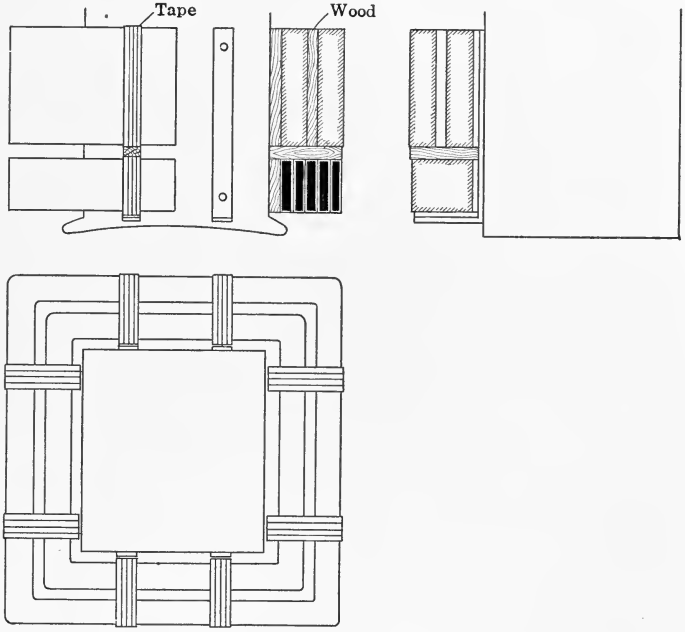


FIG. 37.—Ventilated field coils.

40. Field Coil Insulation.—Two examples of field coil insulation are shown in Fig. 36. Diagram *A* shows an example of a coil which is carried in a cardboard spool while diagram *B* shows an example of a coil which is carried in an insulated metal spool. In both cases the coils, after being wound in the spool and taped up, are baked in a vacuum and then impregnated with compound. This compound is a better insulator than the air which it replaces it is also a better conductor of heat.

Figure 37 shows the type of coil which is used to a large extent on machines the armature diameter of which is greater than 20 in. The shunt coils are made of d.c.c. wire, wound in layers; the individual shunt coils are 1 in. thick and are separated by ventilating spaces 1/2 in. wide. The insulation is carried out entirely by wooden spacing blocks so that there is a large radiating surface, and also little insulation to keep in the heat. The coils are made self-supporting by being impregnated with a solid compound at about 120° C.

The insulation on the individual turns of the series coil consists of one layer of cotton tape 0.006 in. thick and half lapped; this coil, when made of strip copper as shown, is not impregnated but is dipped in finishing varnish.

CHAPTER V

THE MAGNETIC CIRCUIT

41. The Magnetic Path.—Fig. 38 shows two poles of a multipolar D.-C. generator, each pole of which has an exciting coil of T_f turns through which a current I_f flows. Due to this excitation a magnetic flux is produced and the mean path of this flux is shown by the dotted lines. This magnetic flux consists of two parts, one, ϕ_a, which crosses the air gap and is therefore cut by the armature conductors, and the other, ϕ_e, which does not cross the air gap and is called the leakage flux.

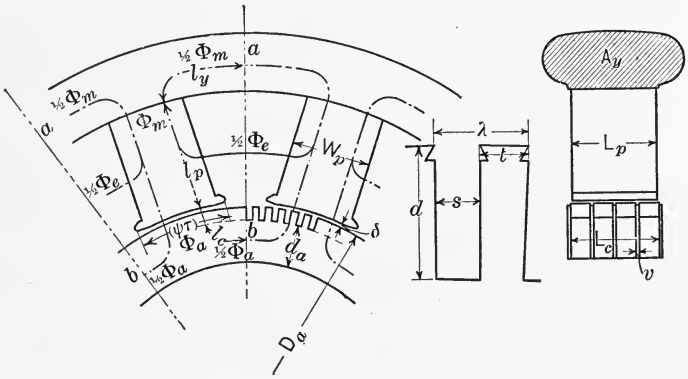


FIG. 38.—The paths of the main and of the leakage fluxes.

42. The Leakage Factor.—The total flux which passes through the yoke and enters the pole $= \phi_m = \phi_a + \phi_e$ and the ratio $\frac{\phi_m}{\phi_a}$ is called the leakage factor and is greater than 1.

43. The Magnetic Areas.—The meanings of the symbols used in this article will be understood by reference to Fig. 38.

D_a = the external diameter of the armature.

p = the number of poles.

τ = the pole pitch $= \frac{\pi D_a}{p}$.

ψ = the per cent. enclosure of the pole $= \frac{\text{pole arc}}{\text{pole pitch}}$.

L_c = the axial length of the armature core.

m = the number of vent ducts in the center of the core.
v = the width of each vent duct.
L_g = the gross length of the iron in the core $= L_c - mv$.
L_n = the net length of the iron in the core $= 0.9\ L_g$; it is less than L_g by the amount of the insulation between laminations.
N = the total number of slots in the armature.
λ = the slot pitch.
s = the slot width.
t = the tooth width.
d = the depth of the slot.
d_a = the depth of the armature core below the slot.
W_p = the pole waist.
L_p = the axial length of the pole.
Then:
A_g = the apparent gap area per pole $= \psi\tau L_c$.
A_{ag} = the actual gap area per pole $= \frac{A_g}{C}$ where C is a constant greater than 1, called the Carter coefficient. This constant takes into account the effect of the slots in reducing the air-gap area.
A_t = the tooth area per pole $= \psi\frac{N}{p}tL_n$; only those teeth which are under the poles are effective.
A_c = the area of the armature core $= d_a L_n$.
A_p = the pole area $= W_p L_p$ when the pole is solid; when built up of laminations the pole area $= (W_p L_p \times$ const.) where the const. is a stacking factor and $= 0.95$ approximately.
A_y = the yoke area.

44. The Carter Coefficient.[1]—Fig. 39 shows the path of the magnetic flux across the air gap. If it were not for the armature slots and vent ducts the air-gap area per pole would be $\psi\tau L_c$ which is called the apparent gap area. The actual gap area per pole $= \left(\frac{x}{\lambda}\psi\tau L_c\right)$ where $\frac{\lambda}{x}$, Fig. 39, is the Carter coefficient.

$x = t + fs$ where f depends on the slot width s and on the air-gap thickness δ, and is got from Fig. 40 for different values of the ratio $\frac{s}{\delta}$; then C, the Carter coefficient $= \frac{\lambda}{x} = \frac{t+s}{t+fs}$.

[1] *Elec. World and Engineer*, Nov. 30, 1901.

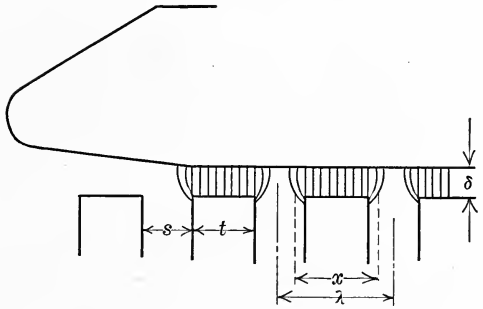


FIG. 39.—Distribution of flux in the air-gap.

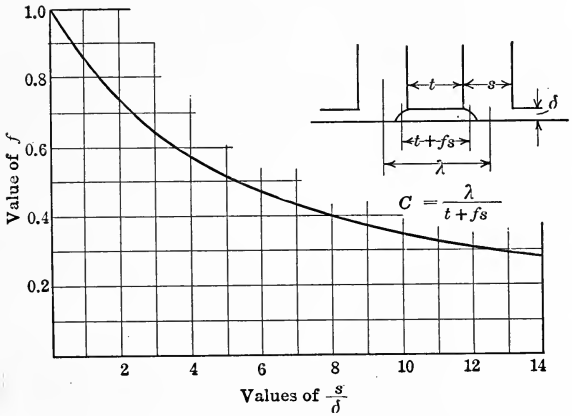


FIG. 40.—The Carter fringing constant.

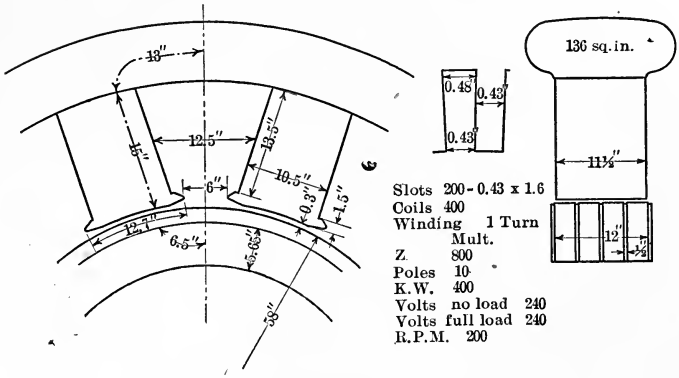


FIG. 41.—Magnetic circuit.

It is required to find the value of the Carter coefficient for the machine drawn to scale in Fig. 41.

$s = 0.43$ in.

$t = 0.48$ in.

$\delta = 0.3$ in.

$\frac{s}{\delta} = 1.44$

$f = 0.78$ from Fig. 40

$$C = \frac{0.48 + 0.43}{0.48 + 0.78 \times 0.43} = 1.12$$

There is a small amount of fringing at the pole tips which tends to increase the air-gap area, but its effect is counterbalanced by the fact that at the pole tips, δ, the thickness of the air gap, is increased.

The Carter coefficient for the vent ducts can be found in the same way as for the slots, but since its value is nearly always $= 1$ the calculation is seldom made.

45. The Flux Densities.—The flux in the different parts of the magnetic circuit is shown in Fig. 38, then:

B_g = the apparent flux density in the air gap $= \frac{\phi_a}{A_g}$.

B_{ag} = the actual flux density in the air gap $= CB_g$.

B_t = the apparent flux density in the teeth $= \frac{\phi_a}{A_t}$.

B_c = the flux density in the armature core $= \frac{\phi_a}{2A_c}$.

B_p = the flux density in the pole $= \frac{\phi_m}{A_p}$.

B_y = the flux density in the yoke $= \frac{\phi_m}{2A_y}$.

The flux density in the teeth at normal voltage is generally about 150,000 lines per square inch, and at such densities the permeability of the iron in the teeth becomes comparable with that of air, so that a considerable amount of flux passes down the slots, vent ducts, and the air spaces between the laminations. If the assumption is made that the teeth have no taper, and that the lines of force are parallel both in the teeth and in the air paths, and if

B_{at} = the actual flux density in the teeth,

B_s = the flux density in the air path consisting of the slots, vent ducts, and air spaces between laminations,

B_t = the apparent flux density in the teeth, then

ϕ_a = the total flux per pole entering the armature $= B_t A_t$,
$= B_{at} A_t + B_s A_s$, where A_s is the area of the air path per pole; therefore

$$B_t = B_{at} + B_s \left(\frac{A_s}{A_t}\right)$$
$$= B_{at} + B_s \left(\frac{\lambda L_c - t L_n}{t L_n}\right).$$

For a given number of ampere-turns between the two ends of the slot, assuming the slot to be 1 in. deep,
B_s = 3.2 (ampere-turns); formula 1, page 6.
B_{at} = the value of flux density corresponding to the given number of ampere turns, found from Fig. 42.
B_t, corresponding to the given number of ampere-turns, is found by substitution in the above formula.

The relation between B_t and B_{at} is plotted in Fig. 43 for different values of the ratio $\frac{\lambda}{t}$ and for the magnetization curve in Fig. 42, the assumption being made that $L_n = 0.8\ L_g$. For example, suppose that in a particular case the width of tooth is equal to that of the slot so that $\frac{\lambda}{t} = 2$, that the ratio $\frac{L_n}{L_c} = 0.8$, and that the actual flux density in the tooth is 160,000 lines per square inch. Then the ampere-turns necessary to send this flux through 1 in. of the tooth is 2250, from Fig. 42; the flux density in the air path due to 2250 ampere-turns is $3.2 \times 2250 =$ 7200 lines per square inch, see formula 1, page 6, and

$$B_{gt} = 160{,}000 + 7200 \left(\frac{\lambda - 0.8t}{0.8t}\right) \text{ where } \lambda = 2t$$
$$= 170{,}800 \text{ lines per square inch.}$$

46. Calculation of the No-load Saturation Curve.—It is required to find the number of ampere-turns necessary to send a certain flux ϕ_a across the air gap of the machine shown in Fig. 38.

The m.m.f. between points a and b is $T_f I_f$ ampere-turns and this must be equal to $AT_y + AT_p + AT_g + AT_t + AT_c$, where

AT_y is the ampere-turns necessary to send the flux $\frac{1}{2}\phi_m$ through the length l_y of the yoke. The value of B_y is known and the corresponding number of ampere-turns required for each inch of the yoke path is found from Fig. 42. This value of ampere-turns per inch multiplied by the length l_y gives the value of AT_y.

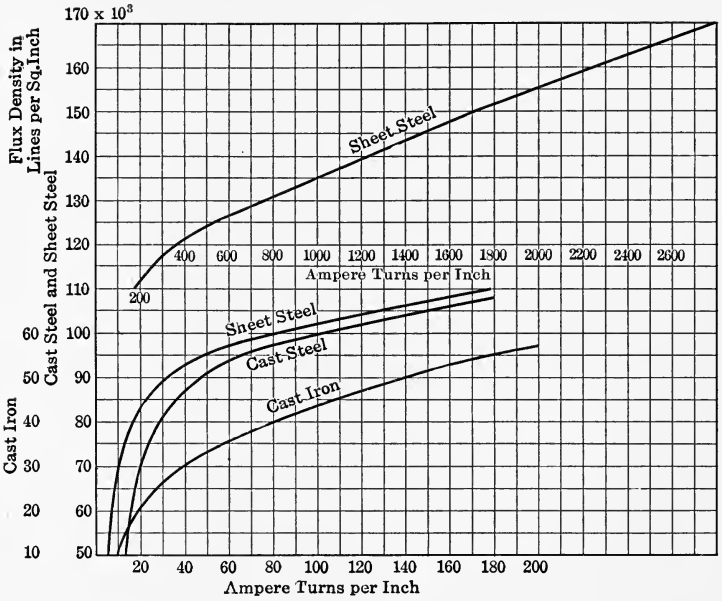


FIG. 42.—Magnetization curves.

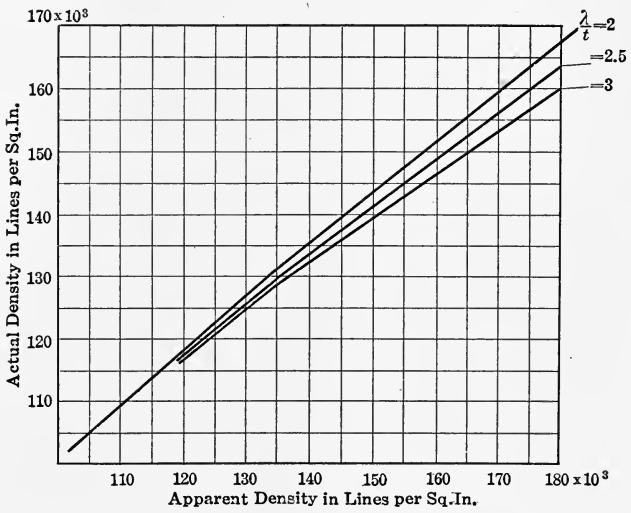


FIG. 43.—Densities in armature teeth.

AT_p is the ampere-turns necessary to send the flux ϕ_m through the length l_p of the pole and is found in a similar manner.

AT_g is the ampere-turns necessary to send the flux ϕ_a across one air gap. To find this value it is necessary to find first of all the value of C, the Carter coefficient, and then the actual flux density in the air gap, namely $B_{ag} = CB_g$.

$$B_{ag} \text{ lines per square inch} = 3.2 \frac{AT_g}{\delta} \qquad (3)$$

where δ is the air gap thickness in inches.

AT_t is the ampere-turns necessary to send the flux ϕ_a through the length d of the tooth. The value of B_t, the apparent tooth

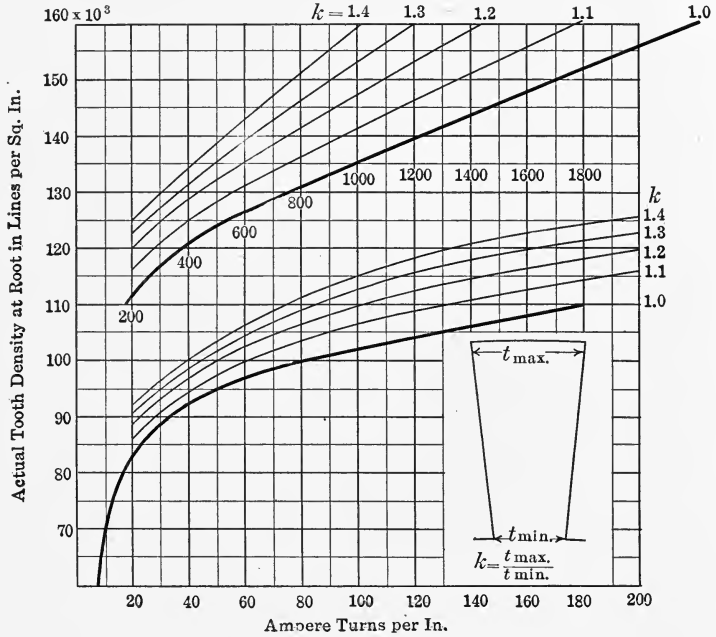


FIG. 44.—Magnetization curves for sheet steel.

density, is readily found, and the value of B_{at}, the actual flux density, can be found by the use of the curves in Fig. 43. The value of ampere-turns per inch corresponding to this actual density can then be taken from Fig. 42. This latter quantity when multiplied by d gives the value of AT_t.

When the teeth are tapered as shown in Fig. 44, so that the flux density is not uniform through the total depth of the tooth,

the problem becomes more difficult. It is necessary to divide the tooth length d into a number of small parts, find the average flux density in each of these parts and the corresponding value of ampere-turns per inch; the average value of these latter quantities multiplied by d gives the value of AT_t. This process is slow, but in Fig. 44 is plotted a series of curves whereby, if the actual flux density at the top and bottom of the tooth is known, the average ampere-turns per inch can be found directly.

AT_c is the ampere-turns necessary to send the flux $\frac{1}{2}\phi_a$ through the length l_c of the core and is found by the use of the curves in Fig. 42.

Example.—Fig. 41 shows a dimensioned sketch of a 10-pole, 400-kw., 240-volt, 200 r. p. m. generator; it is required to draw the no-load saturation curve for this machine.

$$E = Z\,\phi_a \frac{\text{r.p.m.}}{60} \frac{\text{poles}}{\text{paths}} 10^{-8} \text{ volts}$$

$$\text{therefore } \phi_a = \frac{240 \times 60 \times 10 \times 10^8}{800 \times 200 \times 10}$$

$$= 9 \times 10^6 \text{ at 240 volts, no-load.}$$

The magnetic areas:

τ	=the pole pitch	$= \frac{\pi \times 58}{10} = 18.2$ in.
ψ	=the per cent. enclosure	$= 0.7$.
L_g	=the gross iron	$= 10.5$ in.
L_n	=the net iron	$= 9.45$ in.
λ	=the slot pitch	$= 0.91$ in. at top of slot.
		$= 0.86$ in. at bottom of slot.
t	=the tooth width	$= 0.48$ in. at top.
		$= 0.43$ in. at bottom.
A_g	=the apparent gap area	$= 0.7 \times 18.2 \times 12 = 153$ sq. in.
C	=the Carter coefficient	$= 1.12$ from Art. 44, page 45.
A_t	=the minimum tooth area per pole	$= 0.7 \times \frac{200}{10} \times 0.43 \times 9.45$
		$= 57$ sq. in.
A_c	=the core area	$= 5.65 \times 9.45 = 53.5$ sq. in.
A_p	=the pole area	$= 0.95 \times 10.5 \times 11.5 = 114$ sq. in.
A_y	=the yoke area	$= 136$ sq. in.

The flux per pole:

ϕ_a	=the useful flux per pole	$= 9 \times 10^6$.
l_f	=the leakage factor	$= 1.16$, see page 53.
ϕ_m	=the total flux per pole	$= 10.5 \times 10^6$.

The flux densities:

B_g	=the apparent gap density	=59,000 lines per sq. in.
B_t	=the apparent tooth density	=158,000 lines per sq. in.
B_{at}	=the actual tooth density	=150,000 lines per sq. in., from Fig. 43.

B_c = the core density = 84,000 lines per sq. in.
B_p = the pole density = 93,000 lines per sq. in.
B_y = the yoke density = 39,000 lines per sq. in.

The excitation:

AT_g = the gap ampere-turns $= \frac{1.12 \times 59{,}000 \times 0.3}{3.2} = 6200$

AT_t = the tooth ampere-turns $= 1300 \times 1.6 = 2080$

Use curves in Fig. 44 with a tooth taper of 1.12 and a flux density of 150,000 lines per sq. in.

AT_c = the core ampere-turns $= 20 \times 6.5 = 130$
AT_p = the pole ampere-turns $= 40 \times 15 = 600$
AT_y = the yoke ampere-turns $= 75 \times 13 = 980$

Total ampere-turns for 240 volts at no-load = 9990

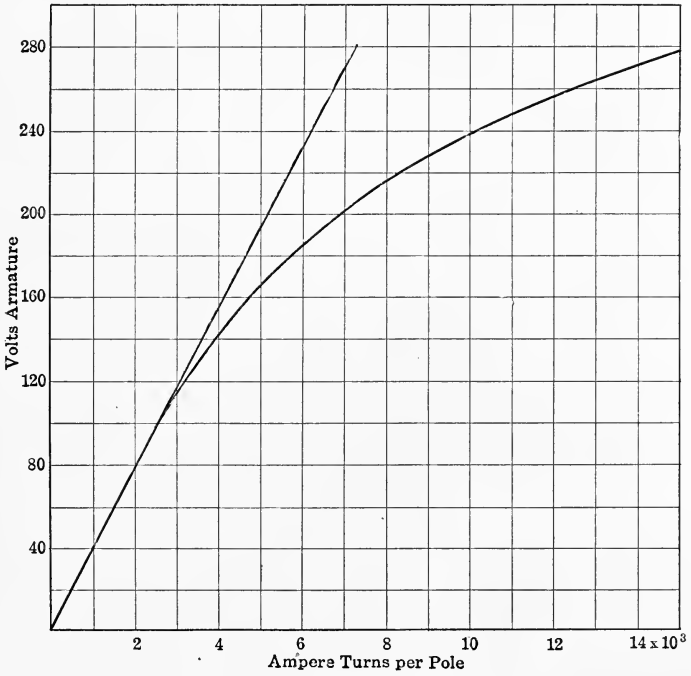


FIG. 45.—No-load saturation curve.

In the above calculation two factors have been omitted which would cause the excitation to be slightly larger than that calculated, namely, the excitation required to send the flux across the joint between the pole and the yoke, and also the extra excitation required for the yoke due to the fact that, near the

contact surface between the pole and the yoke, the yoke density is high, being equal to that of the pole.

Two other points at different voltages are figured out and the results are tabulated as shown in the following table.

No-load voltage			240		280		210	
Flux per pole			9×10^6		10.5×10^6		7.9×10^6	
Leakage factor			1.16		1.16		1.16	
	Length	Area	Density	AT	Density	AT	Density	AT
Air gap	0.30	$\frac{153}{1.12}$	59,000	6200	.....	7200	.....	5400
Min. tooth	1.60	57	158,000	.....	184,000	.....	138,000	.....
			150,000	2080	170,000	3400	134,000	1200
Core	6.5	53.5	84,000	130	98,000	450	73,000	78
Pole	15.0	114	93,000	600	108,000	2400	81,000	270
Yoke	13.0	136	39,000	980	45,000	1460	34,000	680
Total amp.-turns per pole			.....	9990	.....	14950	.....	7628

From these figures the no-load saturation curve in Fig. 45 is plotted.

47. Calculation of the Leakage Factor.—Fig. 46 shows part of a machine which has a large number of poles. The total leakage flux per pole $=\phi_e=\phi_{e1}+\phi_{e2}+\phi_{e3}+\phi_{e4}$, where

$\phi_{e1}=$ the leakage flux in paths 1, between the inner faces of the pole shoes.

$\phi_{e2}=$ the leakage flux in paths 2, between the flanks of the pole shoes.

$\phi_{e3}=$ the leakage flux in paths 3, between the inner faces of the poles.

$\phi_{e4}=$ the leakage flux in paths 4, between the flanks of the poles.

The m.m.f. across paths 1 and $2=2(AT_g+AT_t+AT_c)$
$=2(AT_{g+t})$, since AT_c can be neglected; therefore, the flux across one path 1

$$=3.2\times2(AT_{g+t})\frac{L_s h_s}{l_1} \text{ from formula 1, page 6;}$$

$$=6.4\times(AT_{g+t})\frac{L_s h_s}{l_1}$$

and $\phi_{e1}=13(AT_{g+t})\frac{L_s h_s}{l_1}$ since there are two paths 1, per pole.

The flux across one path $2 = 3.2 \times 2(AT_{g+t}) \int_0^{\frac{W_s}{2}} \frac{h_s dy}{l_1 + \pi y}$

$$= 6.4 \times (AT_{g+t}) \frac{h_s}{\pi} \log_e \left(\frac{l_1 + \frac{\pi W_s}{2}}{l_1} \right)$$

$$= 4.7(AT_{g+t}) h_s \log_{10} \left(1 + \frac{\pi W_s}{2l_1} \right)$$

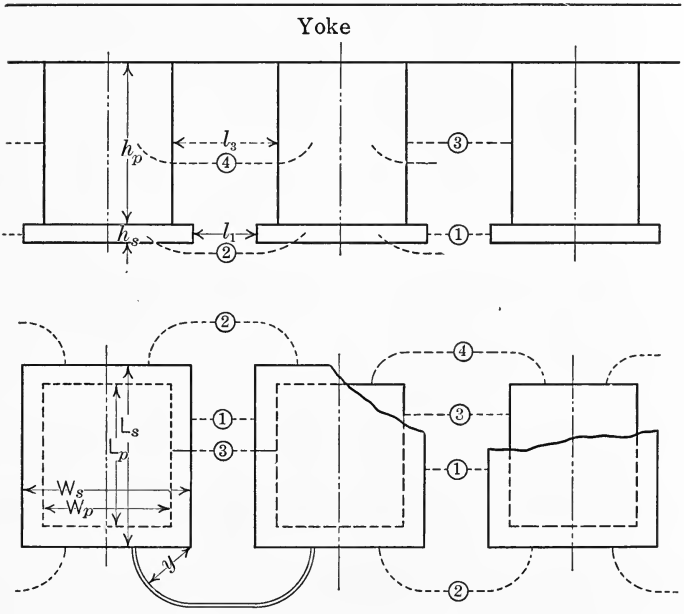


FIG. 46.—The leakage paths.

and $\phi_{e2} = 19(AT_{g+t}) h_s \log_{10} \left(1 + \frac{\pi W_s}{2l_1} \right)$ since there are four paths 2, per pole.

The m.m.f. across the paths 3 and 4 varies from zero at the bottom of the poles to $2(AT_{g+t})$ at the shoe, and the average value is taken as AT_{g+t} ampere-turns, so that

$$\phi_{e3} = 6.5(AT_{g+t}) \frac{L_p h_p}{l_3}$$

and $$\phi_{e4} = 9.5(AT_{g+t}) h_p \log_{10} \left(1 + \frac{\pi W_p}{2l_3} \right)$$

Given the complete data on a magnetic circuit, the value of AT_{g+t}, the ampere-turns to send the flux ϕ_a across one gap and tooth, can be found, and then the value of $\phi_e = \phi_{e_1} + \phi_{e_2} + \phi_{e_3} + \phi_{e_4}$ can be obtained by substitution in the above formulæ.

$$\text{The leakage factor} = \frac{\phi_a + \phi_e}{\phi_a}$$

It is required to find the leakage factor for the machine shown in Fig. 41.

$h_s = 1.5$ in.
$L_s = 11.5$ in.
$l_1 = 6$ in.
$W_s = 12.7$ in.
$h_p = 13.5$ in.
$L_p = 11.5$ in.
$l_3 = 12.5$ in.
$W_p = 10.5$ in.

then $\phi_{e_1} = 13(AT_{g+t}) \left(\frac{1.5 \times 11.5}{6}\right) = 37(AT_{g+t})$

$\phi_{e_2} = 19(AT_{g+t})\ 1.5 \log_{10} \left(1 + \frac{\pi \times 12.7}{2 \times 6}\right) = 18(AT_{g+t})$

$\phi_{e_3} = 6.5(AT_{g+t}) \left(\frac{13.5 \times 11.5}{12.5}\right) = 80(AT_{g+t})$

$\phi_{e_4} = 9.5(AT_{g+t})\ 13.5 \log_{10} \left(1 + \frac{\pi \times 10.5}{2 \times 12.5}\right) = 46(AT_{g+t})$

and ϕ_e = the total leakage flux per pole $= 181(AT_{g+t})$

The value of AT_{g+t} from the table on page 51.

$= 6200 + 2080$
$= 8280$ ampere turns

therefore $\phi_e = 181 \times 8280$
$= 1{,}500{,}000$

and ϕ_a the flux per pole which crosses the gap
$= 9.0 \times 10^6$ from the table on page 51;

therefore the leakage factor $= \frac{\phi_a + \phi_e}{\phi_a}$

$$= \frac{9{,}000{,}000 + 1{,}500{,}000}{9{,}000{,}000}$$

$= 1.16$

For a first approximation the following values of the leakage factor may be used:

Four-pole machines up to 10-in. armature diameter	1.25
Multipolar machines between 10 and 30-in. diameter	1.2
between 30 and 60-in. diameter	1.18
greater than 60-in. diameter	1.15

These values apply to the type of machine shown in Fig. 28.

CHAPTER VI

ARMATURE REACTION

48. Armature Reaction.—In Fig. 47, *A* shows the magnetic field that is produced in the air gap of a two-pole machine by the m.m.f. of the main exciting coils.

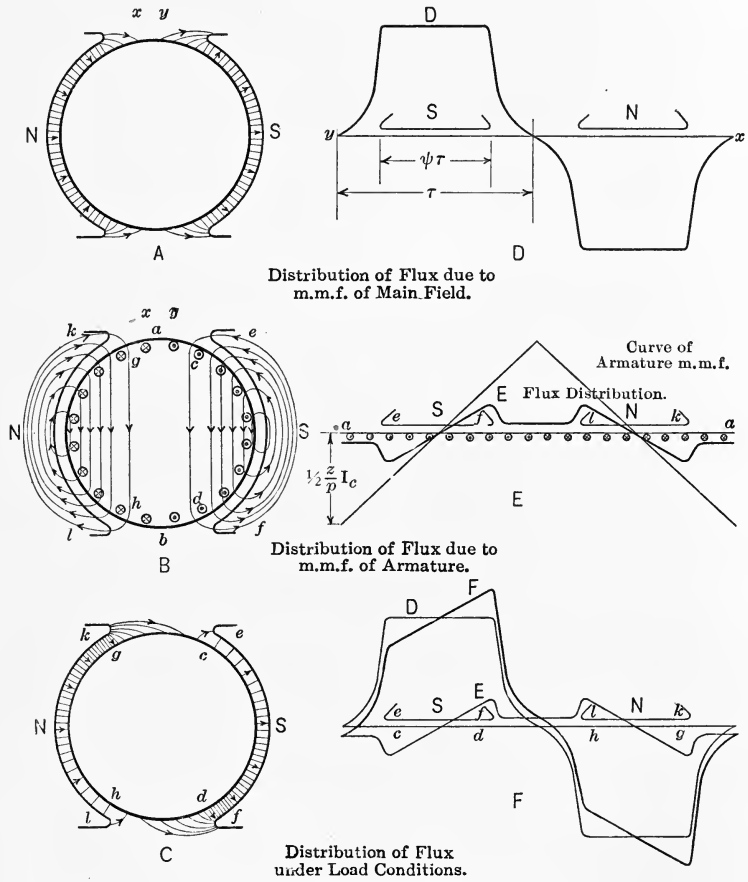


FIG. 47.—Flux distribution curves.

B shows the armature carrying current and the magnetic field produced thereby when the brushes are in the neutral

position and the main field is not excited. The m.m.f. between a and b, called the cross-magnetizing ampere-turns per pair of poles, due to the current I_c in each of the Z conductors $=\frac{1}{2}\ ZI_c$ ampere-turns, and that between c and d and also that between g and $h=\frac{1}{2}\ \phi ZI_c$ ampere-turns. Half of this latter m.m.f. acts across the path ce and the other half across the path fd since the reluctances of the paths ef and cd are so low that they may be neglected. Therefore, the cross-magnetizing effect at each pole tip $=\frac{1}{2}\ \phi\frac{Z}{p}I_c$ for any number of poles (4)

C shows the resultant magnetic field when, as under operating conditions, both the main and the armature m.m.fs. exist together. The flux density, compared with the value shown at A, is increased at the pole tips d and g and decreased at the pole tips c and h.

A convenient method of showing the flux distribution in the air gap is shown in diagrams D, E and F, Fig. 47, which are obtained by assuming that the diagrams A, B and C are split at xy and opened out on to a plane, and that the flux density at the different points is plotted vertically.

D shows the flux distribution due to the main m.m.f. acting alone.

E shows the flux distribution due to the armature m.m.f. acting alone.

F shows the resultant distribution when both the main and the armature m.m.fs. exist together and is obtained by adding the ordinates of curves D and E. It is permissible to add these ordinates of flux density together provided that the paths df and gk do not in the meantime become highly saturated. These paths, however, include the gap and teeth, and the flux density in the teeth due to the main field is about 150,000 lines per square inch at normal voltage, which is well above the point of saturation, so that an increase in m.m.f., such as that at f due to the armature m.m.f., will produce an increase in flux density at pole tip f of only a small amount; while a decrease in m.m.f. of the same value at pole tip e will produce a decrease in flux density at that pole tip of a much larger amount; thus the total flux per pole will be decreased.

It is usual to consider the effect of armature reaction as being due to a number of lines of force acting in the direction shown in diagram B, Fig. 47, and this diagram shows that the same

number of lines is added at the one pole tip as is subtracted at the other pole tip. A truer representation is that shown in Fig. 48.

Since the lines of force of armature reaction meet a high reluctance at *d* some of them take the easier path through *hmc*. These latter lines are in the opposite direction to those of the main field and are, therefore, demagnetizing.

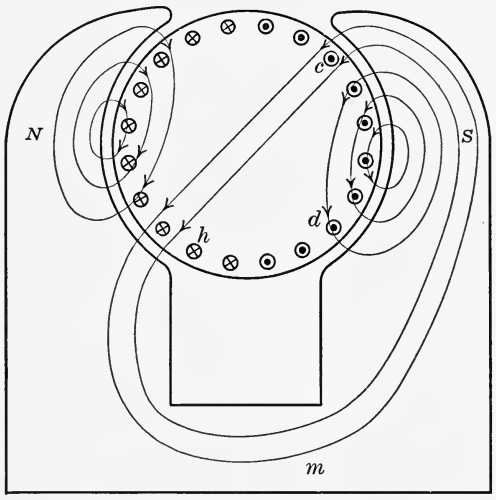


FIG. 48.—Demagnetizing effect of armature reaction with the brushes at the neutral point.

49. Distribution of Flux in the Air Gap at Full Load.[1]—Fig. 49 is part of the development of a multipolar machine with p poles, and curve D shows the flux distribution in the air gap due to the main m.m.f. acting alone. The armature m.m.fs. across df and ce each $= \frac{1}{2} \phi \frac{Z}{p} I_c$ ampere-turns and curve G shows the distribution of the armature m.m.f.

Curve 1, Fig. 50, is the no-load saturation curve of the machine and curve 2 is that part of this saturation curve for the tooth, gap and pole face, so that if oy is the ampere-turns per pole required to send the no-load flux through the magnetic circuit of the machine then ox is that necessary to send this same flux through the length of one gap, one tooth and one pol eface.

Across np, Fig. 49, the m.m.f. at full-load is the same as at

[1] The method adopted in this article is a slight modification of that proposed by S. P. Thompson, Chapter XVII, Dynamo Electric Machinery, Vol. I.

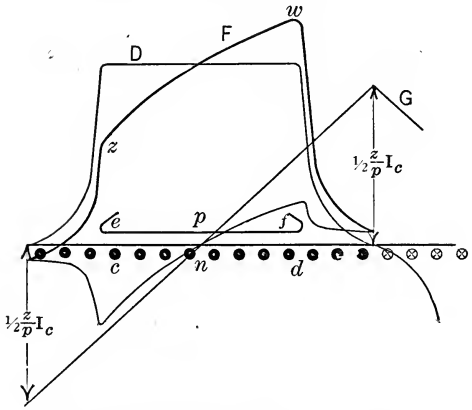


FIG. 49.—Flux distribution at full-load.

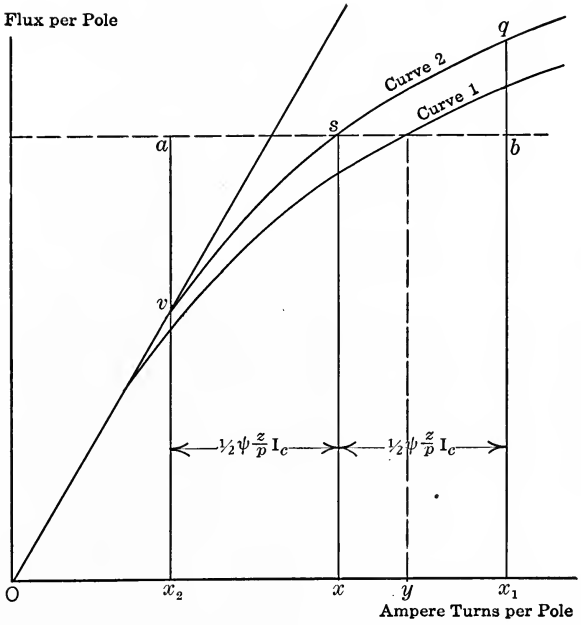


FIG. 50.—No-load saturation curves.

no-load and therefore the flux density in the air gap at n is unchanged.

Across df the m.m.f. at full-load is no longer ox, Fig. 50, but $= ox_1$, where $xx_1 = \frac{1}{2}\psi \frac{Z}{p} I_c$ = the m.m.f. across df due to the armature; therefore the flux density in the air gap at d at full-load is increased over its value at no-load in the ratio $\frac{qx_1}{sx}$, Fig. 50, and is so plotted at dw, Fig. 49.

Across ce the m.m.f. at full-load is no longer ox, Fig. 50, but $= ox_2$, where $xx_2 = \frac{1}{2}\psi \frac{Z}{p} I_c$, and therefore the flux density in the air gap at c at full-load is less than that at no-load in the ratio $\frac{vx_2}{sx}$, Fig. 50, and is so plotted at cz, Fig. 49.

Thus, in Fig. 49, curve D shows the distribution of the flux in the air gap at no-load and curve F that at full-load. The total flux per pole is less at full-load than at no-load in the ratio of the area enclosed by curve F to that enclosed by curve D, which ratio is practically the same as area $\frac{x_2vqx_1}{x_2abx_1}$, Fig. 50.

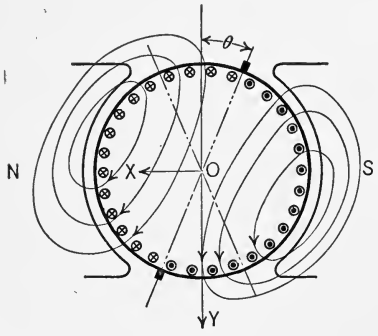


Fig. 51.

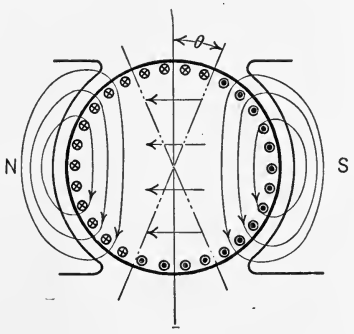


Fig. 52.

Demagnetizing and cross magnetizing effect of the armature.

50. Armature Reaction when the Brushes are Shifted.—Fig. 51 shows the armature carrying current and the magnetic field produced thereby when the brushes are shifted through an angle θ so as to improve the commutation. The armature field is no longer at right angles to the main field and the easiest way in which to consider its effect is to assume that it is the resultant of two components, one in the direction OY which is

called the cross-magnetizing component, the effect of which has already been discussed, and another in the direction OX which is called the demagnetizing component because it is directly opposed to the main field. Fig. 52 shows the armature divided up so as to produce these two components, and it will be seen that the demagnetizing ampere-turns per pair of poles

$$= I_c \frac{Z}{p} \times \frac{2\theta}{180}$$

or the demagnetizing ampere-turns per pole

$$= \tfrac{1}{2} \frac{Z}{p} \times I_c \times \frac{2\theta}{180} \qquad (5)$$

The angle θ for preliminary calculations is usually taken as 18 electrical degrees so that $\frac{2\theta}{180} = 0.2$.

51. The Full Load Saturation Curve.—It is required to draw this curve for the machine which is drawn to scale in Fig. 41 and to which the following data applies.

Rating: 400 kw., 240 volts, 1670 amp., 200 r.p.m.
Poles 10
Coils 400
Winding ..... one turn multiple
Total conductors 800
Current per conductor 167
Per cent. pole enclosure 0.7
Volts drop at full-load across armature, brushes and series field 8.7
θ, the angle of advance of the brushes ..... 18 degrees

Armature ampere-turns per pole $= \frac{800 \times 167}{2 \times 10}$ 6700

Demagnetizing ampere-turns per pole $= \tfrac{1}{2}\left(\frac{800}{10} \times 0.2 \times 167\right) = 1340$

Cross-magnetizing ampere-turns at each pole tip $= \tfrac{1}{2}\left(\frac{800}{10} \times 0.7 \times 167\right) = 4700$

Curve 1, Fig. 53, the no-load saturation curve, is taken directly from Fig. 45.

Curve 2, Fig. 53, that part of the saturation curve for the tooth and gap, is plotted from the figures in the table on page 51.

The m.m.f. required to send the no-load flux through the magnetic circuit is 9990 ampere-turns, of which 8280 ampere-turns are required for the gap and tooth. The voltage generated due to this flux is 240.

At full load the m.m.f. at one pole tip
$=8280+4700=12980$ ampere-turns,
and that at the other pole tip
$=8280-4700=3580$ ampere-turns.

The flux crossing the air gap is reduced in the ratio $\left(\frac{\text{area } aefd}{\text{area } abcd}\right)$, Fig. 53, and, due to this reduction in flux, the voltage generated is reduced from 240 to 235.5.

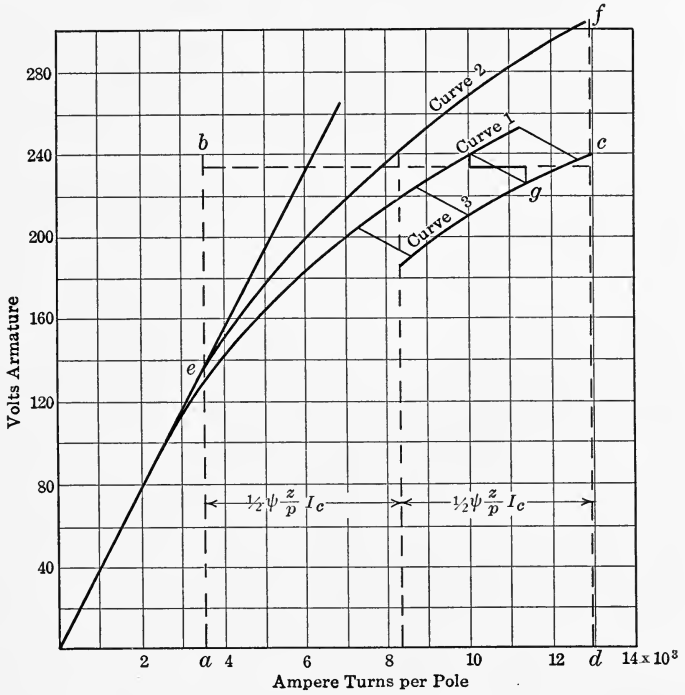


Fig. 53.—No-load and full-load saturation curves.

To maintain the generated voltage at this reduced value of 235.5 volts it is necessary to increase the no-load field excitation of 9990 ampere-turns by 1340, the demagnetizing ampere-turns per pole.

The terminal voltage is less than the generated voltage of 235.5 by 8.7, the voltage required to send the full-load current through the armature, brushes and series field. The full load saturation curve is drawn parallel to the no-load saturation curve through the point g so found.

In order to get the same flux across the air gap at full-load as at no-load the field excitation has to be increased over its no-load value so as to counteract the effect of the m.m.f. of the armature. Due to this increase in excitation the leakage flux is increased, so that the leakage factor is greater at full-load than at no-load, and still more excitation is required on account of the resulting increase in the pole and yoke densities. This latter increase in excitation, however, cannot readily be calculated.

52. Relative Strength of Field and Armature M.M.FS.—Inspection of Fig. 49 will show that if the armature current be increased to such a value that the cross-magnetizing ampere-turns of the armature at the pole tips, namely $\frac{1}{2}\phi\frac{Z}{p}I_c$ ampere-turns, becomes equal to the ampere-turns for the tooth and gap due to the main field excitation, then the flux density will be zero under the pole tip toward which the brushes have been shifted in order to help commutation, so that to obtain a reversing field it is necessary that the ampere-turns of the main field for gap and tooth be greater than $\frac{1}{2}\phi\frac{Z}{p}I_c$.

To get a reasonably strong field for commutating purposes, experience shows that the above value at full load should not be less than 1.7, and the higher the value the better the commutation, other things being equal, but at the same time the more expensive the machine due to the extra field copper required. The quantity $\frac{1}{2}\phi\frac{Z}{p}I_c$ is equal to ϕ (armature ampere-turns per pole) so that

$$\frac{\text{ampere-turns of main field for gap and tooth}}{\text{armature ampere-turns per pole at full load}}$$

$$= 1.7 \times \phi$$

$$= 1.2 \quad \ldots\ldots\ldots\

CHAPTER VII

DESIGN OF THE MAGNETIC CIRCUIT

The problem to be solved in this chapter is, given the armature of a machine and also its rating to design the poles, yoke, and field coils.

53. Field Coil Heating.[1]—Fig. 54 shows one of the poles of a D. C. machine with its field coil. The exciting current I_f passes through this coil and gradually raises its temperature until the point is reached where the rate at which heat is dissipated by the coil is equal to the rate at which it is generated in the coil.

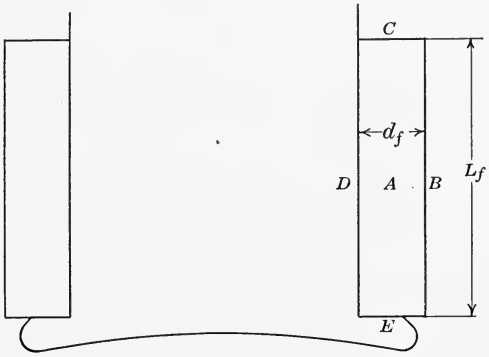


FIG. 54.—D.-C. field coil.

The hottest part of the coil is at A, and the heat has to be carried from this point to the radiating surfaces B, C, D and E, so that there must be a temperature gradient between A and the radiating surface of the coil; Fig. 55 shows the temperature at different points in the thickness of a field coil.

The maximum temperature of the coil limits the amount of current that it can carry without injury, but this temperature is difficult to measure. The external temperature of the coil can be taken by means of a thermometer, and the mean tempera-

[1]A good summary, with complete references, of the work published on field coil heating will be found in a paper by Lister: *Journal of the Institution of Electrical Engineers*, Dec., 1906.

ture can be found by the increase in resistance of the coil, since the resistance of copper at any temperature $t = R_t = R_o(1 + 0.004\ t)$ where R_o is the resistance at 0° C. and t is in centigrade degrees.

It is found that the ratio between the maximum and the mean temperature seldom exceeds 1.2, while that between the mean and the external surface temperature varies from about 1.4 to 3. The latter figure is found in some of the early machines whose field coils were covered with tape and rope. When the coil is insulated as shown in Fig. 36 the ratio of the mean temperature to that of the external surface will be approximately 1.5 if the

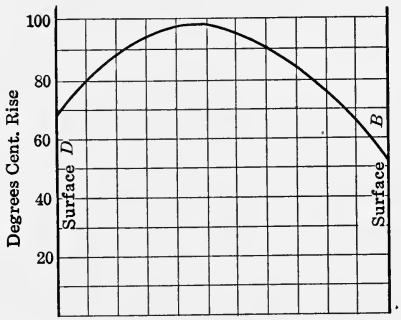


FIG. 55.—Temperature gradient in field coils.

external surface is left bare except for the d.c.c. on the wire, the whole coil impregnated with compound, and the coil about 2 in. thick; the compound is a better conductor of heat than the air which it replaces and is also a better insulator. If two layers of half-lapped tape be put on the external surface of the coil the ratio will increase to about 1.7 and if in addition a layer of cardboard 1/16 in. thick be put on the external surface, as was formerly done to protect the coil, the ratio will exceed 2. These are average figures and may vary considerably since they are affected by the fanning action of the armature, the kind of compound used, the thickness of the insulation on the wire, the radiating power of the poles and yoke on which depends the radiating power of the surface D.

The heating constants for field coils are figured in many different ways, depending on what is taken for the radiating surface. While it is true that all the surfaces, B, C, D and E, are active in radiating heat, yet they are not all equally effective, and for that

reason, and also for convenience, the radiating surface is taken as the external surface B.

For impregnated coils without any insulating material on the external surface other than that on the wire itself, the watts that can be radiated per square inch of the external surface for a temperature rise of 40° C. on that surface, which corresponds to an average temperature rise of 60° C. and a maximum temperature rise of 70° C., varies from 0.5 to 1.0 depending on the length of the coil and also on the peripheral velocity of the armature. The effect of the fanning action of the armature can readily be understood so that the slope of the curves in Fig. 56 needs no explanation. It will be seen from this diagram that a short coil is more effective than a long one because the ratio of the total radiating surface B, C, D and E, to the external surface B on which the constants are based, is greater in the short than in the long coil, and also because only that portion of the long coil which is near the armature is affected by the armature fanning.

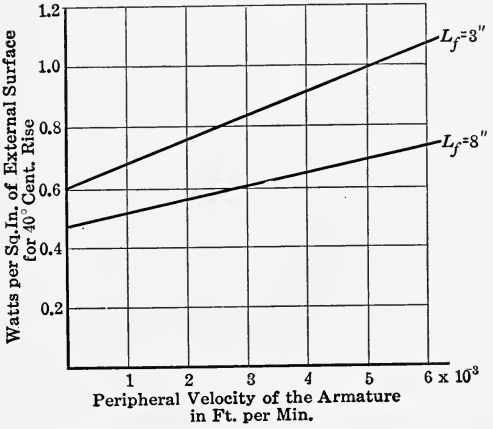


Fig. 56.—Field coil heating constant.

The radiating surface of a coil is often increased by putting in ventilating openings as shown in Fig. 37; this method would seem to double the radiating surface, but the sides of the ventilating opening are not so effective as either the inner or the outer surface of the coil. For this type of field coil, where the individual coils are 1 in. thick and spaced $\frac{1}{2}$ in. apart, the watts per square inch of external surface can be increased 50 per cent. over the values given in Fig. 56.

54. The Size of Wire for Field Coils:

If E_f is the voltage across each field coil,
I_f is the current in the coil,
T_f is the number of turns in the coil,
MT is the length of the mean turn of the coil in inches,
M is the section of the wire used in circular mils,

then the resistance of the coil $= \frac{E_f}{I_f} = \frac{MT \times T_f}{M}$ since the resistance of copper is approximately 1 ohm per circular mil per inch length,

and
$$M = \frac{I_f T_f \times MT}{E_f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

so that for a given machine the size of field wire is fixed as soon as the ampere-turns and the voltage per coil are known.

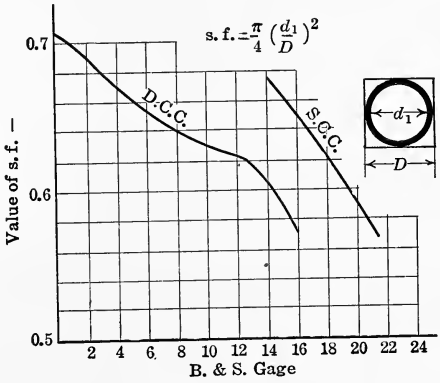


FIG. 57.—Space factor for wire.

55. The Length L_f of the Field Coil.—The watts radiated from the coil shown in Fig. 54 = external surface × watts per square inch. The total section of copper in the coil = $d_f \times L_f \times sf$ square inch. where *sf.*, the space factor of the wire, is the section of the wire divided by the space that the wire takes up in the coil and is got from Fig. 57.

The section of the wire in the coil $= \frac{d_f L_f \times sf}{T_f}$ square inches

$$= \frac{d_f \times L_f \times sf \times 1{,}270{,}000}{T_f} \text{ circular mils.}$$

$$= M$$

The watts loss per coil $= \frac{MT \times T_f}{M} \times I_f^2$ and, substituting for M,

$$= \frac{MT \times T_f^2 \times I_f^2}{d_f \times L_f \times sf \times 1{,}270{,}000}$$

The watts loss per coil also = external surface of coil × watts per square inch.

= external periphery of coil $\times L_f \times$ watts per square inch.

Therefore, equating these two values together,

$$L^2_f = \frac{\text{mean turn} \times (I_f T_f)^2}{\text{ext. periphery} \times \text{watts per sq. in.} \times d_f \times sf \times 1{,}270{,}000}$$

and $$L_f = \frac{I_f T_f}{1000} \sqrt{\frac{\text{mean turn}}{\text{ext. periphery} \times \text{watts per sq. in.} \times d_f \times sf \times 1.27}} \quad (8)$$

In order to have an idea as to the value of L_f found from the above equation assume the following average values:

sf, the space factor $= 0.6$
d_f, the coil depth $= 2.0$ in.
watts per square inch $= 0.6$.
external periphery $= 1.2 \times$ mean turn.

then L_f, the radial length of coil space, $= \frac{I_f T_f}{1000}$ approximately.

56. Weight and Depth of Field Coils.—The weight of the field coil $= 0.32 \times MT \times L_f \times d_f \times sf$. pounds, where 0.32 is the weight of a cubic inch of copper,

$$\text{and } L_f = \frac{I_f T_f}{1000} \sqrt{\frac{\text{mean turn}}{\text{ext. periphery} \times \text{watts per sq. in.} \times d_f \times sf \times 1.27}}$$

$$= I_f T_f \frac{\text{a constant}}{\sqrt{d_f}} \text{ approximately, for a given machine,}$$

therefore the weight of the field coil

$$= 0.32 \times MT \times I_f T_f \times \frac{\text{a constant}}{\sqrt{d_f}} \times d_f \times sf$$

$$= \text{a constant } \sqrt{d_f} \text{ for a given machine.}$$

This may be interpreted as follows: the larger the value of d_f, the shorter the length L_f, the smaller the radiating surface, and therefore the lower the value of permissible loss per coil. Since the section of the field coil wire is fixed, because, as shown in Art. 54, it depends only on the ampere-turns and the voltage per coil, a lower permissible loss can only be ob-

tained by a smaller value of I_f and therefore by a larger value of T_f and a more expensive coil. It would seem then that the thinner the field coil the cheaper the machine, but it must not be overlooked that, as the value of d_f becomes less, and therefore the cost of the field copper decreases, the value of L_f increases and therefore the cost of the poles and yoke also increases. The value of d_f for minimum cost of field system must take this into account and can readily be determined by trial; an average value for d_f is 2 in.

57. Procedure in the Design of the Field System for a Given Armature.

(1) Find the air gap clearance as follows:

AT_{g+t}, the ampere-turns per pole for gap and teeth,

$=1.2$ (armature AT per pole) for generators,

$=1.2$ (armature AT per pole) + demagnetizing AT per pole, for shunt motors; see page 61.

From the armature data, AT_t, the ampere-turns per pole for the teeth, can be found:

AT_g, the gap ampere-turns, $=\frac{B_g \times C \times \delta}{3.2}$ from formula 3, page 48, from which δ can be found.

(2) Draw the saturation curves.

Before this can be done it is necessary to determine approximately the dimensions of the magnetic circuit, which is done as follows:

It is assumed that the no-load excitation $= 1.25(AT_{g+t})$, and that $L_f = \frac{\text{no-load amp.-turns}}{1000}$ approximately, see Art. 55, page 66; this coil length is increased 30 per cent. to allow for the series field should such be required.

The section of the pole is got by assuming that the pole density is 95,000 lines per square inch and that the leakage factor has the value given in the table on page 53; then A_p, the pole area in square inches, $=\frac{\phi_a \times l.f.}{95{,}000}$.

The yoke area is found in a similar way by assuming that the yoke density is 75,000 lines per square inch for cast steel and 40,000 lines per square inch for cast iron.

The magnetic circuit may now be drawn in to scale and the saturation curves determined; these may require a slight modification as the work proceeds.

(3) Design the shunt field coil.

Find the no-load excitation from the saturation curve.

Find M the size of field coil wire from the formula

$$M = \frac{I_f T_f \times MT}{E_f}; \text{ see Art. 54, page 65.}$$

E_f, the volts per coil, $= \frac{\text{terminal voltage}}{\text{poles}} \times k$

where $k = 0.8$ for compound generators; which leaves 20 per cent. of the terminal voltage to be absorbed by the field-circuit rheostat, so that the shunt excitation may be increased 20 per cent. over its normal value should that be desired at any time.

$k = 1.0$ for shunt motors.

k for shunt generators is determined for each separate case; there should be enough field regulation to allow normal voltage to be maintained from no-load to the desired overload.

L_f is found from the formula

$$L_f = \frac{I_f T_f}{1000} \sqrt{\frac{\text{mean turn}}{\text{ext. periphery} \times \text{watts per sq. in.} \times d_f \times sf \times 1.27}};$$

see Art. 55, page 66.

T_f is the number of turns of wire of section M that will fill up the space $L_f \times d_f$.

(4) Design the Series field coil.

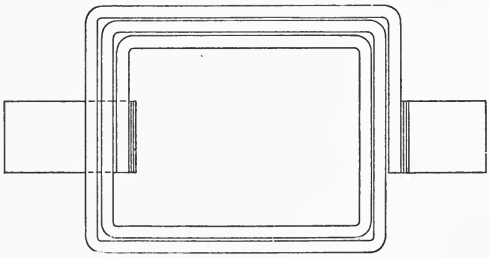


FIG. 58.—Series field coil.

The series excitation at full-load is found from the saturation curves and is the difference between the excitation for the required terminal voltage found from the full-load saturation curve and the shunt excitation at the same voltage.

The number of series turns $= \frac{\text{series excitation}}{\text{full-load current}}$; this value is

generally increased 20 per cent. on new designs because of the difficulty in predetermining exactly the full-load saturation curve; when the coil is shaped as shown in Fig. 58 there must always be a half turn in the coil; the coil shown has $2\frac{1}{2}$ turns.

The current density in the series coil and also the value of watts per square inch external surface are 20 per cent. greater than in the shunt coils of the same machine, because the series coils are closer to the armature and, therefore, better cooled by its fanning effect.

Example.—The armature of a 10-pole, 400-kw., 240-volt no-load, 240-volt full-load, 1670 amp., 200 r.p.m. generator is shown to scale in Fig. 41; it is required to design the field system, which is not supposed to be given.

(1) Find the air-gap clearance.

Armature ampere-turns per pole $= \frac{800 \times 167}{2 \times 10} = 6700$

Ampere-turns per pole (gap + tooth) $= 1.2 \times 6700 = 8100$

Ampere-turns per pole for the tooth are found as follows:

$$\Phi_a = \frac{240 \times 60 \times 10^8}{800 \times 200} = 9 \times 10^6$$

minimum tooth area per pole $= 0.43 \times \frac{200}{10} \times 0.7 \times 9.45$
$= 57$ sq. in.

maximum tooth density $= \frac{9 \times 10^6}{57}$
$= 158{,}000$ lines per square inch, apparent.
$= 150{,}000$ lines per square inch, actual, from Fig. 43.

tooth taper $= k = 1.12$

ampere-turns per pole for the teeth $= 1300 \times 1.6$, from Fig. 44.
$= 2080$

Ampere-turns per pole for the gap $= 8100 - 2080$
$= 6020$

Apparent gap density $= \frac{9 \times 10^6}{18.2 \times 0.7 \times 12}$
$= 59{,}000$ lines per square inch.

$$C \times \delta = \frac{3.2 \times 6020}{59{,}000} = 0.328$$

therefore $C = 1.12$ from Fig. 40, page 44,
and $\delta = 0.29$ (make gap clearance $= 0.3$ in.)

(2) Draw the saturation curves.

No-load excitation $= 1.25 \times 8100$
$= 10{,}100$ amp.-turns approximately.

$L_t = \frac{10{,}100}{1000} = 10$ in. approximately.

Allow 30 per cent. more for the series coil, so that the coil space $= 13$ in.

The pole area $= \frac{9 \times 10^6 \times 1.18}{95{,}000}$

$= 112$ sq. in. approximately.

The yoke area $= \frac{9 \times 10^6 \times 1.18}{2 \times 40{,}000}$

$= 132$ sq. in. approximately.

From the dimensions found above the magnetic circuit is drawn to scale and the no-load and full-load saturation curves are determined. For the machine in question the curves are shown in Fig. 53.

(3) Design the shunt coil.

The no-load excitation $= 9990$ amp.-turns

E_f, the volts per coil $= \frac{240 \times 0.8}{10} = 19$

MT, the mean turn $= 53$ in.

External periphery of the coil $= 61$ in.

Size of shunt coil wire $= \frac{9990 \times 53}{19} = 28{,}000$ circular mils;

use No. 5½ B & S. gauge, a special size between No. 5 and No. 6, which has a section of 29,500 circular mils and a diameter when insulated with d.c.c. of 0.19 in. Where such odd sizes are not available the coil can be made up of the proper number of turns of No. 5 wire in series with the proper number of turns of No. 6, so as to have the same resistance as that of a coil made with wire of a section of 28,000 circular mils.

$L_f = 10.5$ in. assuming that $d_f = 2$ in.

$sf = 0.65$

watts per sq. in. $= 0.6$

The number of layers of wire in a depth of 2 in. $= \frac{2}{0.19} = 10.$

The number of turns per layer in a length of 10.5 in. $= \frac{10.5}{0.19} = 55.$

The number of turns per coil $= 10 \times 55 = 550.$

The shunt current $= \frac{9990}{550} = 18.2$ amp.

The current density in the field coil wire $= \frac{29{,}500}{18.2} = 1600$ cir. mils per amp.

(4) Design the series coil.

The excitation at full-load and normal voltage $= 12{,}800$ amp.-turns.

The shunt excitation at normal voltage $= 9{,}990$ amp.-turns.

Therefore the series ampere-turns at full load $= 2810$

The series turns $= 2.5$

The series current $= \frac{2810}{2.5} = 1120$

The current in the series shunt $= 1670 - 1120 = 550$ amperes

The current density in the series coil $= \frac{1600}{1.2} = 1330$ circular mils per amp.

The size of the series coil wire $= 1330 \times 1120$

$= 1{,}500{,}000$ circular mils

$= 1.2$ sq. in.

The resistance of 2.5 turns of this wire

$$= \frac{53 \times 2.5}{1{,}500{,}000} = 8.8 \times 10^{-5} \text{ ohms}$$

The voltage drop in one series coil $= 8.8 \times 10^{-5} \times 1120$
$= 0.1$ volts

The loss in one series coil $= 0.1 \times 1120$
$= 112$ watts

The permissible watts per square inch external surface $= 0.6 \times 1.2 = 0.72$.

The necessary radiating surface $= \frac{112}{0.72} = 155$ sq. in.

The external periphery of the coil $= 61$ in. approximately.

The length L_f of the series coil $= \frac{155}{61} = 2.5$ in.

The thickness of the series field coil wire $= \frac{\text{section of wire}}{L_f}$

$$= \frac{1.2}{2.5}$$
$$= 0.5 \text{ in.}$$

The section of the wire is made up of four strips in parallel each 0.125×2.5 section so that the coil may be readily bent to shape.

CHAPTER VIII

COMMUTATION

The direction of the current in the conductors of a D.-C. machine at any instant is shown in Fig. 59; therefore, as the armature revolves and conductors pass from one side of the neutral line to the other, the current in these conductors must be reversed from a value I_c to a value $-I_c$, where I_c is the current in each conductor.

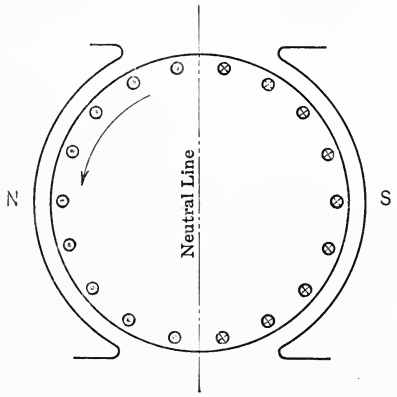


FIG. 59.—Direction of current in a D.-C. armature.

Figure 61 shows part of a full-pitch double-layer multiple winding with two conductors per slot and with the coils M undergoing commutation, and Fig. 60 shows part of the corresponding ring winding. It may be seen from diagrams A and E that the current in coils M is reversed as the armature moves so that the brushes change from commutator segments 1 and 5 to segments 2 and 6.

58. Resistance Commutation.—Let the coils M be in such a position between the poles N and S that, during commutation, they are not cutting any lines of force due to the m.m.f. of the field and armature, and let the contact resistances r_1 and r_2, diagram B, between the brush and segments 5 and 6, be so large

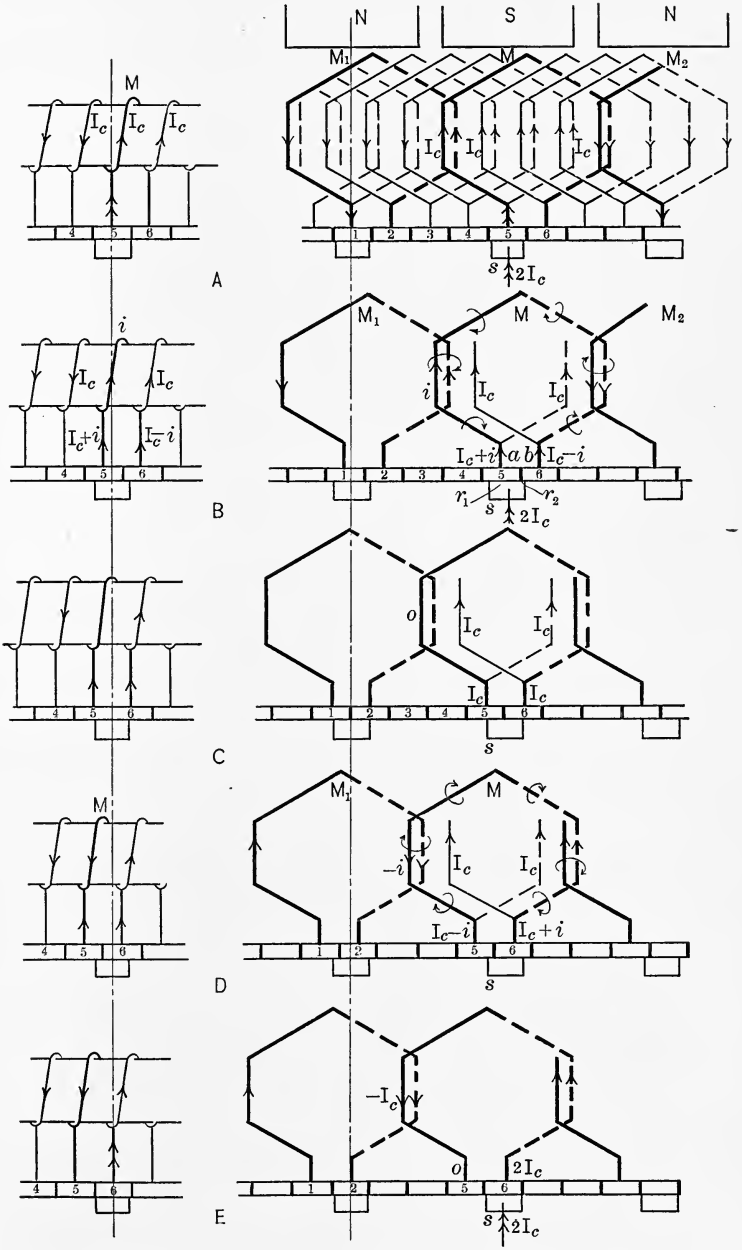


FIG. 60. FIG. 61.

Stages in the process of commutation.

that the effect of the resistance and self induction of the coils M may be neglected.

In the position shown in diagram B the contact area between the brush and segment 5 is large while that between the brush and segment 6 is small, and the current $2I_c$, which passes through the brush, divides up into two parts which are proportional to the areas of contact between the brush and segments 5 and 6 respectively, and are equal to I_c+i and I_c-i.

In the position shown in diagram C the two contact areas are equal and there is no tendency for current to flow round the coil M.

In the position shown in diagram D the contact area between the brush and segment 5 is small while that between the brush and segment 6 is large, and the current $2I_c$ which passes through the brush divides up into two parts as shown in the diagram; the current i in coil M now passes in a direction opposite to that which it had in diagram A. As the contact area with segment 5 decreases, the value of the current i which passes round coil M increases until, at the instant shown in diagram E, this current is equal to $-I_c$.

By this action of the brush the current in the coil M is reversed or commutated while the armature moves through the distance of the brush width, and the variation of current with time is plotted in curve 1, Fig. 62, and follows a straight line law.

59. Effect of the Self-induction of the Coil.—The resistance of the coil undergoing commutation is generally so low that its effect can be neglected, but the effect of self-induction must be considered.

The current i in the coils M sets up lines of force which link these coils. As this current reverses the lines of force also reverse, as shown in diagrams B and D, and the change of flux generates an e.m.f. in each coil M which is generally called its e.m.f. of self-induction. It is important to notice however that part of the flux which links one of the coils, say M, is due to current in the coil M_1 on one side and to current in the coil M_2 on the other side, so that this generated e.m.f. is really an e.m.f. of self and mutual induction.

The e.m.f. of self and mutual induction opposes the change of current which produces it, so that at the end of half of the period of commutation the current in the coils M has not become zero but has still the value cd, curves 2, 3 and 4, Fig. 62;

these curves show the variation of current with time for different values of the ratio $\frac{RT_c}{L+M}$ where:

R is the resistance of the total brush contact in ohms,
T_c is the time of commutation in seconds,
L is the coefficient of self-induction of one coil M in henries,
M is the coefficient of mutual induction between coil M and coils M_1 and M_2 in henries.

The equation from which these curves were plotted is derived as follows:

The difference of potential between a and b, diagram B, Fig. 61, $=(I_c+i)r_1-(I_c-i)r_2$ and this must be equal and opposite to the generated voltage $-(L+M)\frac{di}{dt}$

or $(I_c+i)r_1-(I_c-i)r_2+(L+M)\frac{di}{dt}=0.$

If now R and T_c have the values already mentioned, and t is the time measured from the start of commutation

then $r_1=R\left(\frac{T_c}{T_c-t}\right)$

and $r_2=R\left(\frac{T_c}{t}\right)$

therefore $(I_c+i)R\left(\frac{T_c}{T_c-t}\right)+(L+M)\frac{di}{dt}-(I_c-i)R\left(\frac{T_c}{t}\right)=0$

and $\frac{di}{dt}=-\left(\frac{RT_c}{L+M}\right)\left(\frac{I_c+i}{T_c-t}-\frac{I_c-i}{t}\right).$

The results from this equation are plotted in Fig. 62 for different values of the ratio $\frac{RT_c}{(L+M)}$.[1]

60. Current Density in the Brush.—By the use of the values of i plotted in Fig. 62 the value of the current I_c+i flowing from the brush to segment 5 can be determined, and from it the current density in the brush tip s at any instant can be obtained. This quantity is plotted against time in Fig. 63, and it will be seen that, for values of $\frac{RT_c}{L+M}$ less than 1, the current density in the brush tip s becomes infinite, and due to the concentration of energy at this tip sparking takes place.

The criterion for sparkless commutation then is that $\frac{RT_c}{L+M}$

[1] For the solution of this equation see Reid on Direct Current Commutation. *Trans.* of A. I. E. E., Vol. 24, 1905.

be greater than 1, and perfect commutation is defined as such a change of current in the coil being commutated that the current density over the contact surface between the brush and the commutator segment is constant and uniform.

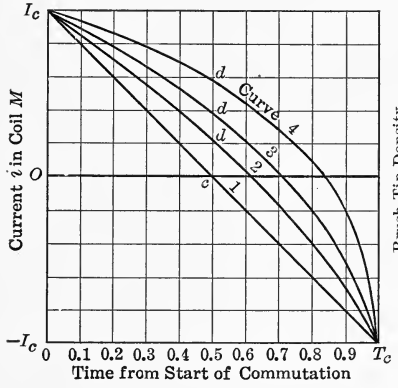


FIG. 62.

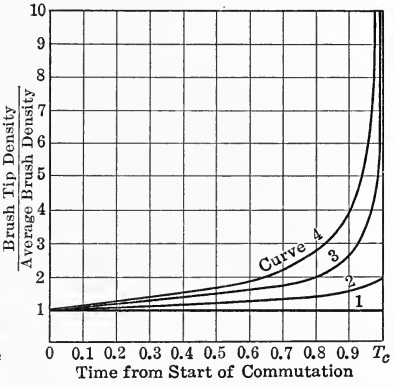


FIG. 63.

Curve 1.— $\frac{RT_c}{L+M}$ = infinity.

Curve 2.— $\frac{RT_c}{L+M} = 2.0$

Curve 3.— $\frac{RT_c}{L+M} = 1.0$

Curve 4.— $\frac{RT_c}{L+M} = 0.5$

Current in the short-circuited coil and current density at the brush tip.

61. The Reactance Voltage.—The above criterion for sparkless commutation is used in practice in a slightly different form for,

if $\frac{RT_c}{L+M}$ must be greater than 1

then R must be greater than $\frac{L+M}{T_c}$

and $2I_cR$ greater than $\frac{2I_c}{T_c}(L+M)$.

This latter quantity is called the average reactance voltage, and is the e.m.f. of self and mutual induction of the coil being commutated on the assumption that the current varies from I_c to $-I_c$ according to a straight line law. This average reactance voltage then should always be less than $2I_cR$ the voltage drop across one brush contact.

62. Brush Contact Resistance.—Curve 1, Fig. 64, shows the

value of the resistance of unit section of brush contact plotted against current density in that contact; the resistance decreases as the current density increases and for higher values than 35 amperes per square inch it varies almost inversely as the current density and the voltage drop across the contact becomes prac tically constant, as shown in curve 2, Fig. 64.

Such curves of brush resistance are obtained by testing the brushes on a revolving collector ring and allowing sufficient time to elapse between readings to let the conditions become stationary.

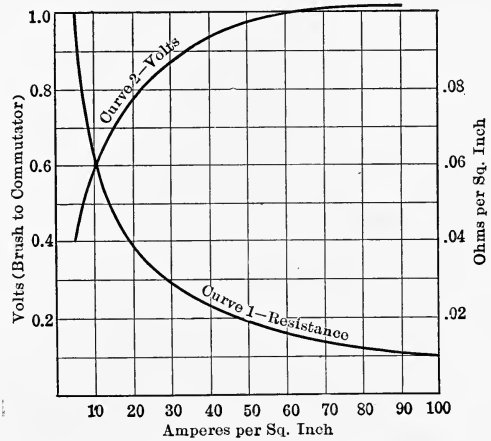


FIG. 64.—Brush-contact resistance curves.

The resistance from ring to brush is generally greater than that from brush to ring by an amount which varies with the material of the brush.

It would seem that, by neglecting the effect of the variation of contact resistance with current density, as was done in Art. 59, the results there obtained would be rendered of little practical importance, it is found, however, that the variation in resistance is largely a temperature effect and that at constant temperature the contact resistance does not vary through such extreme limits, also that, due to the thermal conductivity of carbon, the temperature difference between two points on a brush contact is not very great.

Suppose that, having reached the value of 10 amperes per square inch, and the resistance having become stationary at 0.06 ohms per square inch, see Fig. 64, the current density were

suddenly increased to 40 amperes per square inch; the resistance in ohms per square inch would not fall suddenly to 0.023 but would have a value of about 0.06 and this would gradually decrease until, after about 20 minutes, the resistance would have reached the value of 0.023, the value which it ought to have according to curve 1, Fig. 64. This explains why a machine will stand a considerable overload for a short time without sparking, whereas if the overload be maintained the machine will begin to spark as the brush temperature increases and the contact resistance decreases.

Sparking is cumulative in its effect because slight sparking raises the temperature of the brush contact, which reduces the contact resistance and causes the operation to become worse.

63. Brush Pressure.—The brush contact resistance is found to decrease as the brush pressure increases and as the rubbing velocity decreases, but these effects can be neglected in any study of commutation since the change due to rubbing velocity is small, while the brush pressure is fixed by the service and is made as small as possible. The brush pressure is seldom less than 1.5 lb. per square inch of contact surface because at lower pressures the brushes are liable to chatter, while if the pressure be too great the brushes cut the commutator if they are hard or wear down and smear the commutator if they are soft. A brush pressure greater than 2 lb. per square inch is seldom exceeded except for street car motors, in which case the vibration of the machine itself is excessive, and pressures as high as 5 lb. per square inch have to be used to prevent undue chattering of the brushes.

64. Energy at the Brush Contact.—The criterion for sparkless commutation is that the energy at the brush tip, which is proportional to the current density in that tip, shall not become infinite. The average energy expended at the brush contact must also be limited as may be seen from the following table:

Kind of brush[1]	Current density	Volts across one contact
Very soft carbon.....	50–70 amp. per sq. in.....	0.6–0.4
Soft carbon.....	40–65 amp. per sq. in.....	0.7–0.55
Fairly hard carbon.....	30–45 amp. per sq. in.....	1.1–0.9
Very hard carbon.....	25–40 amp. per sq. in.....	1.5–1.2

[1] Arnold, Die Gleichstrom-machine, Vol. 1, page 351.

The product of amperes per square inch and volts drop across one contact has an average value of 35 watts per square inch. It must be understood that these figures are for machines which operate without sparking and without shifting of the brushes from no load to 25 per cent. overload, but have not necessarily what was defined in Art. 60, page 76, as perfect commutation. The better the commutation the more nearly uniform the current density in the brush contact and the higher the average current density that can be used without trouble developing.

65. Calculation of the Reactance Voltage for Machines with Full-pitch Multiple Windings.—Consider first the case where there are T turns per coil and the brush covers only one segment, as shown in Fig. 65. The winding is of the double layer type and has two coil sides per slot; a section through one slot is shown at S, Fig. 65, for the case where $T=6$.

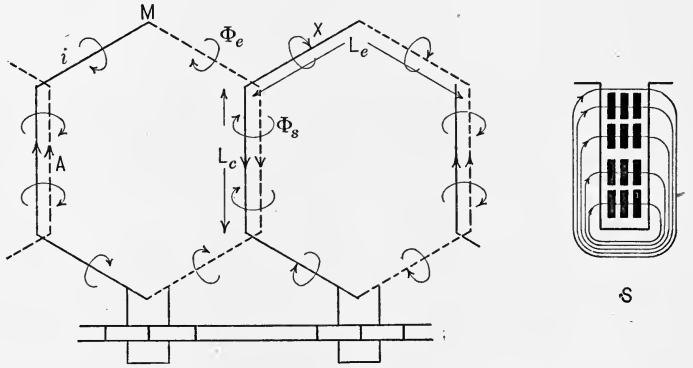


FIG. 65.—Flux circling a full-pitch multiple coil during short-circuit.

Let ϕ_s be the number of lines of force that circle 1 in. length of the slot part of the coil M for each ampere conductor in the group of conductors that are simultaneously undergoing commutation.

and ϕ_e the number of lines of force that circle 1 in. length of the end connections of the coil M for each ampere conductor in the group of end connections that are simultaneously undergoing commutation.

In one of the slots A there are $2T$ conductors, each carrying a current i, so that the flux that circles one side of coil $M = \phi_s \times L_c \times 2T \times i$ lines. In one of the groups of end connections, as at X, Fig. 65, there are T conductors, each carrying a current i, so

that the flux that circles one group of end connections of length $L_e = \phi_e \times L_e \times T \times i$ lines.

The total flux that circles the coil M due to the current i in the coil $= \phi_c$

$$= 2T\ i\ (2\phi_s L_c + \phi_e L_e)$$

and $(L+M) = \dfrac{T \times \phi_c}{i} \times 10^{-8}$ henry

$$= 2\ T^2\ (2\phi_s L_c + \phi_e L_e) \times 10^{-8} \text{ henry}$$

T_c, the time of commutation, is the time taken by the commutator to move through the distance of the brush width and is equal to $\dfrac{60}{\text{r.p.m.}} \times \dfrac{\text{segments covered by the brush}}{\text{total number of commutator segments}}$

$$= \frac{60}{\text{r.p.m.}} \times \frac{\text{segments covered by the brush}}{S}$$

therefore, when the brush covers only one segment, the average reactance voltage $= \dfrac{2I_c}{T_c}(L+M)$

$$= 2I_c \frac{(2\phi_s L_c + \phi_e L_e)\, 2T^2 \times \text{r.p.m.} \times S \times 10^{-8}}{60}.$$

66. The Effect of Wide Slots and Brushes.—Neglect for the present the effect of the end connection flux, which is small compared with the slot flux, and compare the four cases shown in Fig. 66.

A shows part of a winding which has 6 coil sides per slot and a brush which covers 1 commutator segment.

B shows part of a winding which has 2 coil sides per slot and a brush which covers 1 commutator segment.

C shows part of a winding which has 6 coil sides per slot and a brush which covers 3 commutator segments.

D shows part of a winding which has 2 coil sides per slot and a brush which covers 3 commutator segments.

The slots in cases *A* and *C* are three times as wide as those in *B* and *D* and the coils shown black are those which are undergoing commutation.

In *B* the time of commutation is the same as in *A*, but the flux ϕ_s is three times as large since the reluctance of its path is proportional to the ratio $\dfrac{\text{slot depth}}{\text{slot width}}$, the reluctance of the iron part of the path being neglected. The reactance voltage is therefore three times as large.

In C the time of commutation is three times as long as in A, and the flux ϕ_s also is three times as large since there are three times as many conductors undergoing commutation at the same instant. The reactance voltage is therefore the same in each case.

In D the time of commutation is the same as in C and so also is the flux ϕ_s, since the reluctance of three short paths in series is the same as that of a single path three times as long. The reactance voltage is therefore the same in D as it is in C or A.

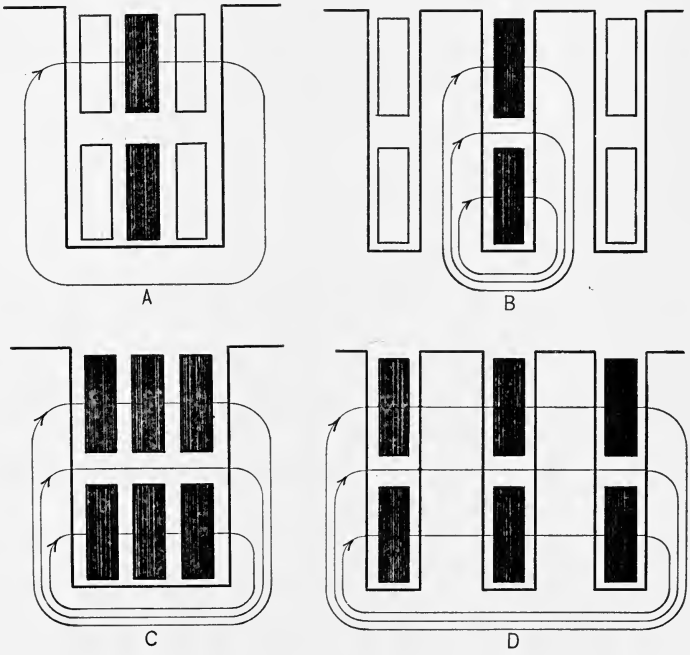


FIG. 66.—Effect of slot and brush width on the reactance voltage.

From the above discussion the following conclusions can be drawn—namely, that when the brush covers one commutator segment only, the reactance voltage is greater the narrower the slot; compare for example, cases A and B; also that an increase in the brush width has no effect on the reactance voltage as long as the group of conductors simultaneously commutated is not greater than the total number of conductors per slot (compare A and C), but decreases the reactance voltage if the group of

conductors simultaneously commutated is greater than the total number of conductors per slot (compare B and D).

The method for determining the brush width is taken up in Art. 72; it is pointed out here, however, that the brush generally covers more than one commutator segment, and under these conditions the case of a very narrow slot with the brush covering one commutator segment, namely case B, Fig. 66, can be neglected; with this restriction the formula for reactance voltage on page 80 can be used for all slot and brush widths.

[1]It has been found experimentally that, for the shape of coil in general use in D.-C. machines and for slots which have the ratio $\frac{\text{slot depth}}{\text{slot width}}=3.5$, a value which is seldom exceeded in non-interpole machines, the value of ϕ_s may be taken as 10 lines per ampere conductor per inch length of core, and ϕ_e may be taken as 2 lines per ampere conductor per inch length of end connection.

By substituting the above values in the formula for reactance voltage on page 80, the following result is obtained for the reactance voltage of a full pitch double layer multiple winding namely:

Average reactance voltage

$$=\frac{2I_c(2\times 10\times L_c+2\times L_e)\times 2T^2\times \text{r.p.m.}\times S\times 10^{-8}}{60}$$

$$=1.33\times S\times \text{r.p.m.}\times I_c\times T^2(L_c+0.1L_e)10^{-8} \qquad (9)$$

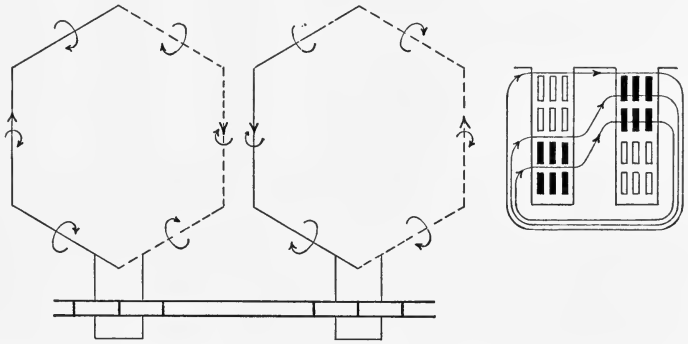


FIG. 67.—Flux circling a short-pitch multiple coil during short-circuit.

67. Short Pitch Multiple Windings.—It was pointed out in Art. 15, page 15, that, when a short-pitch winding is used, the conductors of the coils which are short circuited at any instant are not in the same slot, thus Fig. 67 shows the short pitch diagram

[1] Hobart, Continuous Current Dynamo Design, page 108.

which corresponds to the full-pitch one shown in Fig. 65, and it will be seen that, while the end connection flux is the same in each case, the slot flux ϕ_s has only half the value for a short-pitch winding that it has for a full-pitch winding so long as the pitch is short enough to prevent the conductors which are short-circuited at any instant from lying in the top and bottom of the same slot.

For a short-pitch double-layer multiple winding the value of the average reactance voltage

$$=1.33\times S\times \text{r.p.m.}\times I_c\times T^2\times\left(\frac{L_c}{2}+0.1\,L_e\right)\times 10^{-8}. \qquad (10)$$

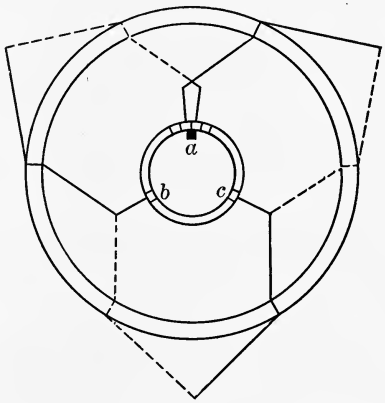


FIG. 68.—Coils in a series winding that are short-circuited by one brush.

68. Calculation of the Reactance Voltage for Machines with Series or Two Circuit Windings.—Fig. 68, which is a reproduction of Fig. 21, shows that when only one + and one − brush are used each brush short circuits $\frac{p}{2}$ coils in series, so that the average reactance voltage in such a case

$$=1.33\times S\times \text{r.p.m.}\times I_c\times T^2(L_c+0.1\,L_e)10^{-8}\times\frac{p}{2}. \qquad (11)$$

When, however, the same number of sets of brushes are used as there are poles, so that brushes are also placed at b and c, there is a short commutation path round one coil which is short-circuited by two brushes at the same potential, in addition to the long path around $\frac{p}{2}$ coils in series and, so far as this short path alone is concerned, the average reactance voltage

$$=1.33\times S\times \text{r.p.m.}\times I_c\times T^2(L_c+0.1\,L_e)10^{-8}.$$

The value of the reactance voltage that should be used as a criterion for commutation is somewhere between these two values, and the results are still further complicated by what is known as selective commutation which was described in Art. 17, page 19, so that in practice it is usual to use the former of the two equations, which is pessimistic; the commutation will generally be about 20 per cent. better than that indicated by the value of reactance voltage so found.

69. Formulæ for Reactance Voltage.—Collecting together the results obtained in Arts. 66, 67 and 68, the following formulæ are obtained:

The average reactance voltage

$= 1.33 \times S \times \text{r.p.m.} \times I_c \times T^2 (L_c + 0.1\ L_e) 10^{-8}$ for full-pitch multiple windings

$= 1.33 \times S \times \text{r.p.m.} \times I_c \times T^2 \left(\frac{L_c}{2} + 0.1\ L_e\right) 10^{-8}$ for short-pitch multiple windings

$= 1.33 \times S \times \text{r.p.m.} \times I_c \times T^2 (L_c + 0.1\ L_e) \frac{p}{2} \times 10^{-8}$ for series windings.

It is pointed out in Art. 99, page 117, that, in order to have an economical machine, the core length L_c should lie between the values (0.9 to 0.6) $\times$[pole pitch].

The length L_e of the end connections is directly proportional to the pole pitch and $= 1.4$ (pole pitch) approximately, so that L_e has a value between (1.6 and 2.4) $\times [L_c]$ and an average value $= 2L_c$.

Substituting this average value in the above formulæ for reactance voltage the following approximate formula is obtained: The average reactance voltage

$$RV = k \times S \times \text{r.p.m.} \times I_c \times L_c \times T^2 \frac{\text{poles}}{\text{paths}} 10^{-8} \qquad (12)$$

where S = the number of commutator segments

r.p.m. = the speed of the machine in revolutions per minute

I_c = the current in each armature conductor

T = the number of turns per coil

L_c = the frame length in inches

$\frac{\text{poles}}{\text{paths}} = 1$ for multiple and $= \frac{p}{2}$ for series windings

$k = 1.6$ for series and full-pitch multiple windings

$= 0.93$ for short-pitch multiple windings.

CHAPTER IX

COMMUTATION (Continued)

70. The Sparking Voltage.—Down to this point it has been assumed that the coils in which the current is being commutated are in such a position between the poles that, during commutation, they are not cutting any lines of force due to the m.m.f. of the field and the armature. Under these conditions the value of the reactance voltage when sparking begins is called the sparking voltage, and depends on the brush resistance, as shown in Art. 61, page 76.

In practice the brushes are generally shifted from the above neutral position in such a direction that the coils which are undergoing commutation are in a magnetic field, and an e.m.f. E_s is generated in them, due to the cutting of this field, which opposes the e.m.f. of self and mutual induction and so causes the commutation to be more nearly perfect. In such cases the value of the resultant of this generated voltage and of the reactance voltage, when sparking begins, is called the sparking voltage.

As the load on a machine increases the reactance voltage increases with it, and in order that the commutation may be perfect at all loads the voltage E_s must increase at the same rate; this can only be the case if the distance the brushes are shifted from the neutral varies with the load.

Suppose that the brushes are in such a position that commutation is perfect at 50 per cent. overload, and that they are fixed there; then at no-load there will be no reactance voltage to counteract, but there will be a large generated voltage E_s which will cause a circulating current to flow in the short-circuited coil, and sparking will take place if E_s is larger than the sparking voltage.

Modern D.-C. machines are expected to carry any load from no-load up to 25 per cent. overload without sparking and also without shifting of the brushes during operation. To accomplish this result the brushes are shifted from the neutral position, in such a direction as to help commutation when the machine is loaded, until the machine is about to spark at no-load; the voltage E_s, which is generated in the short-circuited coil, will then

be a little less than the sparking voltage. When the machine is carrying 25 per cent. overload the reactance voltage will be greater than the generated voltage E_s by an amount which is just a little less than the sparking voltage, and half way between

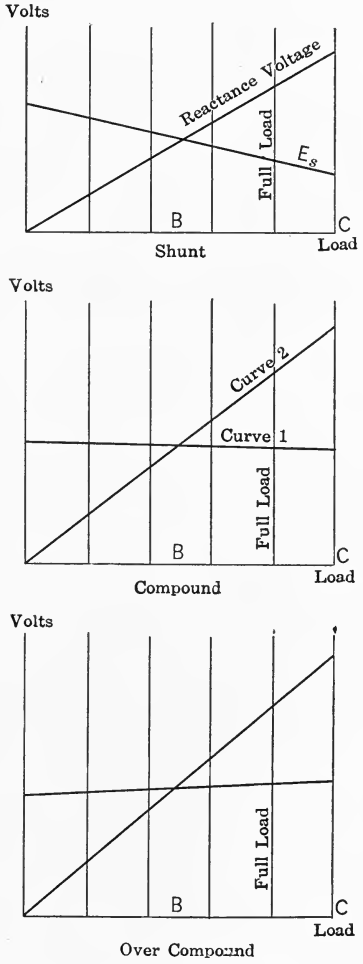


FIG. 69.—Variation of the voltage in the short-circuited coil with load.

these two points E_s will be equal and opposite to the reactance voltage and the commutation will be perfect.

From the above it would seem that the reactance voltage at 25 per cent. overload could have a value equal to about twice the sparking voltage, but in obtaining this result it has been assumed

that the generated voltage E_s remains constant at all loads; as a matter of fact, it decreases with increase of load due to the cross-magnetizing effect of the armature, as pointed out in Art. 48, page 54. To prevent this decrease in the value of E_s from becoming too large, the relation between the field and armature strengths is fixed by making the field ampere-turns per pole for gap and tooth greater than 1.2 (the armature ampere-turns per pole) + the demagnetizing ampere-turns per pole; see Art. 52, page 61.

The diagrams in Fig. 69 show the variation of E_s and of the reactance voltage with load. Curve 1 shows the relation between E_s and the load, and curve 2 shows that between the reactance voltage and load. The brushes are shifted until the value of E_s at no-load is equal to the sparking voltage; at load B the commutation is perfect, and at load C the reactance voltage is greater than E_s by the sparking voltage. These curves show clearly the superiority of the overcompound machine as far as commutation is concerned.

71. Minimum Number of Slots per Pole.—Fig. 70 shows three of the stages in the commutation of the current in a machine which has six coil sides per slot. The commutator segments are evenly spaced, while the coils, being in slots, are not. It will be seen that, between the instant when the brush breaks contact with coil A and the instant when it breaks contact with coil C, the slot in which these coils lie has moved through the distance x, so that, if the magnetic field in which the coils undergo commutation is just right for coil A, it will be a little too strong for coil B, and much too strong for coil C; this latter coil will therefore be so badly commutated that sparking will result and will show up on the commutator in the blackening of every third commutator segment due to the poor commutation of the coil to which it is connected. The distance x = the slot pitch − width of one commutator segment.

The distance through which a slot moves while the conductors which it carries are undergoing commutation is limited by making the number of slots per pole such that there are not less than 3.5 slots in the space between the poles; this space is generally 30 per cent. of the pole pitch, so that the number of slots per pole, corresponding to 3.5 slots in the space between poles, is 12, and is the smallest number of slots per pole that should be used. For large machines there are seldom less than 14 slots per pole.

72. The Brush Width.—It was pointed out in Art. 66, page 82, that the brush width has very little effect on the reactance voltage because, while the number of adjacent coils that are simultaneously undergoing commutation increases with the brush width, the time of commutation also increases at the same rate.

Figure 71 shows the distribution of the magnetic field in the space between the poles of a loaded D.-C. generator and also the

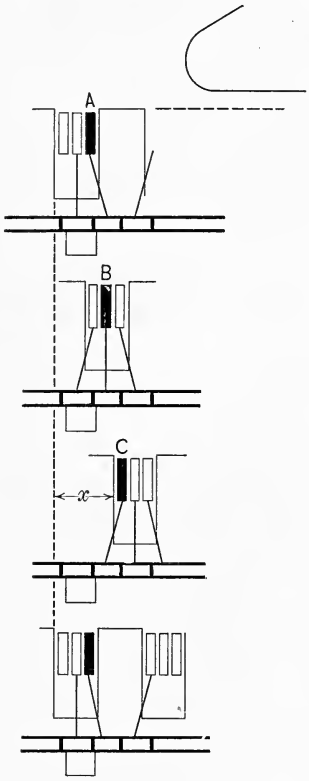


FIG. 70.—Variation of the position of the short-circuited coil when there are several coils per slot.

position of a wide brush. It will be seen that, in order to keep the tip *a* of the brush from under the pole, where the magnetic field is too strong, the tip *b* has to be in a magnetic field which is not a reversing field, so that at the start of commutation in any coil the current in that coil will increase, as shown in Fig. 72, and the effective time of commutation will be less than the

apparent time in the ratio $\frac{cd}{ce}$. To limit this effect the brush should not cover more than 28 per cent. of the space between poles or 0.28×0.3 times the pole pitch.

The brush arc measured on the armature surface should there-

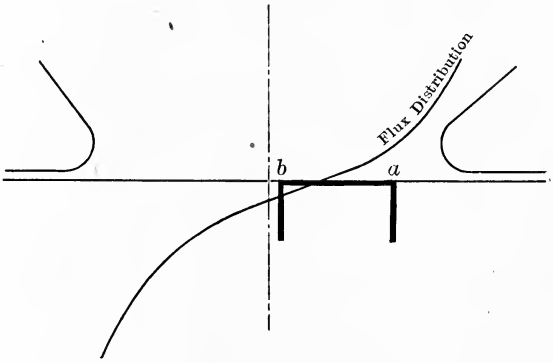


FIG. 71.—Flux distribution at full-load in the space between poles.

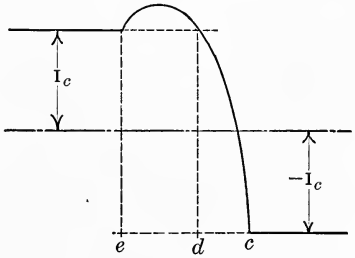


FIG. 72.—Variation of the current in the short-circuited coil when the brush is too wide.

fore not be greater than $\frac{\text{pole pitch}}{12}$ or, measured on the commutator surface, should not be greater than

$$\frac{\text{pole pitch}}{12} \times \frac{\text{dia. commutator}}{\text{dia. armature}}.$$

There is still another limit to the brush arc. When the brushes are shifted from the neutral position in order to help commutation under load there is an e.m.f. generated in the short-circuited coils, namely E_s. Thus, in the case shown in Fig. 73, where the brush covers five segments, the voltage generated between a and b is that of five coils in series and the resistance of the path to the

circulating current that will flow is comparatively low, being that of one coil and two contacts. To prevent trouble due to this circulating current it is necessary to limit the brush arc so that it shall not cover more than three commutator segments.

73. Limits of the Reactance Voltage.—In the discussion in Art. 61, page 76, it is shown that the reactance voltage should not exceed the voltage drop across one brush contact when the brushes are in such a position that the short-circuited coil is not

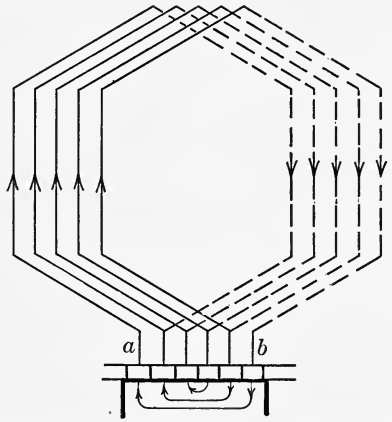


FIG. 73.—Circulating currents at no-load in a wide brush.

cutting any lines of force due to the m.m.f. of the field and armature, and the discussion in Art. 70, page 85, shows that higher values may be used when the brushes are shifted forward so as to help commutation.

Experiment shows that higher values of the reactance voltage may be used than the limits indicated by theory and the following, found from experience, may be used in design work.

For machines which must operate without destructive sparking at all loads from no-load to 25 per cent. overload with brushes in a fixed position, the reactance voltage at full load should not exceed 0.7 (volts drop per pair of brushes) with the brushes on the neutral position, nor should it exceed the volts drop per pair of brushes for machines with the brushes shifted so as to help commutation.

These figures apply to machines which are built so that:—
The number of slots per pole is greater than 12;

The brush arc covers less than 1/12 of the pole pitch when measured on the armature surface, and also does not cover more than three commutator segments;

The pole arc is not greater than 70 per cent. of the pole pitch;

AT_{g+t} is greater than 1.2 (armature AT per pole) + the demagnetizing AT per pole.

These figures are intimately connected with one another and also with the reactance voltage; for example, the brushes may be made wider and the main field weaker than indicated, but in such cases the reactance voltage must also be decreased otherwise trouble will develop.

In addition it must be noted that, as pointed out in Art. 68, page 84, the commutation of a machine with a series winding is about 20 per cent. better than represented by the value of the reactance voltage obtained from the formula.

For machines with short-pitch windings on the other hand, the operation is about 30 per cent. worse than represented by the value of the reactance voltage obtained from the formula, because, as shown in Art. 15, page 15, the effective width of the space between poles is less than the actual width by the distance of one slot pitch.

Fig. 74.—Field due to the armature m.m.f.

74. Limit of Armature Loading.—Fig. 74 shows part of a multipolar machine; the current in the armature conductors is represented by crosses and dots and the lines of force of armature reaction are also shown. As the number of armature ampere-turns per pole increases the field ampere-turns per pole must also be increased so as to prevent too great a distortion of the main field and consequently poor commutation.

There is, however, one part of the armature field which is not counteracted by the main field, namely, the field out on the end connections; this is stationary in space and therefore cut by the coils which are undergoing commutation. The e.m.f. due to the cutting of this field acts in such a direction as to oppose commutation, and in order to prevent this voltage from having such a

value as to cause trouble it is advisable to limit the number of armature ampere-turns per pole to about 7500; a higher value than this must be accompanied by a low reactance voltage otherwise trouble will develop.

75. Interpole Machines.—It was pointed out in Art. 70, page 85, that commutation can be helped by shifting the brushes so that the short-circuited coils are in a magnetic field, and it was also shown that, if the e.m.f. generated due to this field was to be equal and opposite at all times to the reactance voltage, the strength of the field should be proportional to the load.

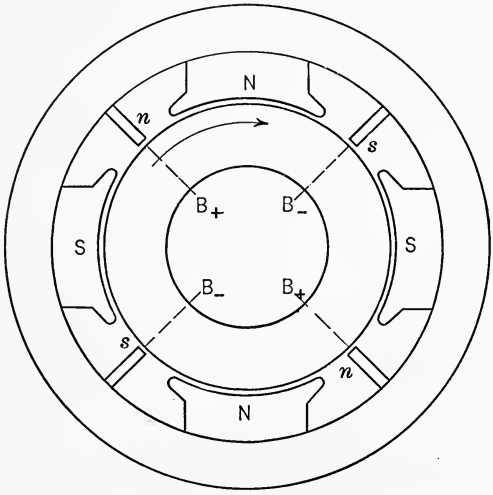


FIG. 75.—Magnetic circuit of a four-pole interpole machine.

Figure 75 shows an interpole generator diagrammatically; n and s are auxiliary poles which have a series winding, so that their strength increases as the load increases. In a generator the brushes would be shifted forward in the direction of motion so that B_+ would come under the tip of the N pole and B_- under the tip of the S pole; instead of that, in the interpole machine, the auxiliary n pole is brought to the brush B_+ and the auxiliary s pole to the brush B_-.

Before the interpole can send a flux across the air gap in such a direction as to help commutation it must have a m.m.f. equal to that of the cross magnetizing effect of the armature which

$= \frac{1}{2}\frac{Z}{p} I_c$ ampere-turns, see Art. 48, page 55,

= the armature ampere-turns per pole,

and in addition a m.m.f. to send a flux across the air gap large enough to generate an e.m.f. in the short-circuited coil which shall be equal and opposite to the e.m.f. of self and mutual induction and to that generated due to cutting the magnetic field out on the end connections.

In order that the flux produced by the interpole may be always proportional to the load, it is necessary that the interpole magnetic circuit do not become saturated.

76. Interpole Dimensions.—W_{ip}, the interpole arc, should be such that, while the current in a conductor is being commutated, the slot in which that conductor lies is under the interpole.

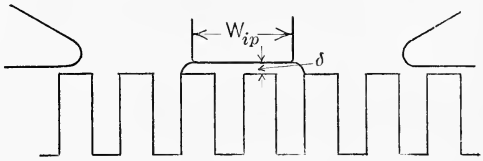


FIG. 76.—Dimensions of interpole.

The distance moved by the coil A, Fig. 70, while it is short-circuited by the brush, is equal to the brush arc referred to the armature surface = brush arc $\times \frac{\text{armature diameter}}{\text{commutator diameter}}$;

the total arc which must be under the influence of the interpole is greater than this by the distance x which, as pointed out in Art. 71, page 87, = the slot pitch − the width of one commutator segment referred to the armature surface.

As shown in Fig. 76, the flux fringes out from either side of the interpole by a distance which is approximately equal to the air-gap clearance, so that the effective interpole arc $= W_{ip} + 2\delta$; this arc must be equal to the slot pitch + brush arc − segment width, all referred to the armature surface.

This arc must also be such that the reluctance of the gap under the interpole shall vary as little as possible for different positions of the armature, otherwise the interpole field will be a pulsating one; if, for example, the arc were equal to the width of one tooth, then the interpole air-gap reluctance would vary from a maximum when a tooth was under the pole to a minimum when a slot was under the pole. If the interpole effective arc be a multiple of the slot pitch, as in Fig. 76, where it is equal to twice the slot pitch, the reluctance of the gap will

be practically constant, because, as one tooth moves from under the pole another comes under its influence.

As a general rule the effective interpole arc is about 15 per cent of the pole pitch and is adjusted so as to be approximately a multiple of the slot pitch; the brush arc is then made to suit the interpole arc so found. To allow space for the interpole and to prevent the interpole leakage flux from being too large, the arc of the main pole is kept down to about 65 per cent of the pole pitch.

The axial length of the interpole, L_{ip}, is found as follows: If B_i is the average interpole gap density, then the voltage generated in each coil while under the interpole

$$= \text{the lines cut per second} \times 10^{-8}$$

$$= 2\ T \times B_i \times L_i \times \pi D_a \times \frac{\text{r.p.m.}}{60} \times 10^{-8}$$

and this should equal the reactance voltage, which, for a full pitch winding, the type used on interpole machines,

$$= 1.6 \times S \times \text{r.p.m.} \times I_c \times L_c \times T^2 \times 10^{-8} \text{volts per coil;}$$

therefore, equating these two values together and simplifying

$$B_i \times L_i = \left(\frac{S \times T \times I_c}{\pi D_a}\right) L_c \times 48$$

$$= \text{ampere conductors per inch} \times L_c \times 48$$

and

$$L_i = L_c \left(\frac{\text{ampere conductor per inch}}{B_i}\right) \times 48$$

The value of B_i is generally chosen about 45,000 lines per square inch at full load, for which value the interpole circuit does not become saturated up to 50 per cent. overload.

The value of ampere conductors per inch seldom exceeds 900; for this value, and for the value of interpole gap density given above

$$L_i = 0.95\ L_c$$

The interpole excitation is not figured out accurately; the usual practice is to put on each interpole a number of ampere-turns which at full load $= 1.5$ (armature AT per pole)

$$= 1.5 \left(\frac{ZI_c}{2p}\right)$$

such a value is sufficient to overcome the armature m.m.f. and also to send sufficient flux across the interpole gap to allow the necessary voltage to be generated. This value of m.m.f. is generally

too large and the final adjustment is made by means of a shunt after the machine has been erected.

77. Flashing Over.—If the voltage between adjacent commutator segments becomes too high the machine is liable to flash over on the commutator from brush to brush, particularly if the commutator is dirty. The voltage between two adjacent bars should not if possible exceed 40; turbo generators have been built in which this value was greater than 60, but such machines are sensitive to changes of load and liable to flash over unless supplied with compensating windings. It must be clearly understood that there is considerable difference between the maximum voltage between commutator bars and the average value, this can be seen from Fig. 49, which shows that when the machine is loaded the flux density in the air gap at point *d* is much higher than the average gap density; at this point the highest voltage per coil will be generated.

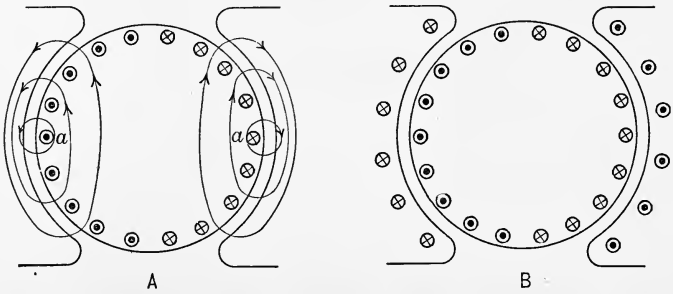


FIG. 77.—Armature field with and without compensating windings.

Flashing over is generally caused by a sudden change of load, the reason being as follows: Fig. 77, diagram A, shows the armature cross field when the machine is loaded; a sudden change in load causes the value of this field to change and a voltage is generated in the armature coils which is proportional to the rate of change of flux and is a maximum in the coil *a*. This voltage increases or decreases that which already exists between adjacent commutator segments and the increase may be sufficient to cause the voltage between adjacent commutator segments to become too high and the machine to flash over.

When the load is very fluctuating in character, such as the load on a motor driving a reversing rolling mill at the instant of reversal of the rolls, at which instant the current changes from

less than full-load current to about three times full-load current in the opposite direction, the average voltage between commutator bars should not exceed 15, or it will be necessary to supply the machine with a compensating winding to prevent flashing over. Fig. 78 shows such a machine. The poles carry a winding on the pole face which is connected in series with the

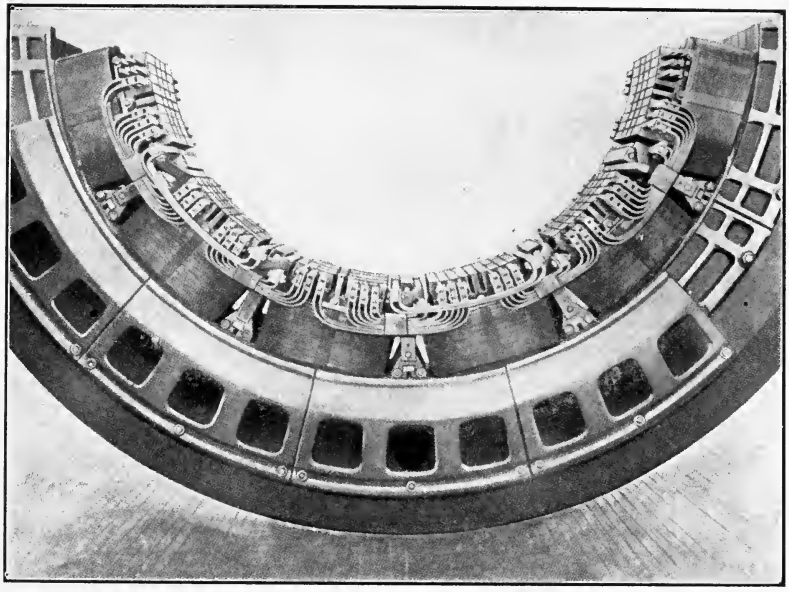


FIG. 78.—Yoke of machine with interpoles and compensating windings.

armature and which has the same number of ampere-turns per pole as there are on the armature. The current in the pole-face conductors passes in the opposite direction to that in the armature conductors under the same pole, so that the armature field is completely neutralized, as shown in Fig. 77, diagram B; there is no crowding of the lines into one pole tip as shown in Fig. 49, and there is no sudden change of armature field with load and therefore no tendency to flash over.

CHAPTER X

EFFICIENCY AND LOSSES

78. The Efficiency of a generator $= \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}$ and that of a motor $= \frac{\text{output}}{\text{input}} = \frac{\text{input} - \text{losses}}{\text{input}}$ where the losses are:

Mechanical losses—windage, brush and bearing friction.

Iron losses—hysteresis and eddy current losses.

Copper losses in field and armature coils.

Commutator contact resistance loss.

79. Bearing Friction.—In a high speed bearing with ring lubrication there is always a film of oil between the shaft and the bushing; bearing friction is therefore an example of fluid friction and the tangential force at the rubbing surface in such a case $= kA_b V_b^n$ lb., where k is a constant which depends on the viscosity of the oil and is found by experiment to be $= 0.036$ for bearings with ring lubrication using light machine oil.

A_b is the projected area of the bearing in square inches = (the bearing diameter d_b in inches $\times$ the bearing length l_b in inches);

V_b is the rubbing velocity of the bearing in feet per minute;

n is found experimentally to vary from 1 for low values of V_b to 0 for very high values and is approximately $= 0.5$ for bearing speeds from 100 to 1000 ft. per minute.

For moderate speed bearings with ring lubrication and with light machine oil the force of friction $= 0.036 A_b V_b^{\frac{1}{2}}$ pounds and the friction loss $= 0.036 A_b V_b^{\frac{3}{2}}$ ft. lb. per minute

$$= 0.81 d_b l_b \left(\frac{V_b}{100}\right)^{\frac{3}{2}} \text{ watts.} \qquad (13)$$

It is important to notice that this loss is independent of the bearing pressure and is therefore independent of the load.

As the pressure on the bearing increases the thickness of the oil film decreases and at a certain limiting pressure it breaks down and the bearing seizes. For electrical machinery the

bearing pressure $= \frac{\text{bearing load}}{A_b}$, should not exceed 80 lb. per square inch when the machine is carrying full load, such a bearing will carry 100 per cent. overload without breakdown.

The bearing loss increases rapidly with the rubbing velocity and after a certain velocity has been reached the bearings are no longer large enough to be self-cooling. For self-cooling bearings the rubbing velocity should not exceed 1000 ft. per minute unless the bearing is specially designed to get rid of the heat, and even this value should only be used for machines which are built so that a large supply of cool air passes continuously over the external surface of the bearing. The bearings of totally enclosed motors for example are poorly ventilated and should not be run with a rubbing velocity greater than about 800 ft. per minute.

The thickness of the oil film in a bearing varies inversely as the temperature of the oil, and this temperature should not exceed 70° C. when measured by a thermometer in the oil well.

80. Brush Friction.—If

μ is the coefficient of friction;

P the brush pressure in pounds per square inch;

A the total brush rubbing surface in square inches;

V_r the rubbing velocity in feet per minute;

then the friction loss $= \mu P A V_r$ ft. lb. per minute.

Approximate values found in practice are:

$\mu = 0.28$ and $P = 2$ lb. per square inch, for which values the

$$\text{friction loss} = 1.25\, A \frac{V_r}{100} \text{ watts.} \qquad (14)$$

81. Windage Loss.—It is difficult to predetermine this loss, but, up to peripheral velocities of 6000 ft. per minute, it is so small that it may be neglected. Fig. 79 shows the windage and bearing friction loss of a motor which has an armature diameter of 100 in. and three bearings each 10 in. by 30 in. The circles show the actual test points and the curve shows the value of the bearing friction loss calculated from the formula, friction loss $= 3 \times 0.81 \times d_b \times l_b \left(\frac{V_b}{100}\right)^{\frac{3}{2}}$ watts. At a speed of 230 r.p.m. the peripheral velocity of the armature is 6000 ft. per minute, up to which speed the windage loss can be neglected; at higher speeds it becomes large because it is proportional to the (peripheral velocity).[3]

Few electrical machines except turbo generators are run at peripheral speeds greater than 6000 ft. per minute, because the cost increases very rapidly for higher speeds on account of the

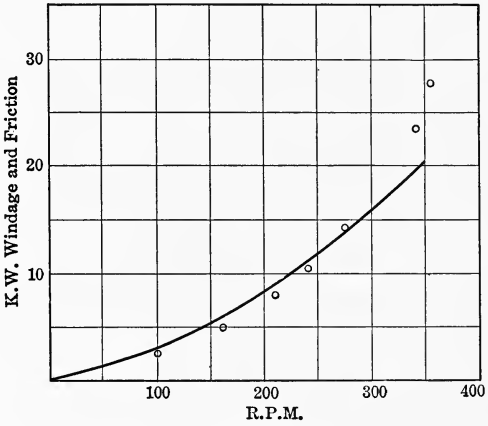


FIG. 79.—Windage and friction loss in a large motor.

special construction required to hold the coils against centrifugal force.

82. Iron Losses.—It may be seen from Fig. 80 that the flux in

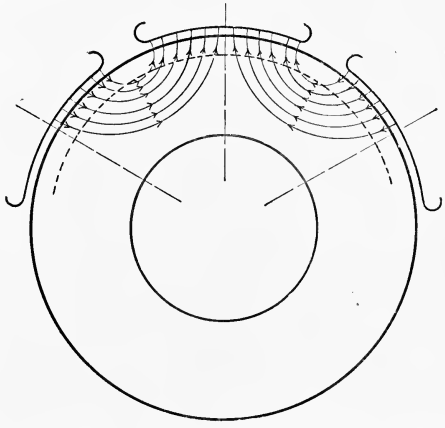


FIG. 80.—Distribution of flux in the armature.

any portion of the armature of a D. C. machine goes through one cycle while the armature moves through the distance of two pole pitches; that is, the flux in any portion of the armature

passes through $\frac{p}{2}$ cycles per revolution, or through $\frac{p}{2} \times \frac{\text{r.p.m.}}{60}$ cycles per second.

The iron losses consist of the hysteresis loss which $= KB^{1.6}fW$ watts, and the eddy current loss which $= K_e(Bft)^2W$ watts where K is the hysteresis constant and varies with the grade of iron;

K_e is a constant which is inversely proportional to the electrical resistance of the iron;
B is the maximum flux density in lines per square inch;
f is the frequency in cycles per second;
W is the weight of the iron in pounds;
t is the thickness of the core laminations in inches.

The eddy current loss can be reduced by the use of iron which has a high electrical resistance. At present, however, most of the grades of very high resistance iron have a lower permeability than those of lower electrical resistance; they also cost more and are more brittle; machines built with such iron are liable to have the teeth break off due to vibration. The reduction in the total loss by the use of high resistance iron is not so great as one would expect, because a large part of the loss is due to losses discussed in the next article and these are not greatly affected by the grade of iron used.

The eddy current loss can be reduced by a reduction in t, the thickness of the laminations; this value is generally about 0.014 in.; thinner iron is difficult to handle.

83. Additional Iron Losses.—Besides the ordinary hysteresis and eddy current loss already mentioned there are additional losses which cannot be calculated, due to the following causes:

(*a*) Loss due to filing of the slots. When the laminations that form the core have been assembled it will be found that in most cases the slots are rough and must be filed smooth in order that the slot insulation may not be cut. This filing burrs the laminations over on one another and provides a low resistance path through which eddy currents can flow; that is, it tends to defeat the result to be obtained by laminating the core.

(*b*) Losses in the spider and end heads due to the leakage flux which gets into these parts of the machine; these losses may be large because the material is not laminated.

(*c*) Loss due to non-uniform distribution of flux in the arma-

ture core. When calculating the value of B_c, the flux density in the core, page 45, it is assumed that the flux is uniformly spread over the core area. This however is not the case and the actual flux distribution is shown in Fig. 80; the lines of force take the path of least reluctance and therefore crowd in behind the teeth until that part of the core becomes saturated, then they spread further out. Due to this concentration of flux, the core loss, which is approximately proportional to (flux density),^{2} is greater than that got by assuming that the flux distribution is uniform through the whole depth of the core.

It is sometimes possible to increase the core depth so as to keep the apparent flux density in the core low and yet make no perceptible reduction in the core loss, because the increased depth of core does not carry its proper share of the flux; for this reason there is seldom much to be gained by making the value of B_c less than 80,000 lines per square inch, which is near the point at which saturation begins and the flux tends to become uniform.

(*d*) Pole face losses. Fig. 39 shows the distribution of flux in the air gap of a D. C. machine and, as the armature revolves and the teeth move past the pole face, e.m.fs. will be induced which will cause currents to flow across the pole face. Experiment shows that when solid pole faces are used the loss due to these currents increases very rapidly as the slot opening becomes greater than twice the length of the air gap; when the slot openings are wider than this the pole face must be laminated.

(*e*) Variation of the iron loss with load. It is shown in Fig. 49 that, when the machine is loaded, the flux density is not uniform in all the teeth under the pole but is stronger at one pole tip than at the other. The effect of the increase of flux density in the teeth under one pole tip in increasing the iron loss is greater than the effect of the decrease of flux density in the teeth under the other tip in reducing the iron loss.

84. Calculation of Core Loss.—Due to the additional losses it is impossible to predetermine the total core loss by the use of fundamental formulæ; core loss calculations for new designs are based on the results obtained from tests on similar machines built under the same conditions. Such test results are plotted in Fig. 81 for machines built with ordinary iron of a thickness of 0.014 in., the slots being made with notching dies so that a certain amount of filing has to be done.

Example of calculation: For the machine shown in Fig. 41 the value of B_{at} the actual flux density in the teeth and B_c the average flux density in the core are given in Art. 46, page 49.

B_{at} = 150,000 lines per square inch;
B_c = 84,000 lines per square inch;
W_t, the total weight of the armature teeth = 385 lb.;
W_c, the total weight of the armature core = 2300 lb.;
The frequency = 16.6 cycles per second;
The loss per pound for the teeth = 6 watts, from Fig. 81;
The loss per pound for the core = 1.8 watts, from Fig. 81;
The total loss = $385 \times 6 + 2300 \times 1.8 = 6450$ watts.

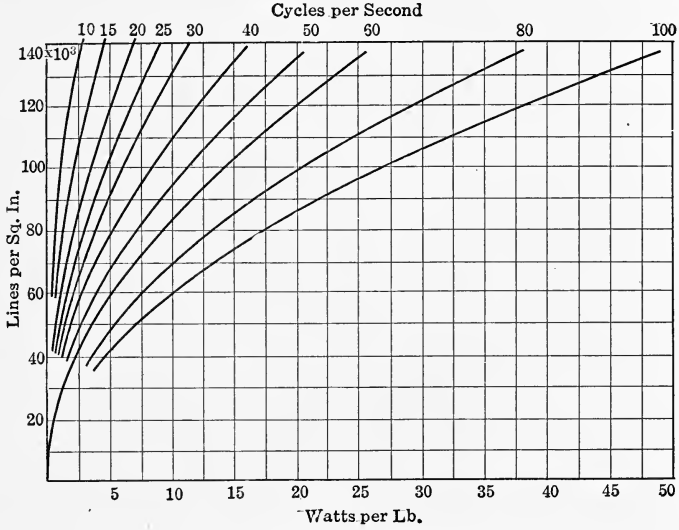


FIG. 81.—Iron loss curves for revolving machinery.

85. Armature Copper Loss.—The resistance of copper at the normal operating temperature of an electric machine is approximately 1 ohm per circular mil cross-section per inch length, so that if

Z is the total number of conductors;
L_b is the length of one conductor in inches;
M is the cross-section of each conductor in circular mils;
I_c is the current in each conductor in amperes;

then the resistance of one conductor in ohms $= \frac{L_b}{M}$

the loss in one conductor in watts $= \frac{L_b}{M} \times I_c^2$

and the total copper loss in the armature in watts $= Z\frac{L_b}{M}I_c^2$ (15)

The value of $L_b =$

1.35 (pole pitch) +2 (armature axial length) + 3 in.

approximately for the type of coil shown in Fig. 33, page 36.

86. Shunt Field Copper Loss.—This loss, which $= E_t I_f$ watts, where E_t is the terminal voltage of the machine and I_f is the current in the shunt coil, is made up of the loss in the field coils and the loss in the field rheostat; the latter for a generator is about 20 per cent. of the total shunt field loss.

87. Series Field Copper Loss.—This loss $= I_a^2 R_s$ watts, where R_s is the resistance of the series field in ohms and I_a the total armature current. When a series shunt is used, so that part of the current I_a passes through the series field and the remainder passes through the shunt, the resistance R_s is the combined resistance of the series field and the series shunt in parallel.

88. Brush Contact Resistance Loss.—This loss has already been discussed in Art. 62, page 76, and $= E_b I_a$ watts, where E_b is the voltage drop per pair of brushes and I_a is the armature current. The volts drop per pair of brushes is approximately constant over a wide range of current, as shown in Fig. 64.

CHAPTER XI

HEATING

89. Cause of Temperature Rise.—The losses in an electrical machine are transformed into heat; part of this heat is dissipated by the machine and the remainder, being absorbed, causes the temperature of the machine to increase. The temperature becomes stationary when the heat absorption becomes zero, that is when the point is reached where the rate at which heat is generated in the machine is equal to the rate at which it is dissipated.

90. Maximum Safe Operating Temperature.—The highest safe temperature at which an electrical machine can be operated continuously is about 85° C. with the present practice in insulating, because, if subjected to that temperature for any length of time, the paper and cloth which are used for coil insulation become brittle, and pulverize due to vibration.

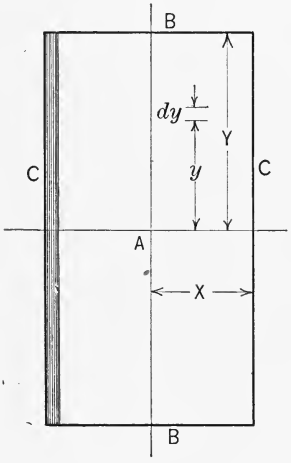


FIG. 82.—The heat paths in an armature core.

The usual heating guarantee is that the machine shall carry full load continuously with a temperature rise of not more than 40° C. This is a conservative guarantee and allows the machine to carry 25 per cent. overload without injury if the air temperature does not exceed 25° C.

If the air temperature is greater than 25° C. a lower temperature rise must be allowed in order that the final temperature shall not be excessive.

91. Temperature Gradient in the Core of an Electrical Machine.—Fig. 82 shows an iron core built up of laminations that are separated from one another by varnish. In this core there is an alternating magnetic flux and the loss in the core for different flux densities and for different frequencies can be found by the help of the curves in Fig. 81, page 102.

The hottest part of the core is at A and the heat in the center of the core has to be conducted to and dissipated by the surfaces B and C.

In order to have an idea as to the relative heat resistances of the paths from A to the surfaces B and C, consider the following propositions.

(*a*) Assume that all the heat passes in the direction Y, then the watts crossing 1 sq. in. of the core at $y = (\text{watts per cubic inch}) \times y$ and since the difference in temperature between two faces a distance dy apart $= \dfrac{(\text{watts per cubic inch}) y\, dy}{1.5}$ degrees centigrade, where 1.5 is the thermal conductivity of iron in watts per 1-in. cube per 1° C. difference in temperature, therefore, the difference in temperature between A and surface B

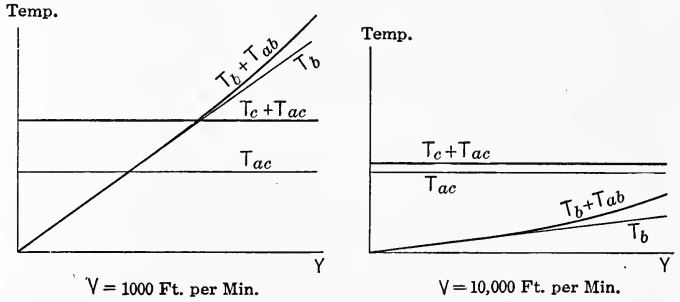


FIG. 83.—Temperatures in an iron core.

$$= T_{ab} = \int_0^Y \frac{(\text{watts per cubic inch}) y\, dy}{1.5}$$

$$= (\text{watts per cubic inch}) \frac{Y^2}{3} \text{ deg. C.}$$

(*b*) If the heat were all conducted in the direction X then, since the conductivity of a core along the laminations is approximately fifty-six times as great as that across the laminations and layers of varnish, the difference in temperature between A and surface C would be

$$= T_{ac} = (\text{watts per cubic inch}) \frac{56\, X^2}{3} \text{ deg. C.}$$

Example; when the frequency is 60 cycles and the flux density is 75,000 lines per square inch then the watts per cubic inch $= 2.3$ and

$$T_{ab} = 0.8Y^2 \text{ deg. C.}$$
$$T_{ac} = 45X^2 \text{ deg. C.}$$

The fact that the conductivity along the laminations is so much better than that across the laminations would indicate that axial ventilation, whereby air is blown across the ends of the laminations, is the most effective. In practice, however, nearly all electrical machines are cooled by means of radial vent ducts, and, in order to discuss intelligently the effect of such ducts, it is necessary to find out the heat resistance between the surfaces B, C and the air.

When air is blown across the surface of an iron core at V ft. per minute, the watts dissipated per square inch of radiating surface for 1° C. rise of the surface temperature is found by experiment to be $= 0.0245(1 + 0.00127\ V)$.[1]

If then, as in case (a), all the heat in the core has to be dissipated by surface B, the difference in temperature between surface B and the air $= T_b$

$$= \frac{\text{watts per square inch on surface } B}{0.0245(1+0.00127\ V)}$$
$$= \frac{(\text{watts per cubic inch})\ Y}{0.0245(1+0.00127\ V)}$$

Similarly in case (b), where it is assumed that all the heat is dissipated by surface C, the difference in temperature between surface C and the air $= T_c$

$$= \frac{(\text{watts per cubic inch})\ X}{0.0245(1+0.00127\ V)}$$

Where $X = 1.5$ in., and for a value of 1 watt per cubic inch, the following table shows the values of T_{ab}, T_{ac}, T_b and T_c for different values of Y and of V.

				$V=1000$		$V=10{,}000$	
X	Y	T_{ac}	T_{ab}	T_b	T_c	T_b	T_c
1.5	1.5	42	0.75	27	27	4.5	4.5
1.5	3.0	42	3.0	54	27	9.0	4.5
1.5	6.0	42	12.0	108	27	18.0	4.5

These results are plotted in Fig. 83 and from them the following general conclusions may be drawn:—

As the core depth Y increases, the path from A to B becomes long compared with that from A to C and, therefore, T_{ab} becomes comparable with T_{ac} in spite of the relatively high conductivity along the laminations.

[1] Ott, Electrician, March 7, 1907.

The area C increases with the core depth while area B remains constant so that the deeper the core the larger the part of the total heat which is conducted across the laminations and dissipated from the surface of the vent ducts.

92. Limiting Values of Flux Density.

The peripheral velocity of a machine

$$\begin{aligned} &= \frac{\pi D_a}{12} \times \text{r.p.m.} \\ &= \frac{\pi D_a}{p} \times \frac{p \times \text{r.p.m.}}{12} \\ &= 10 \times \tau \times f \text{ ft. per minute} \end{aligned} \qquad (16)$$

therefore, for a given frequency, the peripheral velocity of a machine is proportional to its pole pitch.

For a given axial length of core, the longer the pole pitch the greater the flux per pole, and the deeper the core to carry this flux.

Where the peripheral velocity of a machine is low, the core is shallow, the vent ducts have little effect on account of their small radiating surface, and the ventilation is poor; the loss, however, is small because there is not much iron in the core.

Where the peripheral velocity is high the core is deep, the vent ducts are very effective on account of their large radiating surface, and the ventilation is good; the loss, however, is large since much iron is used in the core.

For a given frequency the same flux densities can be used for all peripheral velocities. The following flux densities may be used for D. C. machines with a temperature rise of 40° C., built with iron 0.014 in. thick, the relation between core loss and flux density being as shown in Fig. 81:

Frequency (cycles per second)	Flux density in teeth (lines per square inch)	Flux density in core (lines per square inch)
30	150,000	100,000
40	140,000	85,000
60	125,000	75,000

Core densities higher than 85,000 lines per square inch are seldom used, even for frequencies that are lower than 40 cycles, because at these densities the core becomes saturated and the

cost of the extra field copper required to send the flux through a saturated core is greater than the cost of the extra iron required to keep the core density below the saturation point. There is no such objection, however, to high-tooth densities because a machine with saturated teeth and a short air gap is just as effective in preventing field distortion as a machine with unsaturated teeth and a long air gap; see Art. 52, page 61, and does not require any more excitation.

93. Heating of the End Connections of the Winding.—The end connection heating must be taken up separately from that of the

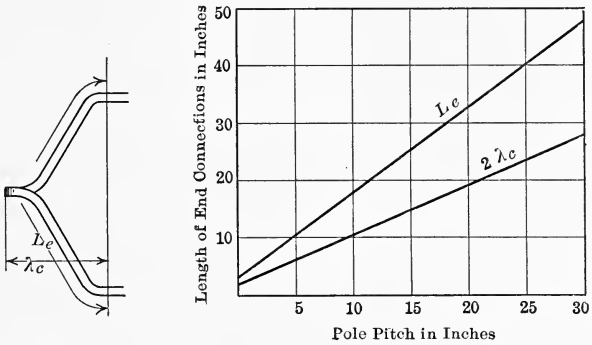


FIG. 84.—Dimensions of coils.

core because the kind of radiating surface is different and also the manner in which it is cooled.

Since the resistance of copper is 1 ohm per circular mil per inch the copper loss in the end connections of one conductor

$$= \frac{L_e}{M} I_c^2 \text{ watts},$$

and the total copper loss in the end connections

$$= Z \frac{L_e}{M} I_c^2 \text{ watts}.$$

The surface by which this loss is dissipated

$$= \pi D_a \times 2\lambda_c \text{ sq. in.} \quad \text{See Fig. 84.}$$

$$= \pi D_a \times \frac{L_e}{1.6} \text{ sq. in.}$$

since L_e, as shown in Fig. 84, is approximately equal to $1.6 \times 2\lambda_c$ for standard machines, therefore the watts per square inch

$$= \frac{ZL_e}{M} I_c^2 \times \frac{1.6}{\pi D_a L_e}$$

$$= \frac{\text{Amp. cond. per in.}}{\text{Circular mils per ampere}} \times \text{a constant.}$$

The temperature rise of the end connections is proportional to the watts per square inch of radiating surface, it is therefore proportional to the ratio $\frac{\text{amp. cond. per in.}}{\text{cir. mils per amp.}}$ and is found from the curves in Fig. 85. Two curves are given, one for machines of large diameter, such as that shown in Fig. 28, and the other for machines of small diameter, such as that shown in Fig. 27. For a given number of watts per square inch a large machine runs cooler than does a small one, because its windings are not so cramped on the ends and the free passage of air into the machine is not restricted by bearing housings.

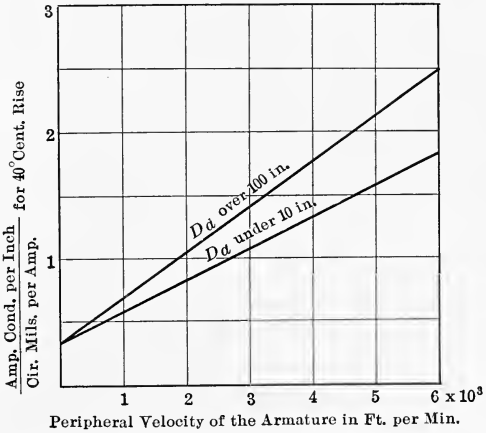


FIG. 85.—Temperature rise of armature coils.

94. Temperature Gradient in the Copper Conductors.—*ab*, Fig. 86, is a conductor which is carrying current and which is embedded in, and insulated from, the iron core. In order to have some idea of the temperature of the copper in the center of the core, consider the following two propositions.

(*a*) Assume that the slot insulation is much thicker than that on the end connections, then the heat in the embedded portion of the winding has to be conducted along the copper and dissipated at the end connections. It is required to find the difference in temperature between *c* and *a*.

The resistance of copper is 1 ohm per circular mil per inch so that the energy crossing $dx = I_c^2 R_x$

$$= I_c^2 \frac{x}{M}$$

where I_c is the current in the conductor in the slot;

R_x is the resistance of length x of the conductor;

M is the section of the conductor in circular mils;

The difference in temperature between two faces a distance dx apart $= \left(I_c^2 \frac{x}{M}\right) \frac{dx}{A} \times \frac{1}{11.1}$ deg. C.

where A is the section of the conductor in square inches

$$= \frac{M}{1.27 \times 10^6}$$

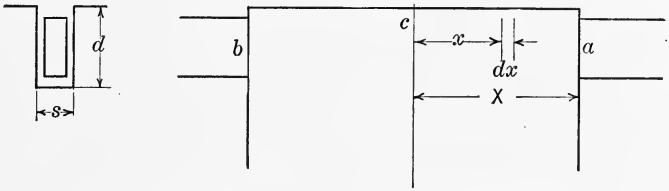


FIG. 86.—Heat paths in an armature conductor.

11.1 is the thermal conductivity of copper in watts per 1 in. cube per 1° C. difference in temperature.

The difference in temperature between the center c and any point $X = \left(I_c^2 \frac{1}{M}\right) \int_0^X \frac{x dx}{A} \times \frac{1}{11.1}$

$$= \frac{I^2_c \times 1.27 \times 10^6}{M^2} \times \frac{X^2}{2} \times \frac{1}{11.1}$$

$$= \frac{5.7 \times 10^4 \times X^2}{(\text{cir. mils per amp.})^2}. \qquad (17)$$

If, for example, the core of the machine is 20 in. long, so that the distance from the center of the core to the end is 10 in., and the circular mils per ampere is 500, then the difference in temperature between the copper at c and that at the end connections = 23° C.

(*b*) Consider now the other case in which it is assumed that the end connections are already so hot that the heat generated in the embedded conductors has to be transmitted through the slot insulation and dissipated by the iron core; it is required to

find the difference in temperature between the copper and the surrounding iron.

The copper loss per 1 in. axial length of slot

$$= \frac{\text{cond. per slot} \times I_c^2}{M}$$

$$= \frac{\text{amp. cond. per slot}}{\text{cir. mils per amp.}}$$

If this heat passes through the insulation then the difference in temperature between the inner and outer layers of the insulation

$$= \left(\frac{\text{amp. cond. per slot}}{\text{cir. mils per amp.}}\right) \times \frac{\text{thickness of insulation}}{2d+s} \times \frac{1}{0.003}. \quad (18)$$

where 0.003 is the thermal conductivity of ordinary paper and cloth insulation in watts per inch cube per 1° C. difference in temperature, and $2d+s$ is the area of the path per 1 in. axial length of slot.

Take for example the following figures:

Ampere conductors per inch $=760$
Slot pitch $=0.88$ in.
Ampere conductors per slot $=760 \times 0.88 = 670$
Circular mils per ampere $=560$
d $=1.6$ in.
s $=0.43$ in.
Thickness of insulation $=0.07$ in., including clearance
Temperature difference between inner and outer layers of insulation

$$= \frac{670}{560} \times \frac{0.07}{3.63} \times \frac{1}{0.003}$$

$=8$ deg. C.

95. Commutator Heating.—The modern D. C. armature is constructed as shown in Fig. 27; the commutator is smaller in diameter than the armature core, and the commutator necks which join the armature winding to the commutator are separated from one another by air spaces, so that when the armature revolves an air circulation is set up as shown by the arrows in Fig. 87, and cool air is drawn over the commutator surface.

The relation between permissible watts per square inch of commutator surface and commutator peripheral velocity, obtained from tests on non-interpole machines, is shown in Fig. 87.

The radiating surface is taken as $\pi D_c F$. It would seem that the radiating surface ought to include that of the commutator necks. It is found, however, that a considerable portion of the commutator neck, such as from a to b, can be closed up without

affecting the commutator ventilation or temperature to any great extent; just how much of the surface of the necks should be considered as radiating surface has not yet been determined experimentally.

The heat to be dissipated is assumed to be due to the brush friction and contact resistance losses. There are also losses which cannot be measured, due to poor commutation and to brush chattering. If the commutation is poor and the brushes

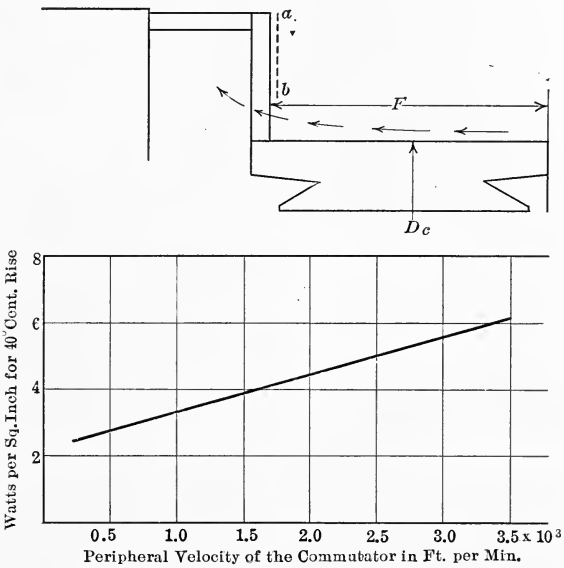


FIG. 87.—Temperature rise of commutators.

chatter badly the temperature rise may be higher than that obtained by the use of the curve in Fig. 87, while in cases where the commutation is exceedingly good and where there is no chattering the temperature rise may be lower.

96. Application of Heating Constants.—When designing a new D. C. machine for a guaranteed temperature rise of 40° C. the core heating is limited by keeping the flux densities below the values given in Art. 92, page 107, and the end connection heating is limited by keeping the ratio $\frac{\text{amp. cond. per in.}}{\text{cir. mils per amp.}}$ below the values given in Fig. 85, page 109. The approximate increase in temperature of the copper at the center of the core over the temperatures of the iron and of the end connections is found by

the use of the formulæ in Art. 94. The design is then compared with designs on similar machines which have already been tested and the densities modified accordingly.

The results of careful tests on similar machines should always take precedence over results obtained from average curves and these curves should always be changing as improvements are made in the methods of ventilation.

CHAPTER XII

PROCEDURE IN ARMATURE DESIGN

97. The Output Equation

$$E = Z\phi_a \frac{\text{r.p.m.}}{60} \times \frac{\text{poles}}{\text{paths}} 10^{-8} \text{ volts}, \quad \text{Formula 2, page 11}$$

$$= Z(B_g \psi L_c \tau) \frac{\text{r.p.m.}}{60} \times \frac{\text{poles}}{\text{paths}} 10^{-8} \text{ volts}$$

and $$q = \frac{ZI_a}{\text{paths}} \times \frac{1}{\pi D_a}$$

therefore $$EI_a = Z(B_g \psi L_c \tau) \frac{\text{r.p.m}}{60} \times \frac{\text{poles}}{\text{paths}} 10^{-8} \times \frac{q \times \text{paths} \times \pi D_a}{Z}$$

$$= B_g \times \tau p \times L_c \times \psi \times \text{r.p.m.} \times q \times \pi D_a \times \frac{10^{-8}}{60}$$

$$= B_g L_c \times \psi \times \frac{\text{r.p.m.}}{60} (\pi D_a)^2 \times 10^{-8} \times q$$

and $$D_a^2 L_c = \frac{\text{watts}}{\text{r.p.m.}} \times \frac{60.8 \times 10^7}{B_g \times \psi \times q} \qquad (19)$$

The value of B_g, the apparent average gap density, is limited by the permissible value of B_t, the maximum tooth density, which value, as shown in Art. 92, page 107, is about

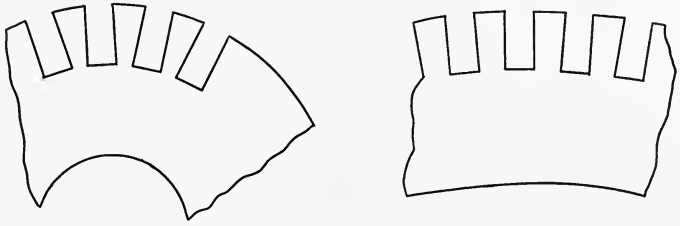


FIG. 88.—Effect of the armature diameter on the tooth taper.

150,000 lines per square inch for frequencies up to 30 cycles. That B_g also depends on the diameter of the machine may be seen from Fig. 88; the smaller the diameter the greater the tooth taper and therefore the lower the gap density for a given density at the bottom of the teeth.

Fig. 89 shows the relation between B_g and D_a for average machines; in cases where the frequency is greater than 30 cycles per second slightly lower values of B_g must be used.

The value of q, the ampere conductors per inch, is limited partly by heating and partly by commutation. Suppose that

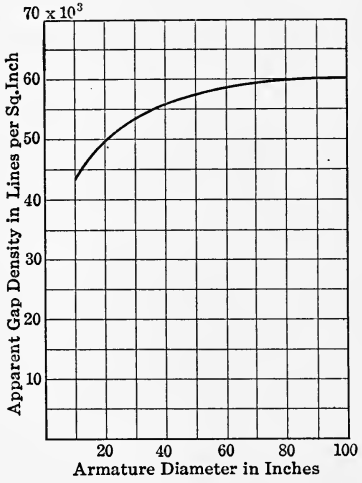


Fig. 89.

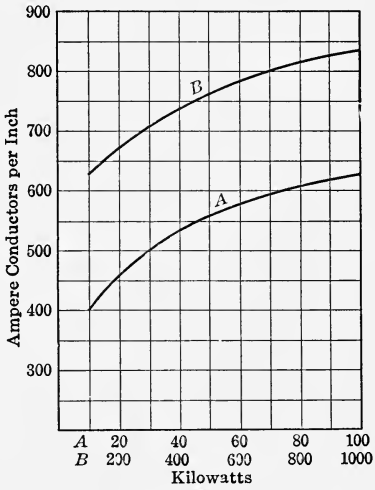


Fig. 90.

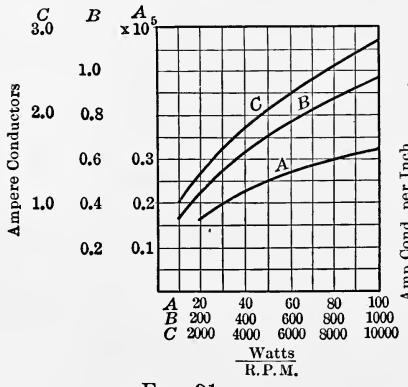


Fig. 91.

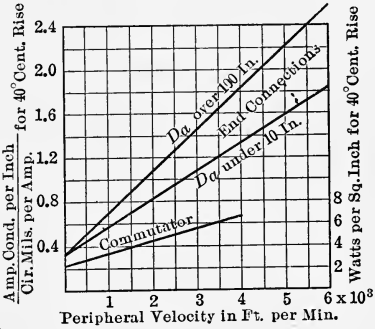


Fig. 92.

Figs. 89–92.—Curves used in preliminary design.

for a given rating the value of q is increased, which can be done by increasing the number of conductors in the machine or by decreasing the diameter. If the number of conductors be increased the reactance voltage will be increased, as may be seen

from formula 12, page 84. If the diameter is decreased the frame must be made longer in order to carry the flux and the slots must be made deeper in order to carry the larger number of ampere conductors on each inch of periphery, both of which changes increase the reactance voltage. In order, therefore, to keep the reactance voltage within reasonable limits it is necessary to limit the value of q.

It is found that q depends principally on the kw. output of the machine and the relation between q and kw. for average machines is plotted in Fig. 90; this curve may be used for preliminary design.

98. The Relation between D_a and L_c.—There is no simple method whereby $D_a{}^2L_c$ can be separated into its two components in such a way as to give the best machine, the only satisfactory method is to assume different sets of values of D_a and L_c, work out the design roughly for each case, and choose that which will give good operation at a reasonable cost.

99. Magnetic and Electric Loading.

$$E = Z\phi_a \frac{\text{r.p.m.}}{60} \times \frac{\text{poles}}{\text{paths}} 10^{-8} \quad \text{volts}$$

$$\text{therefore } EI_a = \frac{ZI_a}{\text{paths}} \times (\phi_a \times \text{poles}) \frac{\text{r.p.m.}}{60} 10^{-8} \quad \text{watts}$$

$$\text{and} \quad \frac{\text{watts}}{\text{r.p.m.}} = \left(\frac{ZI_a}{\text{paths}}\right)(\phi_a \times \text{poles}) \frac{1}{60 \times 10^8}. \tag{20}$$

The term $\frac{ZI_a}{\text{paths}}$ is called the *electric loading* and is the total number of ampere conductors on the periphery of the armature; the larger the value of this quantity the larger the amount of active copper and the smaller the amount of active iron there is in the armature.

The term ($\phi_a \times$ poles) is called the *magnetic loading* and is the total flux entering the armature; the larger its value the larger the amount of active iron and the smaller the amount of active copper there is in the machine.

To get the largest possible output from a given frame both the electric loading and the magnetic loading should be as large as possible.

There is a definite ratio between the magnetic and the electric loading which will give the cheapest machine.

$$\text{The ratio } \frac{\text{magnetic loading}}{\text{electric loading}} = \frac{\phi_a p}{ZI_c}$$

$$= \frac{B_g \phi \tau L_c p}{Z I_c}$$
$$= \frac{B_g \phi L_c}{q}$$

and therefore depends largely on the frame length L_c, which quantity is limited in the following way:

Figure 93 shows one field and one armature coil for a D. C. machine. A pole of circular section is the most economical so far as the field system is concerned because it has the largest area for the shortest mean turn of field coil. If the pole be rectangular in section then that with a square section has the largest area for the shortest mean turn. It will generally be found that the ratio $\frac{\text{pole pitch}}{\text{frame length}}$ lies between the values 1.1 and 1.7.

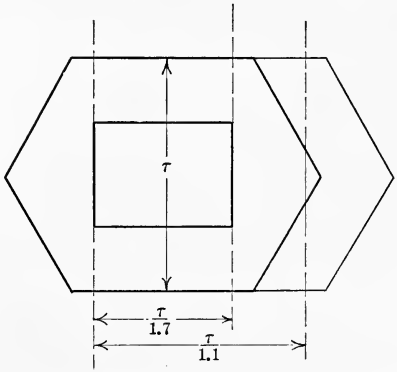


FIG. 93.—Shape of field and armature coils.

The pole pitch is limited by armature reaction; thus in Art. 74, page 91, it was shown that the armature ampere-turns per pole should not exceed 7500. Further, as pointed out in Art. 74, the ratio $\frac{\text{field ampere-turns per pole (gap + tooth)}}{\text{armature ampere-turns per pole}}$ is seldom less than 1.2. An increase in the armature m.m.f. per pole, therefore, requires a corresponding increase in the m.m.f. of the main field and an increase in the radial length of the poles. Rather than allow the armature ampere-turns per pole to exceed 7500, it will generally be found economical to increase the number of poles, so that they may not have too great a radial length.

For 7500 armature ampere-turns per pole, and 900 ampere conductors per inch, the pole pitch is approximately

$$= \frac{7500 \times 2}{900}$$
$$= 17 \text{ in.}$$

L_c therefore, as pointed out above, should not exceed $\frac{17}{1.1} = 15$ in., except in the special case where the peripheral velocity is already so high that the diameter cannot be increased and the rating can only be obtained by the use of an extra long armature.

If L_c, the frame length, $= 15$ in.
B_g, the average gap density, $= 60{,}000$ lines per square inch
ϕ, the pole enclosure, $= 0.7$
q, the ampere conductors per inch, $= 900$

then k, the ratio $\frac{\text{magnetic loading}}{\text{electric loading}} = 700$.

For a machine with a small armature diameter the most economical frame length will be less than 15 in.; for example, a machine 5 in. in diameter and 15 in. long would be more expensive and would give more trouble than one which had the same value of $D_a^2 L_c$, but a diameter of 9 in. and a frame length of 4.5 in., so that, since k depends largely on the length L_c its value varies with the diameter of the machine. Now

$$\frac{\text{watts}}{\text{r.p.m.}} = \text{electric loading} \times \text{magnetic loading} \times \frac{1}{60 \times 10^8}$$
$$= k\,(\text{electric loading})^2 \times \text{a constant} \qquad (21)$$

and for each value of $\frac{\text{watts}}{\text{r.p.m.}}$ there is a value of k, and therefore of electric loading (ZI_c), which gives the most economical machine. Fig. 91 shows the relation between these two quantities for a line of non-interpole machines, and this curve may be used for preliminary design.

It must be understood, and will be seen from the examples given later, that the value of k may vary over a considerable range without affecting the cost of the machine to any considerable extent. The value of k is also affected by the cost of labor and therefore varies under different conditions of manufacture.

100. Formulæ for Armature Design.

$A = \frac{\text{watts}}{\text{r.p.m.}} = k(\text{electric loading})^2 \times \text{a constant}$, formula 21, page 118.

where electric loading $= (ZI_c)$.

B — $D_a^2 L_c = \dfrac{\text{watts}}{\text{r.p.m.}} \times \dfrac{60.8 \times 10^7}{B_g \psi q}$ formula 19, page 114.

C — $\dfrac{\text{pole pitch}}{\text{frame length}} = 1.1 \text{ to } 1.7$ Art. 99, page 117.

D — $E = Z\phi_a \dfrac{\text{r.p.m.}}{60} \times \dfrac{\text{poles}}{\text{paths}} \times 10^{-8}$ formula 2, page 11.

E — Coils $= k\dfrac{p}{2} \pm 1$ for a series winding Art. 22, page 22.

Slots $= k\dfrac{p}{2} \pm 1$ for a series winding Art. 22, page 22.

$= k\left(\dfrac{p}{2}\right)$ for a multiple winding with equalizers Art. 22, page 22.

F — Slots per pole should be
greater than 12 for small machines
greater than 14 for large machines Art. 71, page 87.

G — Reactance voltage $= k \times S \times$ r.p.m. $\times I_c \times L_c \times T^2 \times \dfrac{\text{poles}}{\text{paths}} 10^{-8}$
(formula 12, page 84.)
where k $= 1.6$ for series and full-pitch multiple windings;
$= 0.93$ for short-pitch multiple windings.

H — Reactance voltage $= 0.7$ (volts per pair of brushes) when the brushes are on the neutral;
$= 1.0$ (volts per pair of brushes) when the brushes are shifted from the neutral;
Art. 73, page 90.

The reactance voltage may be 20 per cent. greater than this for machines with series windings and must be 30 per cent. less for those with short-pitch multiple windings.

J — Armature ampere-turns per pole $= \dfrac{ZI_c}{\pi D_a}$ and should be less than 7,500 Art. 74, page 91.

K — $\dfrac{\text{maximum tooth width}}{\text{slot width}} = 1.1$ for large machines;
$= 1.0$ for small machines;
these are approximate values found from practice

L — Flux density is taken from the following table

Cycles per second	Tooth-density lines per square inch	Core-density lines per square inch
30	150,000	100,000
40	140,000	85,000
60	125,000	75,000

Art. 92, page 107.

M — Commutator dia. $=0.6$ armature dia. for large machines;
$=0.75$ armature dia. for small machines.
These are values taken from practice. Except in the case of turbo generators the peripheral velocity of the commutator should not, if possible, exceed 3500 ft. per minute. To keep below this peripheral velocity it may be necessary to use smaller values for the diameter than those given above.

N — Wearing depth of the commutator, namely, the amount that can be turned of the radius without making the commutator too weak mechanically varies from 0.5 in. on a 5-in. commutator to 1.0 in. on a 50-in. commutator.

P — Brush arc should be less than $\frac{\text{pole pitch}}{12} \times \frac{\text{comm. dia.}}{\text{arm. dia.}}$
and should not cover more than 3 segments
Art. 72, page 89.

Q — Watts per square inch brush contact $=35$ approximately
Art. 64, page 78.

R — Brush friction $=1.25\ A\ \frac{V_r}{100}$ watts
formula 14, page 98.

101. Preliminary Design.—To simplify the work of preliminary design the necessary formulæ and curves are gathered together above and the method of procedure is as follows:

Find the electric loading ZI_c for the given rating, from Fig. 91.

Find $q = \frac{ZI_c}{\pi D_a}$ from Fig. 90.

From these two quantities find D_a the armature diameter.

Tabulate three preliminary designs; one for a diameter 20 per cent. larger than that already found and the other 20 per cent. smaller.

Find B_g, the apparent gap density, from Fig. 89.

Find L_c, the frame length, from formula *B*, page 119.

Find p, the number of poles, which should have such a value that $\frac{\text{pole pitch}}{\text{frame length}}$ lies between the values 1.1 and 1.7. Art. 99, page 117.

Find ϕ_a, the flux per pole, $= B_g \psi \tau L_c$ where $\psi = 0.7$ approx.

Find Z, the number of armature face conductors, from formula *D*.

Choose the cheapest winding that will give a reactance voltage below the desired limit; a series winding is generally the cheapest because it requires the smallest number of coils and commutator segments.

Find S, the number of commutator segments, from the value of Z and the type of winding used.

Make D_c, the commutator diameter, = (0.6 to 0.75) × (armature diameter) for a first approximation.

Find the brush arc from formula *P*.

Find the commutator length so that the watts per square inch brush contact shall not exceed 35.

Choose between the different designs.

Example.—Determine approximately the dimensions of a D.-C. generator of the following rating:

400 kw., 240 volts, 1670 amperes, 200 r.p.m.

The work is carried out in tabular form as follows:

Ampere conductors		1.33×10^5, from Fig. 91		
Ampere conductors per inch		733, from Fig. 90		
Armature diameter		58 in.	45 in.	70 in.
Apparent gap density	B_g	58,000	56,500	59,000, from Fig. 89
Frame length	L_c	12 in.	20.5 in.	8 in., formula *B*
Poles	p	10	8	16, formula *C*
Pole pitch	τ	18.2 in.	17.6 in.	13.8 in.
Flux per pole	ϕ_a	8.8×10^6	14.3×10^6	4.55×10^6
Total face conductors	Z	820	505	1590; formula *D*
Winding		one-turn multiple, short-pitch		
Commutator segments	S	410	252	795
Reactance voltage	RV.	1.5	2.0	1.2, formula *G*
Commutator diameter	D_c	35 in.	27 in.	42 in., formula *M*
Brush arc		0.91 in.	0.88 in.	0.69 in., formula *P*
Brush length		10.5 in.	13.5 in.	8.5 in.
Amperes per square inch of brush contact		35	35	35
$\frac{\text{Magnetic loading}}{\text{Electric loading}} = \frac{\phi_a p}{ZI_c}$		640	1090	440

The second machine, which has an armature diameter of 45 in., has the largest flux per pole and therefore the deepest core and the heaviest yoke. It is the longest machine and therefore the most expensive in core assembly. It has the smallest number of coils and commutator segments and is therefore cheapest in winding and commutator assembly.

The third machine, which has an armature diameter of 70 in., has the smallest flux per pole and therefore the shallowest core and the lightest yoke. It is the shortest machine and therefore the cheapest in core assembly. It has the largest number of coils and commutator segments and is therefore the most expensive in winding and commutator assembly.

The first machine probably costs less than either of the other two; it has also a comparatively low reactance voltage and should commutate satisfactorily.

Before completing the design with the 58 in. diameter it is advisable to design the machine roughly with different numbers of poles in the following way:

Poles	8	10	12
Armature diameter	58 in.	—	—
Frame length	12 in.	—	—
Apparent gap density	58,000	—	—
Pole pitch	22.8 in.	18.2 in.	15.2 in.
Flux per pole	11.0×10^6	8.8×10^6	7.4×10^6
Total face conductors	658	820	975
Winding	one-turn multiple, short-pitch		
Reactance voltage	1.5	1.5	1.5
Armature ampere-turns per pole	8550	6900	5700
Commutator diameter	35 in.	35 in.	35 in.
Brush arc	1.14 in.	0.91 in.	0.76 in.
Brush length	10.5 in.	10.5 in.	10.5 in.

The first machine is most expensive in material due to the large flux per pole and therefore the deep core and the large section of yoke.

The third machine is most expensive in labor due to the number of coils and commutator segments that are required.

The reactance voltage is the same in each case but the armature ampere-turns per pole is greatest in the first machine and least in the last, so that, so far as commutation is concerned, the last machine is to be preferred, and the first one should not be used if possible.

Armature Design.—Having determined the approximate

dimensions of the machine, it is now necessary to design the armature in detail, which is done in tabular form in the following way:

Choose the 10-pole design as the most suitable, then
External diameter of armature .....58 in. from preliminary design
Frame length .....12 in. from preliminary design
Center vent ducts .....3—0.5 in. wide
Gross iron in frame length .....10.5 in.
Net iron in frame length .....9.45 in.
Poles 10
Pole pitch .....18.2 in.
Probable flux per pole .....8.8×10^6 from preliminary design
Probable number of face conductors....820 from preliminary design

Winding. The minimum number of slots per pole =14, formula F, therefore the minimum number of total slots =140, and the nearest suitable number is 200

Conductors per slot	=4
Coils	=400
Commutator segments	=400
Winding	= one turn multiple, short pitch
Reactance voltage	=1.5, formula G

$$\text{Ampere conductors per inch} = q = \frac{167\times800}{\pi\times58} = 730$$

$$\frac{\text{Amp. cond. per inch}}{\text{Cir. mils per amp.}} = 1.3 \text{ for } 40^\circ \text{ C. rise, from Fig. 92}$$

Circular mils per ampere 560
Amperes per conductor at full load 167
Section of conductor $=167\times560$ =93.500 circular mils
=0.073 sq. in.

$$\text{Slot pitch} = \frac{\pi\times58}{200} = 0.91 \text{ in.}$$

$$\text{Probable slot width} = \frac{0.91}{2.1} = 0.43 \text{ in., formula } K$$

0.43 assumed slot width
0.064 width of slot insulation, see page 39
0.04 clearance between coil and core
0.326 available width for copper and insulation on conductors.

Use strip copper in the slot as shown in Fig. 34 and put two conductors in the width of the slot; make the strip 0.14 in. wide and insulate it with half lapped cotton tape 0.006 in. thick.

$$\text{Depth of conductor} = \frac{0.073}{0.14} = 0.52 \text{ in.}$$; increase this to 0.55 in. to allow for rounding of the corners.

Slot depth is found as follows:

0.55 depth of each conductor
0.024 insulation thickness on each conductor
0.084 depth of slot insulation, see page 39.

0.658 depth of each insulated coil
2 number of coils in depth of slot

1.316 depth of coil space
0.2 thickness of stick in top of slot

1.516 necessary depth of slot; make it 1.6 in. deep.

Diameter at bottom of slot.....54.8 in.
Slot pitch at bottom of slot.....0.86 in.
Minimum tooth width.....0.43 in.

Tooth area per pole $=\frac{200}{10}\times 0.7\times 0.43\times 9.45=57$ sq. in.

Flux per pole with 800 conductors.....9×10^6 for 240 volts, formula *D*-
Maximum tooth density.....158,000 lines per square inch.
this density is not too high so that the core does not need to be lengthened.

Flux density in the core, assumed.....85,000 lines per square inch

Core area $=\frac{9\times 10^6}{2\times 85{,}000}$.....53 sq. in.

Core depth $=\frac{\text{core area}}{\text{net iron}}$.....5.6 in.

Internal diameter of armature.....43.6 in.

The above data is now filled in on the armature design sheet shown on page 125.

Commutator Design.

Commutator diameter, assumed to be 0.6 (armature diameter) =35 in.
Number of commutator segments.....400

Width of one segment and mica $\frac{\pi\times 35}{400}=0.275$ in.

Brush arc.....0.91 in., formula *P*.
Use a brush 0.75 in. thick set at an angle of 30°, so that the brush arc =0.87 in.

Segments covered by brush.....3.1

Amperes per set of brushes $=\frac{\text{total current}}{\text{poles}}\times 2=334$

Amperes per square inch of brush contact, 35 assumed
Necessary brush length.....11 in.
Brushes per stud, use 6 brushes, each 0.75 in. × 1.75 in.
Commutator length $=6(1.75+0.25)+1=13$ in.
allow 0.25 in. between brushes and 1 in. additional clearance
Peripheral velocity of commutator.....1830 ft. per minute.
Total brush contact area $=10\times 6\times 1.75\times 0.87=91$ sq. in.
Commutator friction loss.....2100 formula *R*.
Volts drop per pair of brushes.....2.5 formula *H*.

Contact resistance loss $= 2.5 \times 1670 = 4200$ watts

Watts per square inch of commutator surface $= \frac{4200 + 2100}{\pi \times 35 \times 13}$
$= 4.4$

Probable temperature rise on commutator $= 40°$ C., from Fig. 92.

Had the commutator temperature rise come out too high, it would have been necessary to have used brushes with lower contact resistance, which might cause the commutation to be poor, or else to have increased the commutator radiating surface.

The above data is now filled in on the design sheet shown below.

102. Armature and Commutator Design Sheet.

Armature

External diameter.....58 in.
Internal diameter.....3.5 in.
Frame length.....12 in.
End ducts.....2–1/2 in.
Center ducts.....3–1/2 in.
Gross iron.....10.5 in.
Net iron.....9.45 in.
Slots, number.....200.
size....0.43 in. × 1.6 in.
Cond. per slot, number.....4.
size...0.14 in. × 0.55 in.
Coils.....400.
Turns per coil.....1.
Total conductors.....800.
Winding, type.....multiple.
pitch.....1–20.
Slot pitch.....0.91 in. − 0.86 in.
Tooth width.....0.48 in. − 0.43 in.
Max. tooth width / Slot width .....1.11.
Core depth.....5.65 in.
Pole pitch.....18.2 in.
Per cent. enclosure.....0.7.
Min. tooth area per pole....57 sq. in.
Core area per pole.....53 sq. in.
Apparent gap area per pole 153 sq. in.

Densities, Iron loss, Excitation

Flux per pole, no-load.....9×10^6.
Maximum tooth density (apparent), 158,000 lines per square inch.
Maximum tooth density (actual), 150,000 lines per square inch.
Core density, 85,000 lines per square inch.

Commutator

Diameter.....35 in.
Face.....13 in.
Bars.....400.
Bar and mica.....0.275 in.
Mica.....0.03 in.
Wearing depth.....1 in.
Brush arc.....0.87 in.
Brush studs.....10.
Brushes per stud.....6 ($\frac{3}{4}'' \times 1\frac{3}{4}''$).
Amperes per square inch contact.37.
Peripheral velocity ft. per min., 1830.
Friction loss, watts,.....2100
Volts per pair of brushes,.....2.5.
Contact resistance loss, watts,.4200.
Watts per square inch surface..4.4.
Temperature rise, deg. C.....40.
Average volts per bar.....6.
Cross-connect every fourth coil.
Size of cross-connectors; half the conductor section.

Copper loss, Commutation

Amp. cond. per inch.....730.
Circular mils per ampere.....560.
Length of conductor.....42 in.
Copper loss.....10 kw.
Volts drop in armature $= \frac{\text{loss}}{I_a} = 6$ (formula 15, page 103).
Armature AT. per pole.....6700.
Ratio $\frac{\text{AT (gap + tooth)}}{\text{arm. AT. per pole}}$.....1.24.
Reactance voltage.....1.5.

Gap density (apparent), 59,000 lines per square inch.
Weight teeth.....385 lb.
core.....2300 lb.
Frequency, cyc. per second, ...16.6.
Iron loss, 6450 watts (Art. 84, page 102).
Air gap clearance.....0.3 in.
Carter coefficient.....1.12.
AT. gap, Art. 46.....6200.
AT. tooth, Art. 46.....2080.

Rating

Kilowatt.....400.
Volts no-load.....240.
Volts full-load.....240.
Amperes.....1670.
R. p. m.....200.
$\frac{\text{Magnetic loading}}{\text{Electric loading}}$.....670.
Output factor $= \frac{\text{KW} \times 10^5}{\text{r.p.m.} \times D_a^2 L_c} = 5.$

CHAPTER XIII

MOTOR DESIGN AND RATINGS

103. Procedure in Design.—The design of a D.-C. motor is carried out in exactly the same way as that of a D.-C. generator.

Example.—Determine approximately the dimensions of a D.-C. shunt motor of the following rating:

30 h.p., 120 volts, 900 r.p.m.

Probable efficiency	=90 per cent.
Full-load current	=208 amperes.
Kilowatt input	=25.
Ampere conductors	$=0.19\times10^5$, from Fig. 91.
Ampere conductors per inch	=480, from Fig. 90.
Armature diameter	=12.5 in.
Apparent gap density, B_g	=44,000, from Fig. 89.
Frame length, L_c	=7.3, from formula B, page 119.
Poles	=4, from formula C.
Pole pitch	=9.8 in.
Flux per pole	$=2.2\times10^6=B_g\phi\tau L_c$.
Total face conductors	=364 for multiple winding.
	=182 for series winding, formula D.

Reactance voltage =1.0 for a full-pitch multiple winding with one turn per coil,
=2.0 for the same winding with two turns per coil,
=2.0 for a series winding with one turn per coil, formula G.

Winding: The series winding is the cheapest since it requires half as many coils and commutator bars as are required for the one-turn multiple winding, and it does not require equalizers as in the case of the two-turn multiple winding.

Commutator diameter =0.75×Armature diameter	=9.5 in., formula M.
Brush arc	=0.61 in.
Brush length	=5.0 in.
Amperes per square inch of brush contact	=35.

While working on the preliminary design it is desirable to find the probable pole area and see if a circular section is suitable for the particular diameter and frame length that have been chosen.

Flux in pole	$=\phi_a\times$leakage factor.
	$=2.2\times10^6\times1.25$ approximately.
	$=2.75\times10^6$ approximately.

Pole area, assuming a pole density of 95,000 lines per square inch

$$= \frac{2.75 \times 10^6}{95000}$$

$= 29$ sq. in.

Pole diameter for a circular section $= 6$ in. which is suitable for a frame length of 7.3 in.

Armature Design.

External diameter	$= 12.5$ in. from preliminary design.
Frame length	$= 7.3$ in. from preliminary design.
Center vent ducts	$= 2 - 0.375$ in. wide, Art 23, page 24.
Gross iron in frame length	$= 6.55$ in.
Net iron in frame length	$= 5.9$ in.
Probable number of face cond.	$= 182$ from preliminary design.

Winding: The minimum number of slots per pole = 12, formula *F*, page 119, therefore:

The minimum number of total slots $= 48$.

The number of slots should $= k\frac{p}{2} \pm 1$ for a series winding, see Art. 22, page 22, and the nearest suitable number of slots $= 49$.

Conductors per slot	= 4.
Coils	= 98 with one dead, see Art. 22, page 22.
Commutator segments	= 97.
Winding	= one-turn series.
Reactance voltage	= 2.1.
Active conductors	= 194.

The remainder of this design is worked out in exactly the same way as that for the generator in Art. 101, page 120, and the final result is tabulated below.

104. Armature and Commutator Design Sheet.

Armature		*Commutator*	
External diameter	12.5 in.	Diameter	9.5 in.
Internal diameter	6.0 in.	Face	6.75 in.
Frame length	7.0 in.	Bars	97.
End ducts	none.	Bar and mica	0.308 in.
Center ducts	2-3/8 in.	Mica	0.03 in.
Gross iron	6.25 in.	Wearing depth	0.5 in.
Net iron	5.6 in.	Brush arc	0.58 in.
Slots, number	49	Brush studs	4.
size	0.4 in. × 0.92 in.	Brushes per stud	3(1/2 in. × 1 3/4 in.).
Conductor per slot, number	4.	Amperes per square inch contact,	35.
size	0.12 in. × 0.32 in.	Peripheral velocity ft. per min.	2240.
Coils	98 (1 dead).	Friction loss, watts	340.
Turns per coil	1.	Volts per pair of brushes	2.0.
Total conductors	194.	Contact resistance loss, watts	416.
Winding, series; pitch	1–13.	Watts per square inch surface	3.75.
Slot pitch	0.8 in. and 0.685 in.	Temperature rise	35° C.
Tooth width	0.4 in. and 0.285 in.		

Armature

Maximum tooth width / Slot width	1.0.
Core depth	2.33 in.
Pole pitch	9.8 in.
Per cent. enclosure	0.7.
Min. tooth area per pole	13.7 sq. in.
Core area	13 sq. in.
Apparent gap area per pole	48 sq. in.

Densities, Iron loss, Excitation

Flux per pole	2.07×10^6.
Maximum tooth density (apparent), 150,000 lines per square inch.	
Maximum tooth density (actual), 142,000 lines per square inch.	
Core density, 80,000 lines per square inch.	
Apparent gap density, 43,000 lines per square inch.	
Weight, teeth	24 lb.
core	96 lb.
Frequency	30 cycles.
Iron loss	630 watts.
Air gap clearance	3/16 in.
Carter coefficient	1.2.
AT. gap	3000.
A.T. tooth	500.

Commutator

Average volts per bar	5.0.
Cross connect	none.
Size of cross-connectors	none.

Copper loss, Commutation

Amperes conductors per inch	515.
Circular mils per ampere	470.
Length of conductor	25 in.
Copper loss, watts	1070.
Volts drop in armature	5.1.
Armature AT. per pole	2600.
Ratio $\frac{\text{AT (gap + tooth)}}{\text{arm. AT. per pole}}$	1.35.
Reactance voltage	2.05.

Rating

H.P.	30.
Volts	120.
Amperes	208.
R.p.m.	900.
Magnetic loading / Electric loading	400.
Output factor	2.5.

105. Motor Ratings for Different Voltages and Speeds.—An open type shunt motor rated at 30 h. p., 120 volts, 208 amperes, 900 r.p.m. is built as follows:

Armature

External diameter	12.5 in.
Frame length	7.0 in.
Slots, number	49.
size	0.4 in. $\times$ 0.92 in.
Conductors per slot, number	4
size	0.12 in. $\times$ 0.32 in.
Coils	98, 1 dead.
Winding, one-turn series	(1.T.S.).

Commutator

Diameter	9.5 in.
Face	6.75 in.
Bars, number	97.

It is required to design armatures for the following open shunt ratings:

500 volts, 950 r.p.m.
220 volts, 1200 r.p.m.
220 volts, 600 r.p.m.

Find the Windings.—In order to use the same poles, yoke and armature parts, it is necessary that the flux per pole be the same in each case.

$$E = Z\phi_a \frac{\text{r.p.m.}}{60} \times \frac{\text{poles}}{\text{paths}} \times 10^{-8}$$

$$= \text{a constant} \left(\frac{Z \times \text{r.p.m.}}{\text{paths}} \right)$$

By the use of the above formula the work is carried out in tabular form as follows:

	E	R.P.M.	Winding	Paths	Total Conductors	Slots	Conductors per slot	Commutator segments
A	120	900	1 T. S. (1 dead)	2	194 active	49	4	97
B	500	950	1 T. S. (1 dead)	2	782 active	49	16	391
			2 T. S. (1 dead)	2	780 active	49	16	195
C	220	1200	1 T. S.....	2	265 active	53	5	cannot wind
			1 T. M.....	4	530 active	53	10	265
D	220	600	1 T. S.....	2	530 active	53	10	265

In design *B*, the number of commutator segments with a one-turn series winding is 391 and the thickness of one copper and one mica segment taken together = 0.077 in. for a commutator 9.5 in. diameter. This value is rather small and a commutator with such a large number of thin segments is liable to develop high bars.

With a two-turn series winding, half the number of segments are required, but the reactance voltage is twice as large so that the machine will probably require high resistance brushes; this will have to be decided after the current rating has been found and the reactance voltage determined.

In design *C*, 265 conductors are required with a one-turn series winding, so that 49 slots cannot be used and a new armature punching must be designed. The minimum number of slots per pole is 12 and of total slots is therefore 48. A winding with 65 slots and four conductors per slot might be used but in that case the slot would be very narrow and the tooth very weak. A one-turn series winding with 53 slots and five conductors per

slot cannot be used since the number of conductors per slot must be a multiple of 2 for a double layer winding; it is possible, however, to use 53 slots with a one-turn multiple winding and 10 conductors per slot.

Find the Size of Slot.—In order that the tooth density be the same in each case it is necessary that the slot area per pole be unchanged. The original machine had 49 slots, 0.4 in. wide; when 53 slots are used they must be made narrower and

$$= \frac{0.4 \times 49}{53}$$

$$= 0.37 \text{ in. wide.}$$

The depth of slot is kept the same and therefore the core density is unchanged.

Find the Size of Conductor.—In order to get the largest possible rating out of the machine the size of conductor should be the largest that can be got into the slot without making the coil too tight a fit.

The size of conductor is given in the following table:

Slot size	Conductors per slot	Size of conductor
0.4 ×0.92	4	0.12 in.×0.32 in.
0.4 ×0.92	16	No. 11 B. & S.
0.37×0.92	10	No. 9 B. & S.
0.37×0.92	10	No. 9 B. & S.

Find the Ampere Rating.—The current in each conductor should be such that the ratio $\frac{\text{amp. cond. per in.}}{\text{cir. mils per amp.}}$ shall not exceed the value found from Fig. 92, page 115, and the work is carried out in tabular form as follows:

R.P.M.	Perip. velocity	$\frac{\text{amp. cond. per in.}}{\text{cir. mils per amp.}}$	Total conductors	Size of conductors
900	2950 ft. per min.	1.07 from Fig. 92	194	0.12″×0.32″
950	3100 ft. per min.	1.12 from Fig. 92	780	No. 11 B. & S.
1200	3920 ft. per min.	1.32 from Fig. 92	530	No. 9 B. & S.
600	1960 ft. per min.	0.83 from Fig. 92	530	No. 9 B. & S.

Amperes per conductor	Ampere conductors per inch	Circular mils per ampere
104	515	470
21.5	430	380
36	490	365
28	380	470

The horse-power of the machine is found as follows:

Volts	Amperes	Assumed efficiency	Horse-power	Reactance voltage
120	208	0.89	30	2.05
500	43	0.89	25	3.6
220	144	0.89	37.5	1.3
220	56	0.87	14	1.0

Discussion of Ratings.—If the flux be constant the volts per conductor is directly proportional to the speed.

The total volume of current in the armature = the total cross-section of copper in the armature × the current density in that section.

The total input into the machine = volts per conductor × total current volume, and so is proportional to r.p.m. × total section of copper × current density, so that in the case of high voltage motors, where the total number of conductors is large and, therefore, the *space factor of the slot*—namely, the $\frac{\text{section of copper in the slot}}{\text{the total slot section}}$ is small—the value of the total section of copper is lower than in the case of a low voltage motor built on the same frame, and therefore the rating has to be lower.

In the case of very slow speed motors the number of conductors is generally large so as to get the desired voltage and the current density is low because of the poor ventilation; for these reasons motors have to be rated down more than in proportion to the speed.

106. Calculation of Efficiency.—It is required to calculate the

efficiency of the machine whose design data is given on page 128. Before this can be determined it is necessary to find the bearing friction and also the excitation loss.

The bearings measure 2.5 in. $\times$ 6.25 in.
The rubbing velocity at 900 r.p.m. = 590 ft. per minute.
The friction loss $= 2 \times 0.81 \times 2.5 \times 6.25 \times (5.9)^{3/2}$, formula 13, page 97.
= 360 watts.

The probable excitation loss may be found as follows:

Ampere-turns (gap + tooth) = 3000 + 500 = 3500, from design sheet.
The probable field excitation = $3500 \times 1.25 = 4400$ ampere-turns.

The length $L_f = \frac{\text{amp.-turns}}{1000} \sqrt{\frac{\text{mean turn}}{\text{ext. periphery} \times \text{watts per sq. in.} \times d_f \times sf \times 1.27}}$

(Formula 8, page 66) where

pole diameter	= 6 in. from preliminary design
d_f	= 2 in., assumed
mean turn	= 25 in.
external periphery	= 31 in.
watts per square inch	= 0.8, from Fig. 56
space factor	= 0.65, approximately, from Fig. 57

therefore L_f for 4400 ampere-turns = 3.5 in.

The excitation loss = external surface of coil $\times$ watts per square inch $\times$ number of coils
$= 31 \times 3.5 \times 0.8 \times 4$
= 350 watts.

The total losses therefore are

Bearing friction	= 360 watts.
Excitation loss	= 350 watts.
Iron loss	= 630 watts, from design sheet.
Armature copper loss	= 1,070 watts, from design sheet.
Commutator friction loss	= 340 watts, from design sheet.
Commutator resistance loss	= 416 watts, from design sheet.
Total loss,	3166 watts.
Output (= 30 h.p.)	= 22,400 watts.
Input	= 25,566 watts.
Efficiency	= 88 per cent.

107. Rating as an Enclosed Motor.—Experiment shows that in the case of a totally enclosed motor the temperature rise of the coils and core of the machine is proportional to the total loss (neglecting bearing friction which seems to be conducted along the shaft and dissipated by the pulley) and is independent of the distribution of this loss. The temperature rise is also found to vary inversely as the external radiating surface and to depend

on the peripheral velocity of the armature in the way shown in Fig. 94.

In the machine which has already been discussed in this chapter the external diameter of the yoke = 28 in.

The length of the frame axially = 24 in. approx.

The external surface = 3340 sq. in.; Fig. 94

The total loss neglecting bearing friction = 2800 watts at a load of 30 h.p.; Art. 106.

Watts per square inch = 0.85

Probable temperature rise = 80° C.

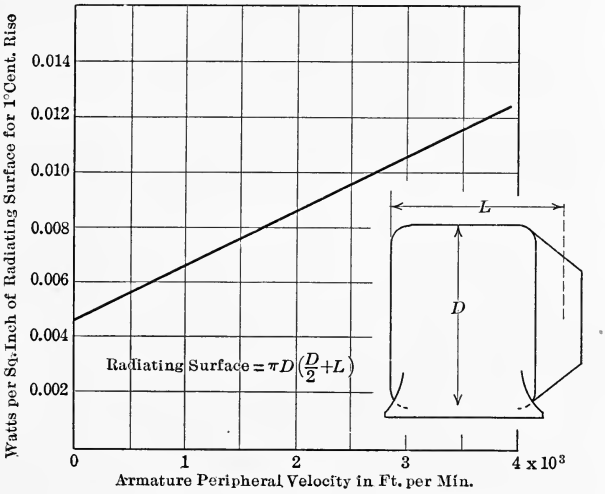


FIG. 94.—Heating curve for enclosed motors.

In order to lower the temperature rise it is necessary to lower the rating so as to cut down the losses. The horse-power is reduced so as to cut down the current, and the speed is increased so as to cut down the flux and, therefore, the core loss and the excitation loss.

In the machine under discussion let the horse-power be reduced 30 per cent. and the speed be increased 20 per cent., the losses will then be changed as follows:

Excitation loss is proportional to the ampere-turns and this will be reduced about 20 per cent., due to the decrease in flux.

Iron loss is found by the use of the curves in Fig. 81 for 20 per cent. lower densities and 20 per cent. higher frequency.

Armature copper loss is proportional to the $(\text{current})^2$ and is

therefore reduced to $\frac{1}{(1.3)^2}$ of its original value, due to the decrease in horse-power.

Brush friction loss will be reduced 30 per cent. because less brush area is required and will be increased 20 per cent. because of the increase in speed.

Contact resistance loss will be proportional to the current since the current density in the brush is kept constant and the volts drop per pair of brushes is unchanged.

The losses then will be as follows:

	30 h.p. 900 r.p.m.	23 h.p. 1080 r.p.m.
Excitation loss	350	290
Iron loss	630	580
Armature copper loss	1070	630
Commutator friction loss	340	315
Contact resistance loss	416	320
Total loss,	2806	2135
Watts per square inch for 1° C. rise	=0.0105	0.0118, from Fig. 94.
Square inch radiating surface	3340	3340
Watts per square inch	0.84	0.64
Probable temperature rise	80° C.	55° C.

108. Possible Ratings for a given Armature.—The following ratings are generally recognized by manufacturers of small motors and are given to show approximately what a motor can do under different conditions of operation.

Continuous Duty, Constant Speed, Open Shunt Motors.—For such machines the usual temperature guarantee is that—the temperature rise shall not exceed 40° C. by thermometer after a continuous full-load run, nor shall it exceed 55° C. after 2 hours at 25 per cent. overload immediately following the full-load run.

The guarantee for commutation is that the machine shall operate over the whole range from no-load to 25 per cent. overload without destructive sparking and without shifting of the brushes.

Continuous Duty, Constant Speed, Screen-covered Shunt Motors.—Due to the resistance of the perforated sheet metal that is used to close up all the openings in the machine to the free circulation of air, the temperature rise will be about 20 per cent. higher than that for the same machine operating as an open motor.

Continuous Duty, Constant Speed, Enclosed Shunt Motors.—For such machines the temperature guarantee is that—the temperature rise inside of the machine shall not exceed 65° C. by thermometer after a continuous full-load run. An overload temperature guarantee is seldom made.

To keep within this guarantee the standard open motor is generally given a 30 per cent. lower horse-power rating so as to reduce the armature current, and is run at 20 per cent. higher speed so as to reduce the flux per pole and therefore the iron loss and the excitation loss.

Elevator Rating, Open Compound Motor.—For such service the usual temperature guarantee is that—the temperature rise shall not exceed 45° C. after a full-load run for 1 hour and that, immediately after the full-load run, the machine shall carry 50 per cent. overload for 1 minute without injury.

To obtain this rating the standard open motor is rated up about 20 per cent. The compound field is generally made about 30 per cent. of the total field excitation at full-load so that the starting current will be less than it would be with a shunt motor of the same rating. For elevator service the brushes must be on the neutral position so that the motor can operate equally well in both directions.

Crane Rating, Totally Enclosed Series Motor.—For such service the usual temperature guarantee is that—the temperature rise inside of the machine shall not exceed 55° C. after a full-load run for half an hour, the machine shall also carry 50 per cent. overload for 1 minute, immediately following the full-load run, without injury.

To obtain this rating the standard open motor is given about twice its normal rating.

A crane motor is operated with brushes on the neutral position.

Hoisting Rating, Open Series Motor.—For such service the usual temperature guarantee is that—the temperature rise shall not exceed 55° C. after a full-load run for 1 hour, the machine shall also carry 50 per cent. overload for 1 minute, immediately following the full-load run, without injury.

To obtain this rating the standard open motor is given about twice its normal rating.

A hoist motor is operated with the brushes on the neutral position.

Variable Speed Motors for Machine Tools.—The size of such a

machine depends on the minimum speed at which it is necessary to give the rated power because a machine tool such as a lathe takes a constant horse-power at all speeds, since the cutting speed of the tool is practically constant. After the minimum speed has been fixed the maximum speed is that at which the peripheral velocity or the reactance voltage becomes too high. If the limit of speed due to reactance voltage is reached before the peripheral velocity of the machine has become dangerous then higher speeds can be obtained, and therefore the speed range of the machine increased, by the use of interpoles.

CHAPTER XIV

LIMITATIONS IN DESIGN

109. Relation between Reactance Voltage and Average Volts per Bar.—The voltage between brushes $= Z\phi_a \frac{\text{r.p.m.}}{60} \times \frac{\text{poles}}{\text{paths}} 10^{-8}$

and the number of commutator segments between brushes $= \frac{S}{p}$

for both series and multiple windings, therefore,
the average voltage between adjacent commutator segments

$$= Z\phi_a \frac{\text{r.p.m.}}{60} \times \frac{\text{poles}}{\text{paths}} \times \frac{\text{poles}}{S} \times 10^{-8}$$

$$= 2TS(B_g \psi \tau L_c) \frac{\text{r.p.m.}}{60} \times \frac{\text{poles}}{\text{paths}} \times \frac{\text{poles}}{S} \times 10^{-8}$$

The reactance voltage $= k \times S \times \text{r.p.m.} \times I_c \times L_c \times T^2 \times \frac{\text{poles}}{\text{paths}} \times 10^{-8}$

where $k = 0.93$ for short-pitch multiple windings, Art. 69, page 84.
$= 1.6$ for series and for full-pitch multiple windings,

therefore $$\frac{\text{average volts per segment}}{\text{reactance voltage}} = \frac{2B_g \psi p \tau}{60kSI_cT}$$

$$= \frac{4B_g \psi}{60kq}$$

$$= \frac{B_g \psi}{15kq} \qquad (22)$$

110. Limitation Due to High Voltage.—A given frame, including yoke, poles and armature parts, has to be wound for different voltages but for the same speed, it is required to find out if there is any upper limit to the voltage for which the machine may be wound.

Since, as shown in Art. 109,

$$\frac{\text{average volts per segment}}{\text{reactance voltage}} = \frac{B_g \psi}{15qk} = \text{a constant, approx.}$$

therefore, for the same reactance voltage in each case, the average volts per segment must be constant and the number of commutator segments must increase directly as the terminal voltage.

As the number of commutator segments increases the thickness of each decreases and the commutator becomes expensive and

is liable to develop high bars, since the probability of such trouble increases with the number of segments.

When the point is reached beyond which it is not considered advisable to increase the number of commutator segments, higher terminal voltages must be obtained by an increase in the number of turns per coil or by a decrease in the number of paths through the winding since

$$E = Z\phi_a \frac{\text{r.p.m.}}{60} \frac{\text{poles}}{\text{paths}} \times 10^{-8}$$

$$= \text{a constant} \times \frac{Z}{\text{paths}} \quad \text{for a given frame and speed,}$$

$$= \text{a constant} \times \frac{ST}{\text{paths}} \quad \text{for a given frame and speed.}$$

In either case the average volts per bar is increased and so also is the reactance voltage; a point will finally be reached beyond which the reactance voltage becomes so high that good commutation is impossible without the use of interpoles.

When interpoles are supplied the voltage between commutator segments may have any value up to about 30 if the load is fairly steady, but when such a value is reached the machine becomes sensitive to changes of load and liable to flash over; the limit can be extended a little further by the use of compensating windings as described in Art. 77, page 95, but very little is known regarding the operation of D.-C. machines under such extreme conditions.

111. Limitation due to Large Current.—When the voltage for which a machine is wound is lowered, the current taken from the machine increases and to carry this current increased brush contact surface must be supplied.

When the brushes are shifted forward so as to help commutation the maximum brush arc $= \frac{\text{pole pitch}}{12} \times \frac{\text{commutator dia.}}{\text{armature dia.}}$ and when this value of brush arc has been reached increased current can be collected from the machine only by increasing the axial length of the brushes and commutator.

The commutator bars are subject to expansion and to contraction as the load, and therefore their temperature, varies, and the difficulty in keeping a commutator true increases with its length. The limit of commutator length must be left to the judgment of the designer since it varies with the type of construction used and with the class of labor available. The type

of commutator shown in Fig. 28 is seldom made longer than 24 in.; longer commutators have been made by putting two such commutators on the same shaft and connecting the corresponding bars on each with flexible links so as to form an equivalent single bar of twice the length.

The brush arc can be increased about 20 per cent. over the value given in the above formula if the brushes are in the neutral position, and low resistance brushes may be used, but neither of these changes can be made unless the reactance voltage of the machine is low or interpoles are supplied to take care of the commutation; such low resistance brushes, as pointed out in Art. 64, page 78, have a larger current carrying capacity than have brushes of higher contact resistance.

112. The Best Winding for Commutation.

$$E, \text{ the generated voltage} = Z\phi_a \frac{\text{r.p.m.}}{60} \times \frac{\text{poles}}{\text{paths}} \times 10^{-8}$$

$$= 2\ ST\phi_a \frac{\text{r.p.m.}}{60}\ \frac{\text{poles}}{\text{paths}} \times 10^{-8}$$

and the reactance voltage

$$= k \times S \times \text{r.p.m.} \times I_c \times L_c \times T^2 \times \frac{\text{poles}}{\text{paths}} \times 10^{-8}$$

where $k = 1.6$ for series and full-pitch multiple windings
$= 0.93$ for short-pitch multiple windings

therefore the reactance voltage

$$= k\ (S \times T \times \text{r.p.m.} \times \frac{\text{poles}}{\text{paths}} \times 10^{-8})\ I_c L_c T$$

$$= 30k \frac{E}{\phi_a} I_c L_c T$$

$$= \frac{30\ L_c E I_a}{\phi_a} \times \frac{kT}{\text{paths}}$$

For a given frame and a given output ϕ_a, L_c and $E \times I_a$ are all constant and under these conditions

$$\text{Reactance voltage} = \text{a constant} \times \frac{k \times \text{turns per coil}}{\text{paths}}$$

therefore the multiple winding is better than the series winding since it has the larger number of paths; the multiple winding with one turn per coil is the best type of full-pitch winding; and the short-pitch multiple winding is better than the full-pitch multiple winding because it has a lower value of k.

The best winding that can be used in any case is therefore the short-pitch, one-turn multiple winding.

113. Limitations due to Speed in Non-interpole Machines.—The best winding that can be used in such a machine is the short-pitch, one-turn multiple winding for which the reactance voltage $=0.93\ S$ r.p.m. $I_c L_c 10^{-8}$

and $I_a = I_c \times$ paths

$$= \frac{RV}{0.93 \times S \times \text{r.p.m.} \times L_c \times 10^{-8}} \times \text{paths}$$

now $E = Z\phi_a \frac{\text{r.p.m.}}{60} \times \frac{\text{poles}}{\text{paths}}\ 10^{-8}$

$= 2\ S(B_g \psi \tau L_c)\ \frac{\text{r.p.m.}}{60} \times 10^{-8}$, for a one-turn short-pitch multiple winding, since poles = paths.

therefore $E \times I_a =$ watts

$$= 2\ S(B_g \psi \tau L_c)\ \frac{\text{r.p.m.}}{60} \times 10^{-8} \times \frac{RV \times \text{paths}}{0.93 \times S \times \text{r.p.m.} \times L_c \times 10^{-8}}$$

$$= \frac{RV \times B_g \times \phi \times D_a}{9} \quad \text{since } p\tau = \pi D_a$$

and $D_a = \frac{\text{watts} \times 9}{RV \times B_g \times \phi}$ for a short-pitch, one-turn multiple winding.

For sparkless commutation from no-load to 25 per cent. overload, with brushes shifted from the neutral and clamped, the reactance voltage at full-load for a short-pitch multiple winding should not exceed 0.75 (volts drop per pair of brushes), formula H, page 119, and the volts drop per pair of brushes should not exceed 3, otherwise it will be difficult to keep the commutator cool, therefore the highest value for RV in the above formula is $0.75 \times 3 = 2.25$ volts; for this value the armature diameter

$$D_a = \frac{\text{watts}}{B_g \times \phi} \times 4$$

$$= \frac{\text{watts}}{B_g} \times 6 \text{ approximately for a short-pitch}$$

one-turn multiple winding and this is the minimum diameter that can be used for the output without the risk of trouble due to commutation.

Since B_g depends principally on the diameter of the machine, as shown in Fig. 89, it is possible to plot the relation between

output in watts and the smallest diameter of machine from which that output can be obtained without sparking and without the use of additional aids to commutation. This relation is plotted in curve 1, Fig. 95.

In order to use a smaller diameter than that given by the above equation it is necessary to use a higher value for the reactance voltage and under these conditions sparking is liable to occur unless interpoles or some other device is used to help commutation.

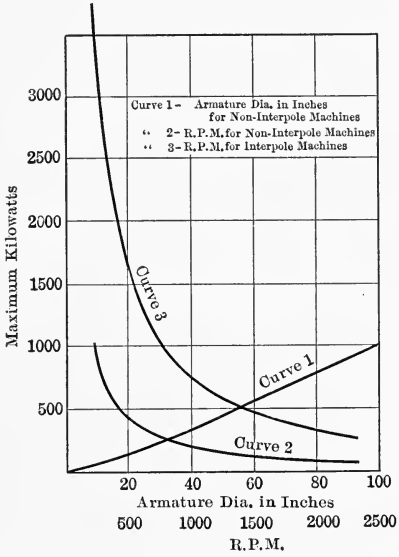


FIG. 95.—Limits of output for D.-C. generators.

114. Limit of Output for Non-interpole Machines.—The peripheral velocity of a D.-C. armature should, if possible, be kept below the value of 6000 ft. per minute, because for higher peripheral velocities the cost of the machine increases rapidly, due to the difficulty in holding the coils of the armature against centrifugal force.

Taking the relation between minimum diameter of armature and output plotted in Fig. 95, and a maximum peripheral velocity of 6000 ft. per minute, the maximum output that can be obtained for a given speed in r.p.m. is figured out and plotted in curve 2, Fig. 95.

115. Limit of Output for Interpole Machines.—It has been shown in Art. 109, page 138, that

$$\frac{\text{average volts per bar}}{\text{reactance voltage}} = \frac{B_g \psi}{15qk}$$

where $k = 1.6$ for a full-pitch multiple winding, the type used for interpole machines.

The average voltage between commutator bars should not exceed 30, so that the reactance voltage, even for interpole machines, should not exceed the value

$$RV = \frac{30 \times 15 \times q \times 1.6}{B_g \times \psi}$$

$$= \frac{1000 \times q}{B_g}$$

since this work is only approximate average values for B_g and for q may be used,

if $B_g = 55{,}000$ lines per square inch

and $q = 800$

then the maximum reactance voltage $RV = 15$.

It was shown in the last Art. that for a short-pitch multiple winding the minimum armature diameter

$$D_a = \frac{\text{watts} \times 9}{RV \times B_g \times \psi}$$

in the same way it can be shown that for a full-pitch multiple winding the minimum armature diameter

$$D_a = \frac{\text{watts} \times 15.5}{RV \times B_g \times \psi}$$

the difference in the constant being due to the fact
that k for a short-pitch winding $= 0.93$
whereas k for a full-pitch winding $= 1.6$.

For an interpole machine the reactance voltage should not exceed 15 and the pole enclosure should be about 0.65; for these values

$$D_a = \frac{\text{watts} \times 1.6}{B_g}$$

Taking this value of D_a, a peripherial velocity of 6000 ft. per minute, and the relation between D_a and B_g shown in Fig. 89, the maximum output that can be obtained for a given speed is figured out and plotted in curve 3, Fig. 95.

116. Limit of Output for Turbo Generators.—The only difference between turbo generators, and the ordinary interpole machine discussed in the last article, is that the construction of

the former is made such that it can be run at peripheral velocities of the order of 15,000 ft. per minute.

For this peripheral velocity and for a reactance voltage of 15, the maximum output that can be obtained for a given speed is figured out and plotted in curve 1, Fig. 96.

Curve 2, Fig. 96, gives the usual speed of steam turbines for different outputs, and it may be seen that for outputs greater than 1000 kw., it is difficult to build generators that can be direct connected to steam turbines, because speeds lower than those in curve 2 can be obtained only by a sacrifice of efficiency.

The output for a given speed can be increased over the value given in curve 1, Fig. 96, by the use of peripheral velocities

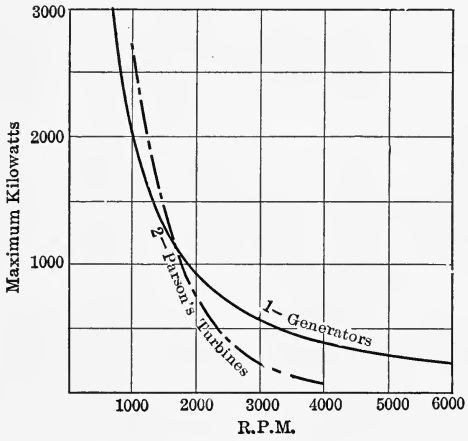


FIG. 96.—Limits of output for D.-C. turbo-generators.

higher than 15,000 ft. per minute and by the use of a higher value of the average volts per bar. To increase that value over 30 volts will probably require the use of compensating windings in addition to interpoles and machines have been built in which this value was as high as 60 but such machines are very sensitive to changes in load and to changes in the interpole field. For such a high value of average volts per bar the reactance voltage will probably be about 30 volts and the interpole difficult to adjust and further, any lag of the interpole field behind the interpole current, when the load and therefore the current changes, will lead to trouble in commutation.

One must be careful in the interpretation of the curves shown

in Figs. 95 and 96. They are derived on the assumption that the machine is limited only by commutation. For many of the outputs within the range of the different curves the voltage or current limitation might be reached before the output limit is reached due to speed. Some of the ratings also would probably require a machine with forced ventilation.

CHAPTER XV

DESIGN OF INTERPOLE MACHINES

117. Preliminary Design.—The preliminary design work on an interpole machine, whereby the principal dimensions are determined approximately, is carried out in the same way as that on the non-interpole machine discussed in Art. 101, page 120, but some slight modifications are made on the constants used.

It was pointed out in Art. 66, page 80, that, the deeper the slots in a D.-C. machine, the greater the slot leakage flux, and therefore the greater the reactance voltage; because of this the slots in non-interpole machines have to be limited in depth so that the value of q, the ampere conductors per inch, cannot greatly exceed that given in Fig. 90. When interpoles are supplied the reactance voltage becomes of less importance and it is possible to use deep slots without the risk of trouble due to poor commutation; with such deep slots a large amount of copper can be put on each inch of the armature periphery, so that the value of q may be made larger than in the case of the non-interpole machine, and is usually about 20 per cent. larger than that given in Fig. 90, page 115.

The ratio $\dfrac{\text{field amp.-turns per pole for tooth and gap}}{\text{armature ampere-turns per pole}}$ is seldom less that 1.2 for machines without interpoles, in order that the magnetic field under the pole tip toward which the brushes are shifted to help commutation may not be too weak. When interpoles are supplied such a commutating field is no longer necessary, and a weaker main field is generally used. Inspection of Fig. 49, page 57, will show that if the cross-magnetizing ampere-turns at the pole tips, namely $\frac{1}{2}\phi\dfrac{Z}{p}I_c$, becomes equal to the ampere-turns per pole for tooth and gap due to the main field excitation, then the effective m.m.f. across the gap and tooth under one pole tip will be zero while that under the other pole tip have twice the no-load value; the field will therefore be greatly distorted. On account of the saturation of the teeth under this latter pole tip the flux density will not be proportional to the m.m.f. and, as pointed out in Art. 48, the total flux per

pole will be reduced. Due to the high flux density under the one pole tip, the armature core loss, which depends on the maximum density in the core, will be higher at full-load than at no-load, and the tendency to flash over, which, as pointed out in Art. 77, page 95, depends principally on the voltage between adjacent commutator segments at any point, will also be greater.

The ratio $\frac{\text{field amp.-turns per pole for tooth and gap}}{\text{armature cross ampere-turns at the pole tips}}$ is found in practice to have a value of about 1.2 for interpole machines and, taking the value of the pole enclosure = 0.65, the ratio

$$\frac{\text{field amp.-turns per pole for gap and tooth}}{\text{armature ampere-turns per pole}} = 1.2 \times 0.65 = 0.8 \text{ approx.}$$

As the armature ampere-turns per pole is increased the field ampere-turns must increase in the same ratio, and the pole length must also increase in order to carry this excitation. When the armature strength reaches the value of 10,000 ampere-turns per pole it will generally be found that an increase in the number of poles, and therefore a decrease in the armature strength per pole and in the length of poles, will give a more economical machine.

The ratio $\frac{\text{magnetic loading}}{\text{electric loading}} = \frac{B_g \psi L_c}{q}$, Art. 99, page 116, and for interpole machines is generally about 10 per cent. smaller than for machines without interpoles; in the above equation ψ is slightly less to give room for the interpoles and q, as pointed out above, is generally 20 per cent. greater.

118. Example of Preliminary Design.—The machine to be taken as an example of interpole design has a rating that lies within curve 2, Fig. 95, that is, it can be built without interpoles. In order to compare the interpole machine with that which has no interpoles two designs are given which were built for the same rating; the non-interpole machine is the oldest.

Example.—Determine approximately the dimensions of a D.-C. non-interpole machine of the following rating: 200 kw.; 115 volts, no-load; 115 volts full-load; 1740 amp.; 500 r.p.m.

The work is carried out in tabular form as follows:

Ampere conductors	$= 0.64 \times 10^5$, from Fig. 91.
Ampere conductors per inch	= 660, from Fig. 90.
Armature diameter	= 31 in.
Apparent gap density	B_g = 53,000 lines per square inch, from Fig. 89.

Frame length	$L_c = 10$ in.; formula B, page 119.
Poles	$p = 6$; formula C.
Pole pitch	$\tau = 16.2$ in.
Flux per pole	$\phi_a = 6.0 \times 10^6 = B_g \psi \tau L_c$.
Total face conductors	$Z = 230$, formula D.
Winding	one-turn multiple, short pitch.
Reactance voltage	$RV = 1.55$; formula G.
Commutator diameter	$= 0.6 \times$ armature diameter $= 19$ in.
Brush arc	$= 0.83$; formula P.
Brush length	$= 20$ in.
Amperes per square inch brush contact	$= 35$

This commutator will be very long and will probably give trouble; the length can be reduced by increasing the diameter so as to allow the use of a wider brush. In the actual machine the commutator diameter was made 21 in.; the brush arc was made 1 in., which is 10 per cent. larger than the value obtained by the use of formula P, page 120, and was possible because the reactance voltage is low; the brush length was cut down to 16 in., for which brush and commutator the amperes per square inch of brush contact is 36.

The final designs for both the interpole and the non-interpole machine are given in the following partial design sheet.

119. Armature and Commutator Design Sheet.—All in inch units.

Armature.			*Commutator.*		
Ex. diameter	31	26	Diameter	21	21
In. diameter	20	14.5	Face	19	$16\frac{3}{4}$
Frame length	10	14	Bars	120	120
End ducts	$2-\frac{1}{2}$	$2-\frac{3}{8}$	Bars and mica	0.55	0.55
Center ducts	$2-\frac{1}{2}$	$4-\frac{3}{8}$	Brush arc	1.0	1.15
Gross iron	9	12.5	Brush studs	6	6
Net iron	8.1	11.2	Brushes per stud.	9 ($\frac{7}{8} \times 1\frac{3}{4}$)	8 ($1 \times 1\frac{3}{4}$)
Slots, number	120	120	Amp. per square inch contact	36	36
Slots, size	0.4×1.34	0.33×2	Perip. velocity	2750	2750
Cond. per slot, number	2	2	Friction loss	3250	3250
Cond. per slot, size	2 (0.12 × 0.5)	2(0.1 × 0.75)	Volts per pair of brushes	2.5	2.0
Coils	120	120	Contact resistance loss	4350	3500
Turns per coil	1	1	Watts per square inch surface	6	6
Total conductors	240	240	Temperature rise.	45° C.	45° C.
Winding, type	Multiple.	Multiple.	Average volts per bar	5.75	5.75
Winding, pitch	Short 1–20	full 1–21			
Slot pitch	0.81 / 0.74	0.68 / 0.578	*Copper loss, Commutation.*		
Tooth width	0.41 / 0.34	0.35 / 0.248	Amp. cond. per inch	710	850
Max. tooth width / Slot width	1.02	1.06	Cir. mils per amp.	500	620
Core depth	4.16	3.75			

Armature			*Commutator*		
Pole pitch	16.2	13.6	Arm. AT. per pole.	5800	5800
Per cent. enclosure.	0.7	0.65	Ratio $\frac{\text{AT.(gap + tooth)}}{\text{arm. AT. per pole}}$	1.12	0.83
Min. tooth area per pole	38.5	36	Reactance voltage	1.62	3.9
Core area per pole	33.6	42			
Apparent gap area per pole	114	124			
Densities, Iron loss, Excitation.					
Flux per pole	5.75×10^6	5.75×10^6	*Type*	Non-inter-pole	Interpole
Max. tooth density (apparent)	150,000	160,000			
Max. tooth density (actual)	141,000	149,000,	*Rating.*		
Core density	85,000	70,000	Poles	6	6
Gap density, (apparent)	50,500	46,500	kw	200	200
Frequency	25	25	Volts no-load	115	115
Air gap clearance	0.3	0.2	Volts full-load	115	115
Carter coefficient	1.12	1.12	Amperes	1740	1740
AT. gap	5400	3250	r.p.m.	500	500
AT. tooth	1000	1600	$\frac{\text{Magnetic loading}}{\text{Electric loading}}$	500	500
			Output factor	4.15	4.25

The reactance voltage in each case is figured from formula *G*, page 119, and is larger for the interpole than for the non-interpole machine, because the former has the longer frame length and has also a full-pitch winding, whereas that in the latter machine is short-pitch.

The actual reactance voltage in the case of the interpole machine will probably be greater than that determined from the formula because the coils are now short circuited while under the interpole, the presence of which lowers the reluctance of the path of that part of the leakage flux which circles the short-circuited coil by crossing the tooth tips.

120. Design of the Field System for an Interpole Machine.—The design of the shunt and series windings presents no new problem and is worked out in the same way as that of the field system designed in Art. 57, page 67.

The shunt winding is usually tapered, as shown in Fig. 97, so that the air has free access to all the field coils.

The series winding is supplied to give the necessary compounding effect. While the demagnetizing ampere-turns per pole in an interpole machine is zero, since the brushes are on the neutral position, yet the effect of the cross magnetizing ampere-turns in reducing the flux per pole, see Art. 49, page 56, is large, because the ratio $\frac{\text{main field AT for gap and tooth}}{\text{armature amp.-turns per pole}}$ is small and the field distorsion is therefore great.

After the shunt and series fields are designed they are drawn in to scale, as shown in Fig. 97, and from that drawing the radial length of the interpole coil space is determined, and then the interpole itself is designed in the following way:

Example.—Design the interpole and its winding for the machine given in the design sheet on page 148.

L_{ip}, the axial length of the interpole, is found from the formula

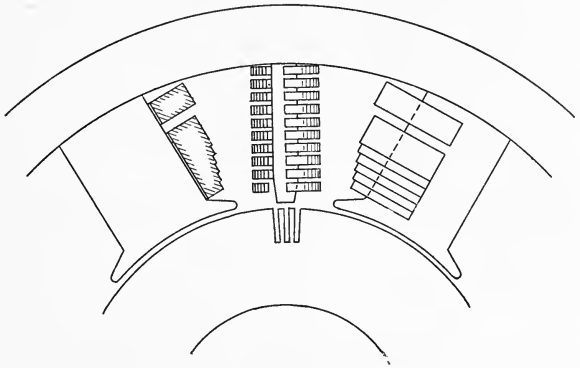


FIG. 97.—Field windings of an interpole machine.

$$L_{ip} = L_c \times \frac{48 \times q}{B_{ip}}$$

page 94, where B_{ip} is assumed to be = 45,000 lines per square inch.

$q = 850$, from design sheet

$L_c = 14$ in. from design sheet

therefore $L_{ip} = 12.5$ in.

W_{ip} the interpole width at the armature surface, is found from the formula $W_{ip} + 2\delta =$ brush arc + slot pitch − segment width all measured at the armature surface, Art. 76, page 93, where the brush arc referred to the armature surface

$$= \text{brush arc} \times \frac{D_a}{D_c}$$

$$= 1.15 \times \frac{26}{21}$$

$$= 1.42 \text{ in.}$$

and slot pitch = segment width at the armature surface, for a machine with one coil per slot.

To prevent pulsation of the interpole field the effective interpole arc should be approximately a multiple of the slot pitch; the slot pitch = 0.68 in. and twice the slot pitch = 1.36 in., which is approximately = 1.42 in. The effective interpole arc is therefore taken as 1.4 in. and the interpole is trimmed down at the tip, as shown in Fig. 97, so that the actual interpole width at the tip

$$= \text{effective width} - 2\delta$$
$$= 1.4 - 0.4$$
$$= 1 \text{ in.}$$

The interpole winding has now to be designed.

The armature amp.-turns per pole = 5800

The interpole amp.-turns per pole = 5800×1.5 approximately,

Art. 76, page 94.

$= 8700$ approximately

The number of turns required on each interpole $= \dfrac{8700}{\text{full-load current}}$

$= 5.$

The full-load current is 1740 amperes, and to carry such a large current very heavy copper would be required for the interpole winding, the winding is therefore divided into two circuits which are put in parallel and each carries half of the total current, namely 870 amperes; each coil has twice the number of turns found above, namely 10.

The external surface of each interpole coil = external periphery $\times$ radial length, where the radial length, = 7.5 in., is taken from Fig. 97 and the external periphery = 38 in. approximately, therefore, the external surface = $38 \times 7.5 = 290$ sq. in.

The permissible watts per square inch = 0.8, from Fig. 56, page 64, and this is increased 50 per cent. because of the ventilated construction, therefore the total permissible loss per coil

$$= 290 \times 0.8 \times 1.5 = 350 \text{ watts.}$$

The loss in each coil $= \dfrac{\text{mean turn} \times \text{turns per coil} \times \text{current}^2}{\text{cir. mil. section}}$

where the mean turn = 34 in. approximately and the current = 870 amperes, therefore

the section of the interpole copper in cir. mils

$$= \frac{\text{mean turn} \times \text{turns per coil} \times \text{current}^2}{\text{loss in each coil}}$$

$$= \frac{34 \times 10 \times 870^2}{350}$$

$$= 740{,}000$$
$$= 0.58 \text{ sq. in.}$$

The actual winding was made with 10 turns per coil of 0.5 sq. in. section made up of ten strips each = 0.1 in. × 0.5 in. so that it could easily be bent to shape. The section was a little less than that obtained by calculation because the required number of ampere-turns have been taken slightly larger than necessary; the interpole current is adjusted after the machine has been set up for test by means of a shunt in parallel with the interpole winding.

CHAPTER XVI

SPECIFICATIONS

121. Specifications.—A specification for an electrical machine is a detailed statement made out by the intending purchaser to tell the bidder what he has to supply. When agreed to by both buyer and seller it generally forms part of the contract. The following is a typical specification for a D.-C. generator.

SPECIFICATION FOR A DIRECT-CURRENT GENERATOR

Type.—Two-wire, compound wound, non-interpole.

Rating.—Rated capacity in kilowatts..... 400
No-load voltage..... 240
Full-load voltage..... 240
Normal full-load current..... 1670
Speed in revolutions per minute..... 200

Construction.—The generator will be of the engine type for direct connection to a steam engine and shall be furnished without shaft. The armature core and the commutator shall be built together on a cast-iron spider arranged to be pressed on the engine shaft.

The poles shall be securely attached to the yoke by bolts, and the yoke must be split so that half of it may readily be removed if necessary.

Armature.—The armature shall be of the slotted drum type. Hardwood wedges, driven into grooves in the teeth, to be used to retain the coils in the slots so as to dispense with the use of band wires on the core. The armature coils shall be formed without joints, then insulated completely, impregnated under pressure, pressed in heated forms and cooled under pressure. The winding must be such that the coils are all individually removable and are all of the same form and dimensions.

Commutator.—The commutator bars shall be of hard drawn copper and must be insulated from one another by mica of such quality and thickness as to ensure even wear on the commutator surface; the wearing depth of the commutator must be not less than 1 in. The ends of the armature conductors shall be thor-

oughly soldered to the necks of the commutator bars. All bolt heads and other projecting parts must be protected in such a way that a person can examine the brushes without the risk of being caught.

Brush Holders and Supports.—Brush holders shall be designed for carbon brushes and so constructed as to give ready access to the commutator. The supports which carry the brush holders must be strong and rigid and constructed so that they may be shifted to adjust the brushes and locked in any desired position.

Workmanship and Finish.—The workmanship shall be first class. All parts must be made to standard gauges and be interchangeable. All surfaces not machined are to be dressed, filled and rubbed down so as to present a smooth finished appearance.

Rheostat.—A suitable shunt-field rheostat of the enclosed plate type will be supplied.

Pressing on Armature.—The armature will be pressed on the shaft by the engine builder to whom the generator builder will supply an accurate gauge of the diameter of the armature bore, the engine builder to make the allowance for press fit.

Keys.—Gauges of the keyways in the armature will be furnished by the generator builder.

Foundation bolts will not be furnished.

Bidders shall furnish plans or cuts with descriptive matter from which a clear idea of the construction may be obtained. They shall also state the following:

Armature net weight;
Total net weight;
Shipping weight;
Efficiency at 1/4, 1/2, 3/4, full, and 1 1/4 load.

The Losses shall include all the losses in the machine except windage and bearing friction, they shall also include the loss in the shunt field circuit rheostat, and in the series shunt should that be supplied.

The constant armature and commutator loss will be found by driving the machine by an independent motor, the output of which may be suitably determined, the machine being run at normal speed and excited so as to generate the full-load voltage + the voltage drop at full-load in the armature, brushes and series field.

The shunt field coil and rheostat loss shall be taken as the terminal voltage $\times$ the no-load exciting current.

The armature copper, brush, and series field losses will be found by passing currents corresponding to the different loads through the armature and measuring the voltage drop so as to include armature, brushes and series field, the armature being stationary. The voltage drop $\times$ the corresponding current will be taken as the loss in these parts, this test to be made immediately after the heat run.

Excitation.—The shunt excitation must be such that the terminal voltage of the machine at full-load may be raised at least 15 per cent. above normal by the operation of the shunt field rheostat.

Temperature.—The machine shall carry the rated capacity at full-load voltage and normal speed continuously, with a temperature rise that shall not exceed 40° C. by thermometer on any part of the armature or commutator, or 60° C. by thermometer or by resistance on any part of the field windings. The machine shall also carry 25 per cent. overload at normal full-load voltage for two hours, immediately following the full-load run, without the temperature rise exceeding 55° C. by thermometer on any part of the armature or commutator or 60° C. by thermometer or resistance on any part of the field system.

The temperature rise should be referred to a room temperature of 25° C.

Commutation.—The machine shall carry continuously any load from no-load to 25 per cent. overload without destructive sparking and without shifting of the brushes.

Insulation.—The machine shall withstand the puncture test recommended in the standardization rules of the American Institute of Electrical Engineers and the insulation resistance of the whole machine shall be greater than 1 megohm.

Testing Facilities.—The builder shall provide necessary facilities and labor for testing in accordance with this specification.

122. Points to be Observed in Writing Specifications.—A specification should have only one interpretation and should therefore contain no such general clauses as perfect commutation or satisfactory operation, because these terms have no generally accepted meaning.

While the general construction of the machine should be described, details which have no particular effect on the operation of the machine should not be embodied in a specification as they merely tend to prevent competition without improv-

ing the operating qualities of the machine, but were certain details such as form wound coils are desired they should be specified.

The recommendations of the Standardization Rules of the American Institute of Electrical Engineers should be followed as closely as possible.

Temperature guarantees, lower than those in general use for the type of apparatus on which the specification is being written, should not be demanded unless absolutely necessary. If it is desired that the rating be liberal, it is better in most cases to call for a machine of greater capacity than that corresponding to the proposed load. Manufacturers build standard apparatus to suit the average trade, and such a standard machine can only be run with a temperature rise below normal by the use of low resistance brushes to keep the commutator loss down and by the use of small air gaps and therefore low excitation to keep the field heating down; both of these changes, however, tend to make the commutation poor, so that the machine built to suit such guarantees is often liberal so far as heating is concerned but closely rated so far as commutation is concerned.

The figures for efficiency should be left for the maker of the machine to supply. The method whereby the efficiency shall be calculated should be stated. In the case of engine type generators the bearings are supplied by the engine builder so that the bearing friction loss cannot readily be measured; for that reason the windage and bearing friction losses are not considered in calculating the efficiency of such a type of machine.

123. Effect of Voltage on the Efficiency.—If two machines are built on the same frame, and for the same output and speed, but for different voltages, the losses will be affected in the following way.

The windage and the bearing friction will be unchanged since they depend on the speed, which is constant.

The excitation loss will be unchanged since the same frame and the same flux per pole are used.

The iron loss will be unchanged since the flux per pole is the same in each case and so also is the frequency.

The brush friction loss will be the lower in the machine with the higher voltage because, for a given output, the current to be collected from the commutator is inversely as the voltage, and

the brush contact surface on which the brush friction depends is directly proportional to the current to be collected.

The contact resistance loss will be lower in the machine with the higher voltage if the machines are wound so as to have the same reactance voltage, because then the same volts drop per pair of brushes will be required for each machine, and the contact resistance loss, which is equal to the volts drop per pair of brushes $\times$ the full-load current, will be proportional to that current and so inversely proportional to the terminal voltage.

The armature copper loss is independent of the voltage if the same total amount of armature copper is used in each case. This may be shown as follows:

Output of machine = volts per cond. $\times Z \times I_c$

and armature copper loss $= \dfrac{L_b \times Z \times I_c^2}{M}$

$$= \frac{L_b}{M \times Z} \times Z^2 I_c^2$$

$$= \frac{L_b}{M \times Z} \times \frac{(\text{output})^2}{(\text{volts per cond.})^2}$$

$$= \frac{L_b \times (\text{output})^2}{(\text{volts per cond.})^2} \times \frac{1}{\text{total copper section}}$$

$$= \text{a constant} \times \frac{1}{\text{total copper section}}$$

for a given frame, rating and speed.

Except in the case of very small machines, the same amount of copper can be got into a machine at any voltage up to 600, so that over this range the higher the voltage the higher the efficiency, on account of the considerable reduction in the commutator loss.

124. Effect of Speed on the Efficiency.—As shown in Art. 97, page 114, $\dfrac{\text{watts}}{\text{r.p.m.}} = \text{a constant} \times D_a^2 L_c \times B_g \times \psi \times q$

$= \text{a constant} \times B_g \times q$ for a given frame.

If then a given machine is increased in speed the frequency will increase, and therefore the flux per pole, and B_g, the average flux density in the air gap, must be decreased, as shown in Art. 92, page 107, while, due to the better ventilation, the value of q must be increased for the same rise in temperature. Over a considerable range of speed the product of B_g and q is approximately constant and therefore the watts output is approximately directly proportional to the r.p.m. With regard to the losses; the

field excitation loss is proportional to the field excitation and, therefore, as shown in Art. 52, page 61, is directly proportional to the armature ampere-turns per pole, which $= \frac{ZI_c}{2p} = q\frac{\text{pole-pitch}}{2}$ and so for a given machine, is directly proportional to q. If then q increases, the excitation loss also increases.

The total armature and commutator loss will increase with the speed because, for the same temperature rise, the permissible loss in the revolving part of a given machine $= (A + B \times \text{r.p.m.})$ where A is the loss that is dissipated by radiation and is independent of the speed.

Thus in a given machine, when the speed is increased, the output is directly proportional to the speed, the excitation loss is increased slightly, and part of the armature and commutator loss is directly proportional to the speed, therefore, the losses do not

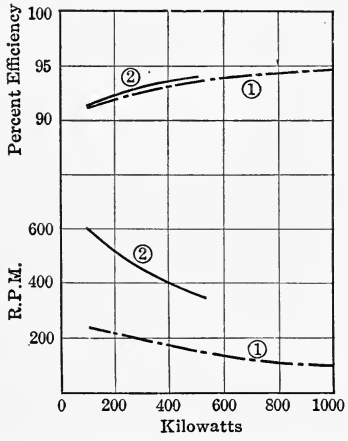


FIG. 98.—Efficiency curves for 550-volt D.-C. generators.

increase as rapidly as the output does and the higher the speed the higher the efficiency until the speed is reached at which a radical change in the design is necessary, such as the addition of interpoles or of compensating windings, when a slight drop is efficiency generally takes place due to the extra loss in these additional parts.

If now the case be taken of two machines which have the same axial length and the same pole-pitch but a different number of poles, and the speed in r.p.m. is made inversely as the number of poles so that the peripheral velocity and the frequency is the

same in each case, then each pole, with the corresponding part of the armature, may be considered as one unit.

The output, excitation loss, armature and commutator loss will all be proportional to the number of poles, so that, for an increase in the watts output with a proportional decrease in the speed, the efficiency is unchanged over a considerable range in speed.

Curves 1 and 2, Fig. 98, show the efficiencies that may be expected from a line of 550 volt D.-C. generators at speeds

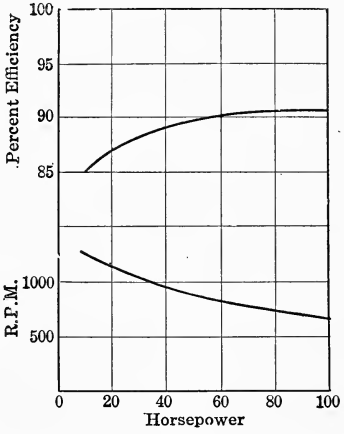


FIG. 99.—Efficiency curves for 220-volt D.-C. motors.

given in the corresponding speed curves 1 and 2; the slow speed machines are direct connected engine type units and the efficiency does not include the windage and bearing friction losses; the high-speed machines are belted units.

Figure 99 shows a similar set of curves for a line of 220 volt D.-C. motors.

CHAPTER XVII

ALTERNATOR WINDINGS

125. Single-phase Fundamental Winding Diagram.—Diagram *A*, Fig. 100, shows the essential parts of a single-phase alternator which has one armature conductor per pole. The direction of motion of the armature conductors relative to the magnetic

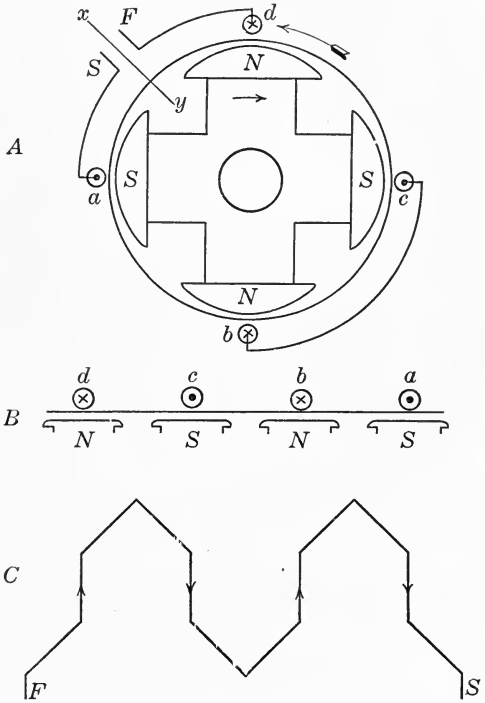


FIG. 100.—Fundamental single-phase winding diagram.

field is shown by the arrow, and the direction of the generated e.m.f. in each conductor is found by Fleming's three-finger rule and is indicated in the usual way by crosses and dots.

The conductors *a*, *b*, *c* and *d* are connected in series so that their voltages add up and the method of connection is indicated

in diagram A. Such a connection diagram, however, becomes exceedingly complicated for the windings that are used in practice and a simpler diagram is that shown at C, Fig. 100, which is got by splitting diagram A at xy and opening it out on to a plane.

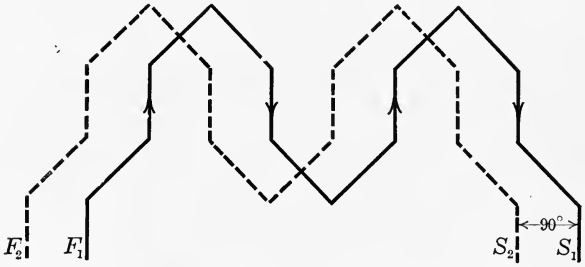


FIG. 101.—Fundamental two-phase winding diagram.

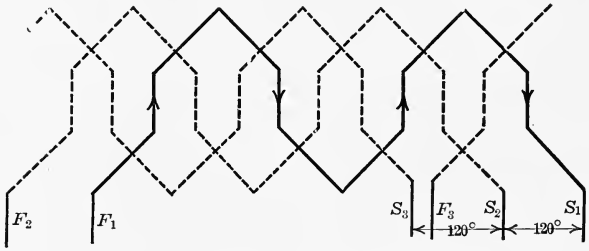


FIG. 102.—Fundamental three-phase winding diagram.

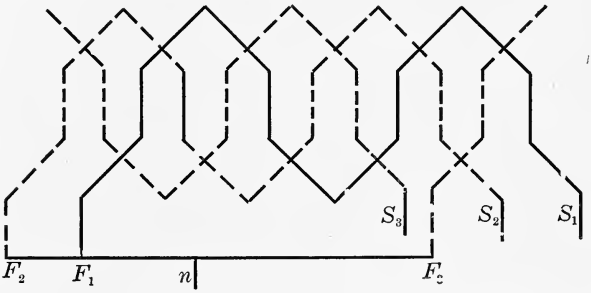


FIG. 103.—Fundamental three-phase Y-connected winding.

Diagram C may be called the fundamental single-phase winding diagram because on it all other single-phase diagrams are based. The letters S and F stand for the start and finish of the winding respectively.

126. The Frequency Equation.—The voltage generated in

any one conductor goes through one cycle while the conductor moves relative to the magnetic field through the distance of two-pole pitches, so that one cycle of e.m.f. is completed per pair of poles;

$$\text{the cycles completed per revolution} = \frac{p}{2},$$

$$\text{the cycles completed per second} = \frac{p}{2} \times \frac{\text{r.p.m}}{60},$$

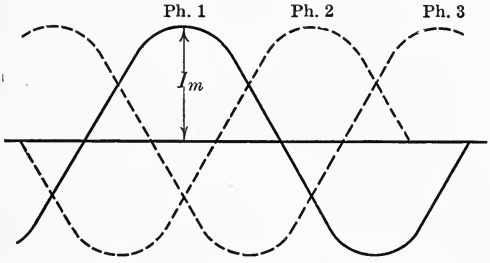


FIG. 104.—Currents in the three phases.

therefore f, the frequency in cycles per second

$$= \frac{p \times \text{r.p.m.}}{120} \tag{23}$$

127. Electrical Degrees.—The e.m.f. wave of an alternator is represented by a harmonic curve and therefore completes one cycle in 2π or 360 degrees; as shown above, the e.m.f. of an

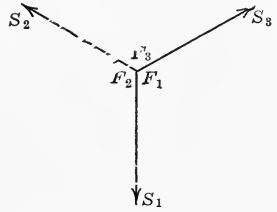


FIG. 105.—Voltage vector diagram for a Y-connected winding.

alternator completes one cycle while the armature moves, relative to the poles, through the distance of two-pole pitches; it is very convenient to call this distance 360 electrical degrees.

128. Two- and Three-phase Fundamental Winding Diagrams.—Fig. 101 shows the fundamental winding diagram for a two-phase machine. A two-phase winding consists of two single-phase windings which are spaced 90 electrical degrees apart so that the

e.m.fs. generated in them will be out of phase with one another by 90 degrees.

Figure 102 shows the fundamental winding diagram for a three-phase machine. A three-phase winding consists of three single-phase windings which are spaced 120 electrical degrees apart so that the e.m.fs. generated in them will be out of phase with one another by 120 degrees.

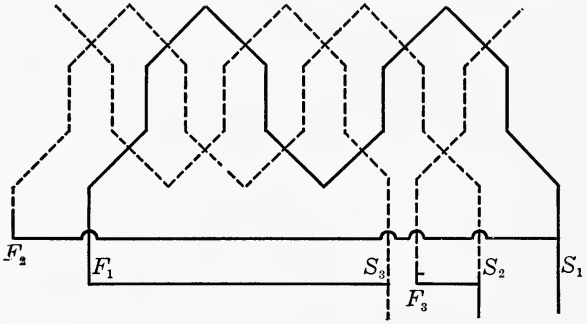


FIG. 106.—Fundamental three-phase Δ-connected winding.

129. Y and Δ Connection.—It will be seen from Fig. 102 that a three-phase winding requires six leads, two for each phase. It is usual, however, to connect certain of these leads together so that only three have to be brought out from the machine and connected to the load.

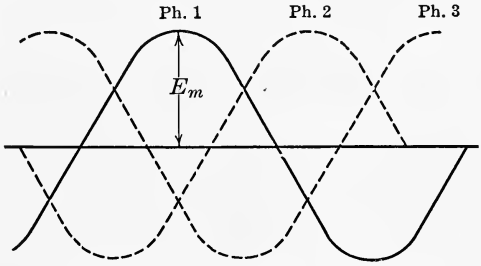


FIG. 107.—E.M.FS. in the three phases.

Figure 103 shows the Y connection used for this purpose. The three finishes of the winding are connected together to form a resultant lead n and the current in this lead at any instant is the sum of the currents in the three phases. The current in each of the three phases at any instant may be found from the curves in

Fig. 104 from which curves it may be seen that at any instant the sum of the currents in the three phases is zero, so that the lead n may be dispensed with and the machine run with the three leads S_1, S_2, and S_3.

Figure 105 shows the voltage vector diagram for a Y-connected winding and from the shape of this diagram the connection takes its name.

Figure 106 shows the Δ connection. The winding is connected to form a closed circuit according to the following table:

S_1 to F_2
S_2 to F_3
S_3 to F_1

It would seem that, since the windings form a closed circuit, the e.m.fs. of the three phases would cause a circulating current to flow

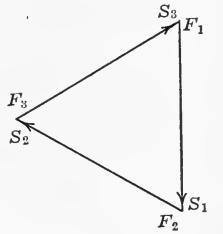


Fig. 108.—Voltage vector diagram for a Δ-connected winding.

in this circuit; however, the three e.m.fs. are 120 degrees out of phase with one another and an inspection of Fig. 107 will show that the resultant of three such e.m.fs. in series is zero at any instant.

Fig. 108 shows the voltage vector diagram for a Δ-connected winding, the phase relation of the three voltages is the same as in Fig. 105.

130. Voltage, Current and Power Relations in Y- and Δ-Connected Windings.—Let M and N, Fig. 109, represent two phases of a three-phase winding, the voltages generated therein are out of phase with one another by 120 degrees and are represented by vectors in diagram B.

If the phases are connected in series, so that F_2 is connected to S_1, then the voltage between F_1 and S_2 = the voltage from F_1 to S_1 + the voltage from F_2 to S_2 = E_r, diagram C, and is equal to E, the voltage per phase.

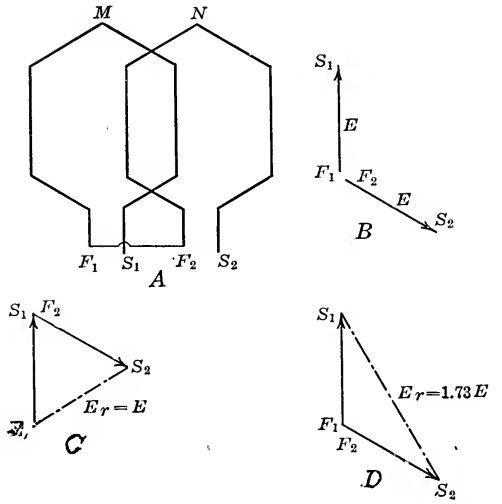


FIG. 109.—Voltage relations in three-phase windings.

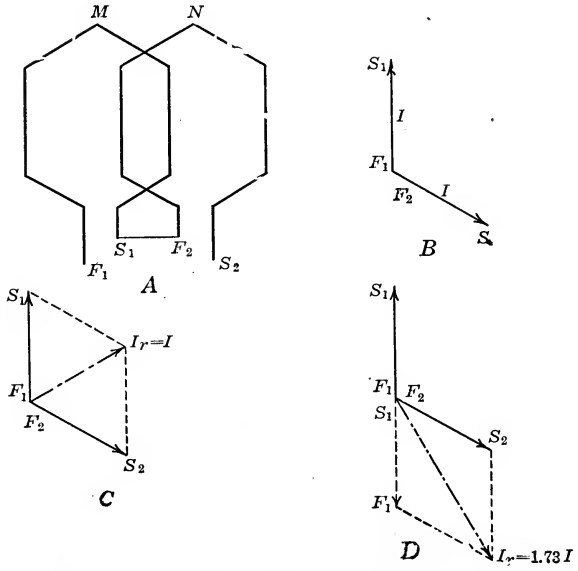


FIG. 110.—Current relations in three-phase windings.

If, however, as in a Y-connected winding, F_2 is connected to F_1, then the voltage between S_1 and S_2 = the voltage from S_1 to F_1 + the voltage from F_2 to S_2 = E_r, diagram D, which is equal to 1.73 E.

In a Y-connected machine therefore, the voltage between

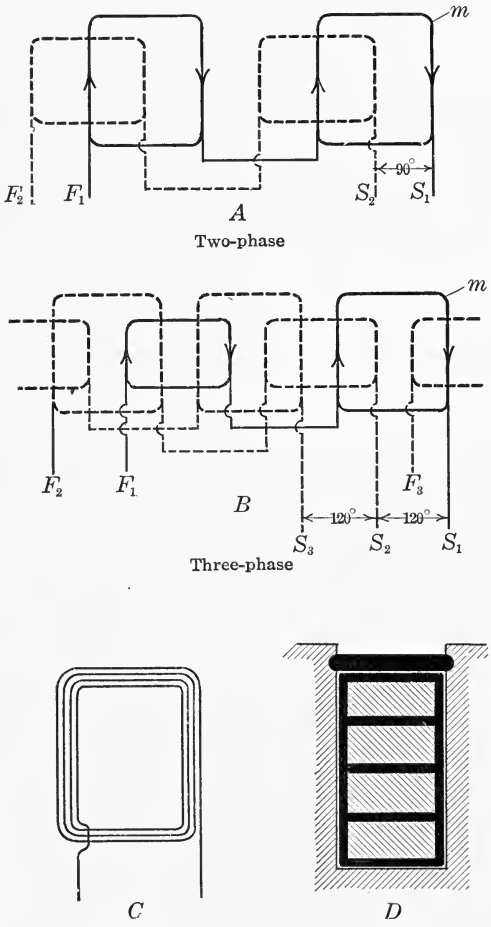


Fig. 111.—Four-pole chain winding.

terminals is 1.73 times the voltage per phase, while the current in each line is the same as the current per phase.

Let M and N, Fig. 110, represent two phases of a three-phase winding, the currents therein are out of phase with one another by 120 degrees and are represented by vectors in diagram B.

If the two phases are connected in parallel, so that F_2 is connected to F_1, then the current in the line connected to F_1F_2 = the current from F_1 to S_1 + the current from F_2 to S_2 = I_r, diagram C, and is equal to I, the current per phase.

If, however, as in a Δ-connected winding, S_1 is connected to F_2, then the current in the line connected to S_1F_2 = the current

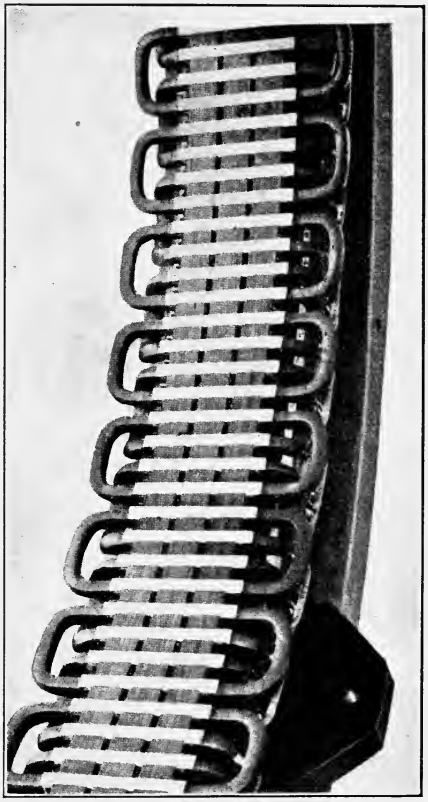
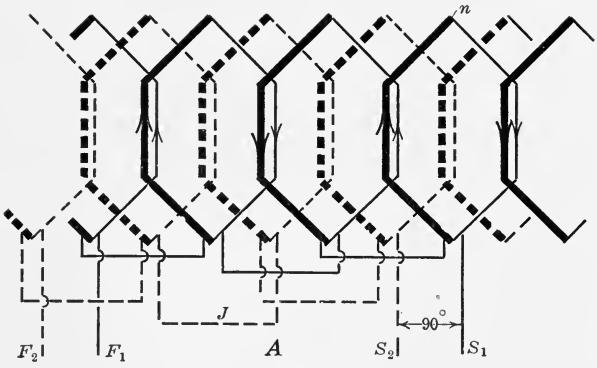


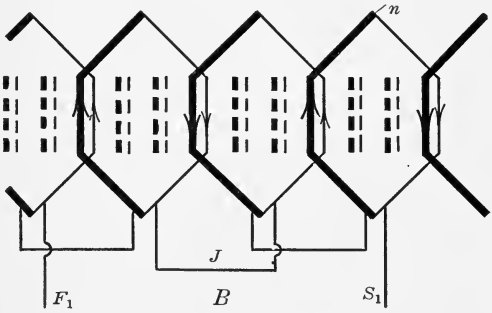
FIG. 112.—Three-phase chain winding with one slot per phase per pole.

from S_1 to F_1 + the current from F_2 to S_2, which is equal to the current from F_2 to S_2 − the current from F_1 to S_1 = I_r, diagram D, = 1.73I.

In a Δ-connected machine therefore, the current in each line is 1.73 times the current in each phase while the voltage between terminals is the same as the voltage per phase.



Two Phase



Three Phase, one phase shown

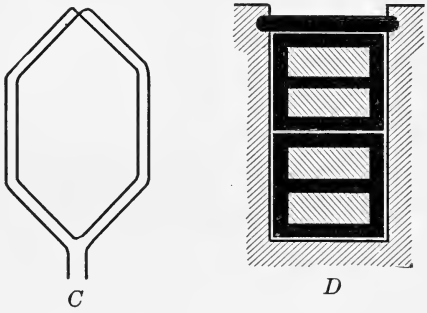


FIG. 113.—Four-pole double layer winding.

The power delivered by a three-phase alternator

$= 3\ E\ I \cos\theta$

$= 1.73\ E_t\ I_l \cos\theta$ for either Y- or Δ-connected machines

where E is the voltage per phase,

I is the current per phase,

E_t is the voltage between terminals,

I_l is the current in each line,

θ is the phase angle between E and I.

131. Windings with Several Conductors per Slot.—When there are more than one conductor per slot a slight modification must be made on the fundamental winding diagrams. Consider for example the case where there are four conductors per slot.

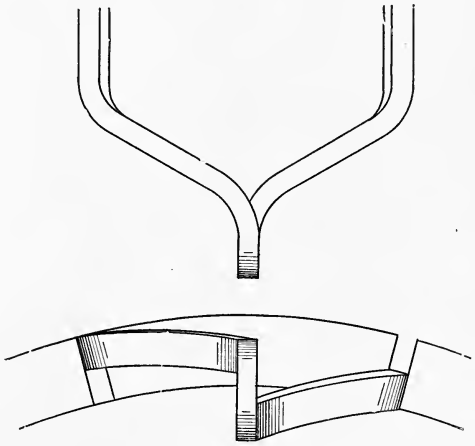


FIG. 114.—Coil for a double layer winding.

One method of connecting up the winding is shown in Fig. 111, which shows the two- and three-phase diagrams. Each coil m consists of four turns of wire as shown in diagram *C;* these wires are insulated from one another and are then insulated in a group from the core so that a section through one slot and coil is as shown in diagram *D*. On account of its appearance this type of winding is called the *Chain Winding.*

Figure 112 shows part of a machine, which is wound according to diagram *B*, Fig. 111; the method whereby one coil is made to jump over the other is clearly shown.

Another method of connecting up the winding is shown in Fig. 113, which shows the two- and three-phase diagrams. Each

coil n consists of two turns of wire as shown in diagram *C;* these wires are insulated from one another and are also insulated from the core so that a section through one slot and coil is as shown in diagram *D.*

The coils are shaped as shown in Fig. 114; one side of each coil, represented in the winding diagrams by a heavy line, lies in the top of a slot, while the other side, represented by a light line, lies in the bottom of a slot about a pole-pitch further over on the armature. The whole winding lies in two layers and is therefore called the *Double-Layer Winding.*

132. Comparison between Chain and Double-Layer Windings.

Chain	Double-layer
The number of conductors per slot may be any number.	The number of conductors per slot must be a multiple of two.
The number of coils is half of the number of slots.	The number of coils is the same as the number of slots.
There are several shapes of coil, therefore a large outlay is necessary for winding tools, and a large number of spare coils must be kept in case of breakdown.	The coils are all alike, therefore the number of winding tools is a minimum and so also is the number of spare coils that must be kept.
The end connections of the winding are separated by large air spaces.	The end connections are all close together and therefore more liable to breakdown between phases than in the chain winding.

The chain winding is the easier to repair because, in order to get a damaged coil out of a machine, fewer good coils have to be removed than in the case of the double layer winding; this may be seen from the chain winding and the corresponding double-layer winding shown in Figs. 118 and 119.

The chain winding requires the larger initial outlay for tools but the winding itself is the cheaper because there are not so many coils to be formed and insulated.

The amount of the slot section that is taken up by insulation is less with the chain than with the double-layer winding, as may be seen by comparing diagrams *D,* Figs. 111 and 113.

133. Wave Windings.—The connections from coil to coil, marked *J* in diagram *A,* Fig. 113, and called jumpers, must have the same section as the conductors in the winding. When there are only two conductors per slot the conductors are large in section and the jumpers become expensive; in such a case the wave winding is generally used because it requires very few

jumpers. Such a winding is shown in Fig. 115, which shows the two- and three-phase diagrams.

134. Windings with Several Slots per Phase per Pole.—Modern alternators have seldom less than two slots per phase per pole. The principle advantages of the distributed winding are that the wave form is improved and the total radiating surface of the coils increased; the self-induction of the winding is reduced but that, as shown in Art. 209, page 282, is in some cases a disadvantage.

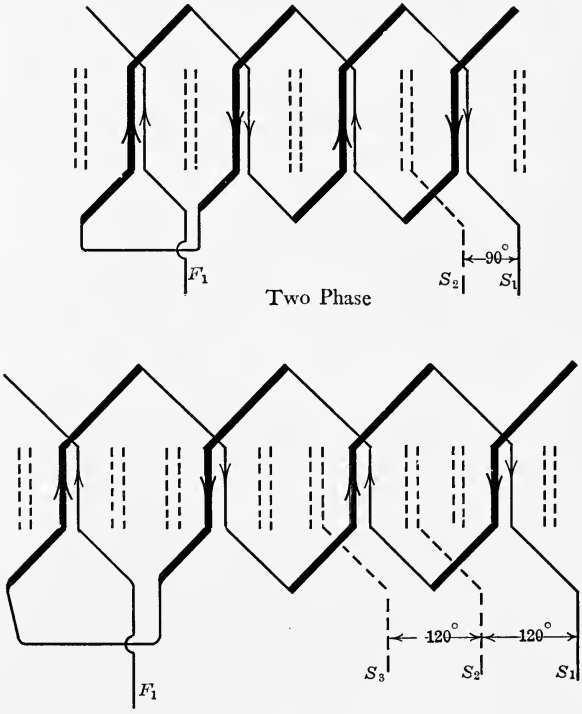


FIG. 115.—Four-pole wave winding.

135. Windings With Several Circuits per Phase.—The windings shown down to this point have all been single-circuit windings, that is, windings in which all the conductors of one phase are connected in series with one another. It is, however, often necessary to use more than one circuit.

Suppose, for example, that a large number of small alternators are to be built for stock, the winding used would be such that by

a slight change in the connections, it could be made suitable for a number of standard voltages.

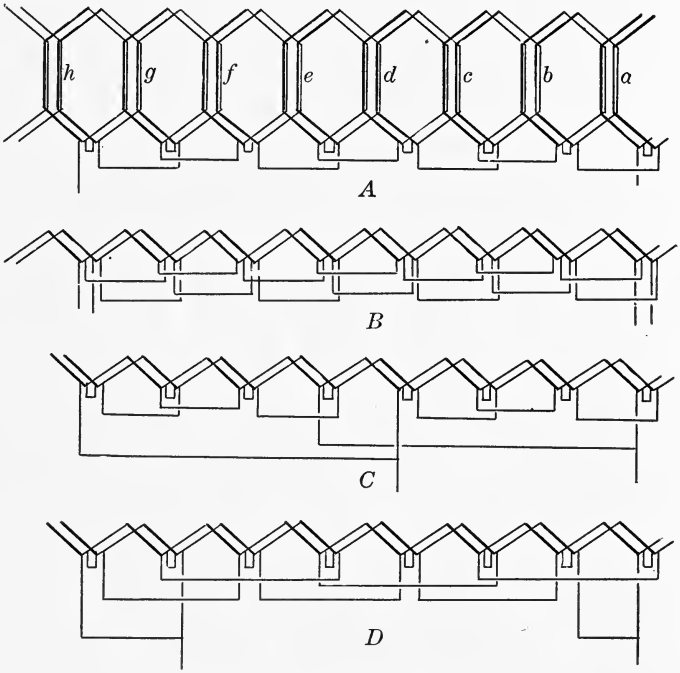


Fig. 116.—Two circuit windings.

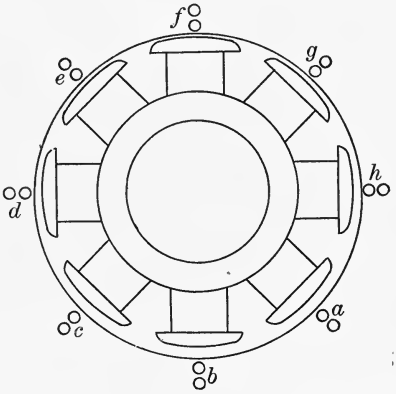


Fig. 117.—Alternator with an eccentric rotor.

Figure 116 shows the winding diagram for one phase of an eight-pole three-phase machine with two slots per phase per pole.

Diagram *A* shows a single circuit connection.

Diagrams *B*, *C* and *D* show possible two circuit connections.

When a winding is divided up into a number of circuits in parallel it is necessary, in order to prevent circulating currents,

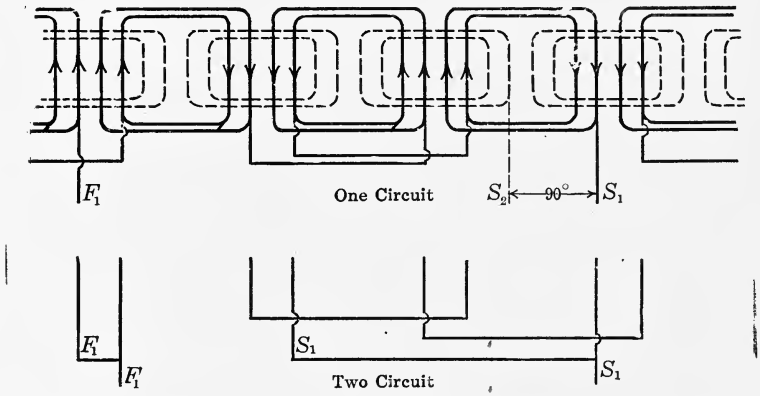


FIG. 118.—Four-pole, two-phase, chain winding with eight slots per pole.

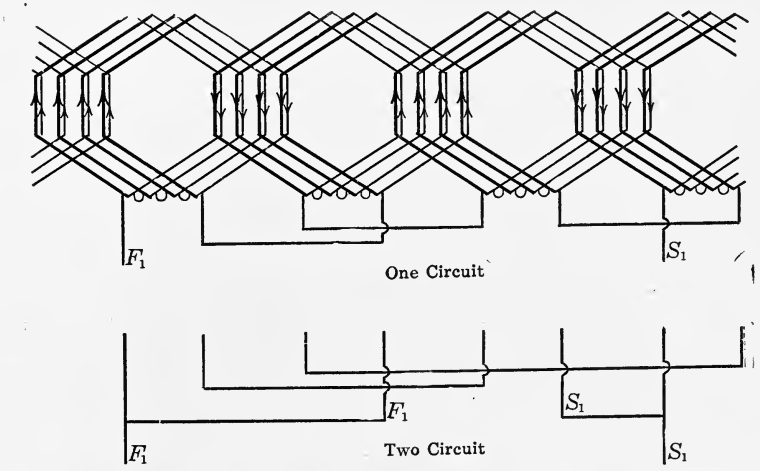


FIG. 119.—Four-pole, two-phase, double layer winding with eight slots per pole, one phase shown.

that the voltages in the different circuits in parallel be equal to and in phase with one another.

Diagram *B* does not meet this condition because the voltages in the two circuits shown, while of equal value, are out of phase,

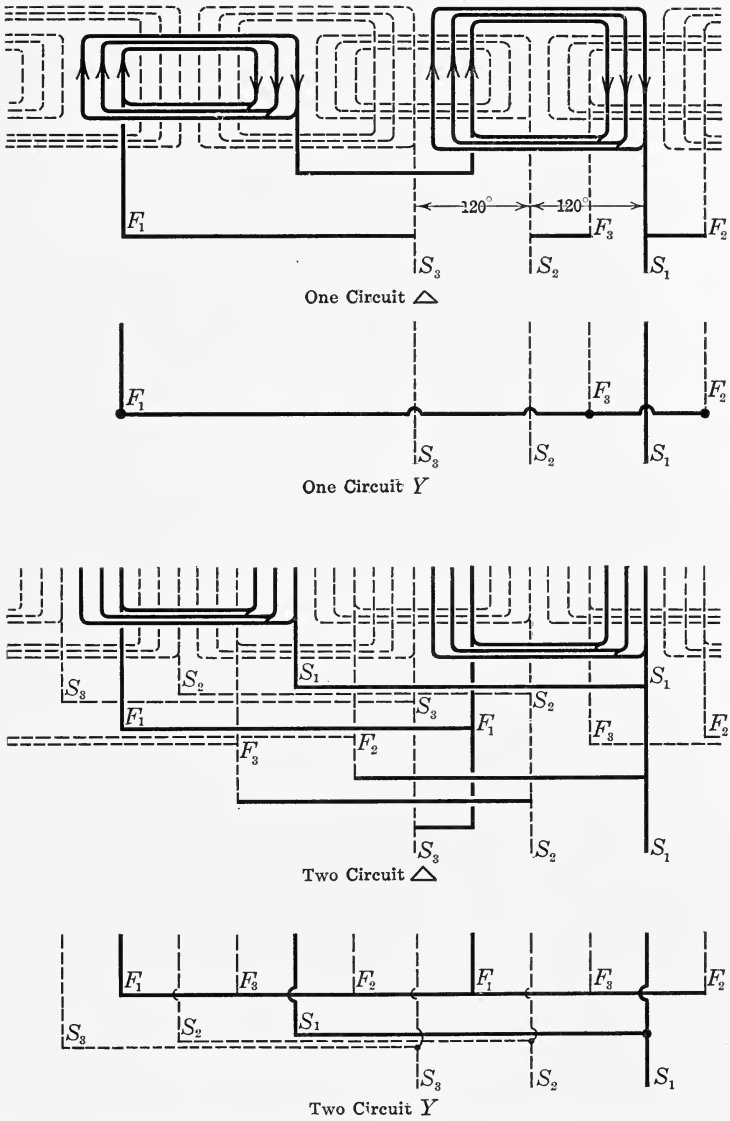


FIG. 120.—Four-pole, three-phase, chain winding with nine slots per pole.

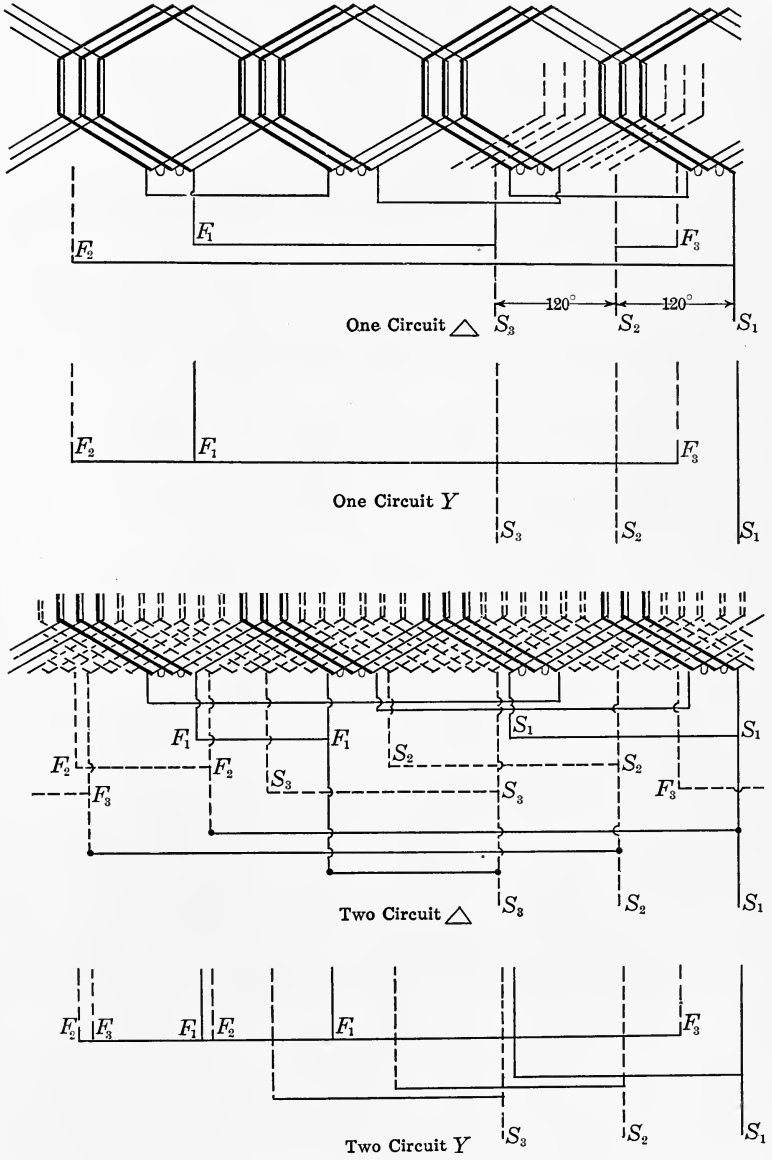


FIG. 121.—Four-pole, three-phase, double layer winding with nine slots per pole.

with one another by the angle corresponding to one slot-pitch, namely, by 30 degrees.

The winding shown in diagram *D* is to be preferred to that in diagram *C* for the following reason. Fig. 117 shows an eight-pole machine the field and armature of which are eccentric due to poor workmanship in erection. The voltage generated in a circuit made up of conductors in slots *a*, *b*, *c* and *d* is smaller than that

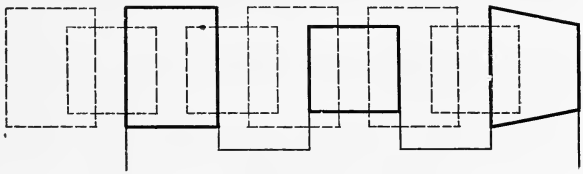


FIG. 122.—Six-pole, three-phase, chain winding with three slots per pole.

generated in a similar circuit wound in slots e, *f*, *g* and *h*, so that if these two circuits be put in parallel a circulating current will flow. Diagram *C*, Fig. 116, shows an example of such a winding.

In diagram *D* the connection is such that each of the two circuits takes in conductors from under all of the poles so that, no matter how different the air gaps become, the voltages in the two circuits are always equal.

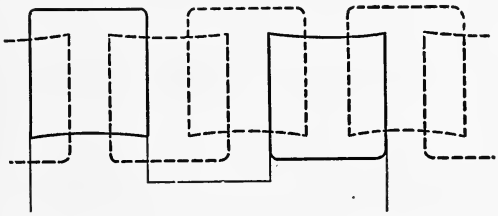


FIG. 123.—Three-phase chain winding with coils all alike.

136. Examples of Winding Diagrams.—Figs. 118, 119, 120 and 121 show typical alternator winding diagrams and should be carefully studied. Many other examples might have been shown but the subject is too wide to take up in greater detail.

When the number of groups of coils is odd in a machine with a chain winding, it is impossible to have the same number of groups of long as of short coils, and one group must be put into the machine the coils of which have one side long and the other

side short; such a winding is shown in Fig. 122 for a six-pole machine with three phases, one slot per phase per pole, and one coil per group.

By the use of specially shaped coils, as shown in Fig. 123, it is possible to reduce the number of coil shapes that are required for a chain winding. In the particular case shown the coils are all alike.

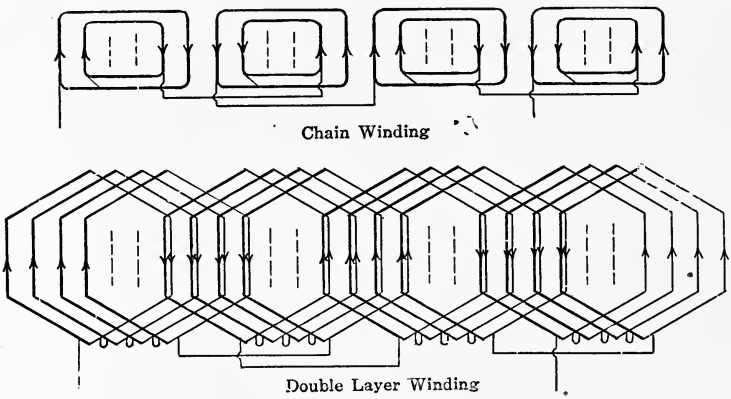


FIG. 124.—Four-pole, single-phase winding with four active slots per pole.

In a single-phase machine it is generally advisable, for the reason explained in Art. 145, page 187, to make the winding cover not more than two-thirds of the pole-pitch. Such a winding is shown in Fig. 124 for a machine with six slots per pole, of which only four are used.

CHAPTER XVIII

THE GENERATED ELECTRO-MOTIVE FORCE

137. The Form Factor and the E.M.F. per Conductor.

If ϕ_a is the flux per pole

and p the number of poles

then one armature conductor cuts $\phi_a p$ lines of force per revolution

$$\text{or } \phi_a p \frac{\text{r.p.m.}}{60} \text{ lines per second}$$

$$\text{and the average e.m.f. in one conductor} = \phi_a p \frac{\text{r.p.m.}}{60} 10^{-8} \text{ volts.}$$

The form factor of an e.m.f. wave is defined as the ratio

$$\frac{\text{effective voltage}}{\text{average voltage}} \text{ and for a sine wave of e.m.f. this value} = \frac{\frac{1}{\sqrt{2}} E_{max}}{\frac{2}{\pi} E_{max}}$$

$$= 1.11$$

$$\text{The effective e.m.f. per conductor} = \phi_a p \frac{\text{r.p.m.}}{60} 10^{-8} \times \text{form factor}$$

$$= 1.11 \ \phi_a p \frac{\text{r.p.m.}}{60} 10^{-8} \text{ for sine wave e.m.f.}$$

$$= 2.22 \ \phi_a f \ 10^{-8} \qquad (24)$$

$$\text{since } f, \text{ the frequency} = \frac{p \times \text{r.p.m.}}{120}$$

138. The Wave Form.—Fig. 125 shows the shape of pole face that is in general use and curve A shows the distribution of flux in the air gap under such a pole face. The e.m.f. in a conductor is proportional to the rate of cutting lines of force and has therefore a wave form of the same shape as the curve of flux distribution.

Curve A is not a sine wave but can be considered as the resultant of a number of sine waves consisting of a fundamental and harmonics. The frequency and magnitude of these harmonics depend principally on the ratio of pole arc to pole-pitch. In Fig. 125 this ratio is 0.65 and the fundamental and harmonics which go to make up the flux distribution curve are shown to scale, higher harmonics than the seventh being neglected.

139. Trouble due to Harmonics.—The fundamental and harmonics that go to make up an e.m.f. wave act as if each had a separate existence. If the circuit to which this e.m.f. is applied consists of an inductance L in series with a capacity C so that the impedence of the circuit

$$= 2\pi f L - \frac{1}{2\pi f C},$$

then the current in this circuit consists of

$$\text{a fundamental} = \frac{E_1}{2\pi f_1 L - \frac{1}{2\pi f_1 C}}$$

and harmonics of the form $\dfrac{E_n}{2\pi f_n L - \dfrac{1}{2\pi f_n C}}$

where E_n is the effective value of the nth harmonic. If now, f_n has such a value that $2\pi f_n L = \dfrac{1}{2\pi f_n C}$, so that the circuit is

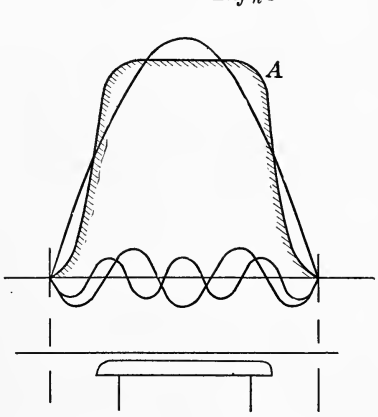


FIG. 125.—E. M. F. wave of an alternator.

in reasonance at this frequency, then the nth harmonic of current will be infinite, and the nth harmonic of e.m.f. across L and C individually will also be infinite.

The above is an ideal case; in ordinary circuits the current cannot reach infinity on account of the resistance that is always present, nevertheless, if the circuit is in resonance at the frequency of the fundamental or of any of the harmonics, dangerously high voltages will be produced between different points in the circuit. The constants of a circuit are seldom such as to

give trouble at the fundamental frequency, trouble is generally due to high frequency harmonics. It is desirable then to eliminate harmonics from the e.m.f. wave of a generator as far as possible and several of the methods adopted are described below.

140. Shape of Pole Face.—The pole face is sometimes shaped as shown in Fig. 126, that is, the air gap is varied from a minimum

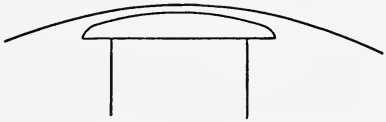


FIG. 126.—Pole face shaped to give a sine wave e. m. f.

under the center of the pole to a maximum at the pole tip, so as to make the flux distribution curve approximately a sine curve; then the e.m.f. wave from each conductor will be approximately a sine wave.

141. Use of Several Slots per Phase per Pole.—Fig. 127 shows part of a three-phase machine which has six slots per pole or

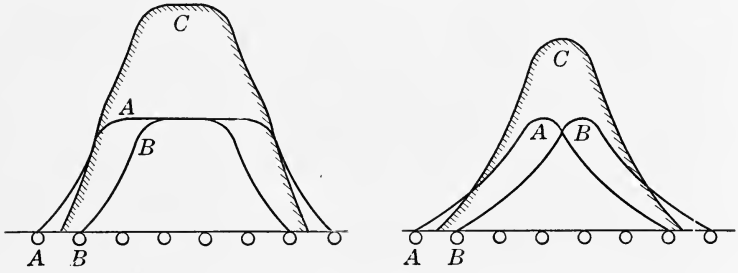


FIG. 127.—E. M. F. wave of a three-phase alternator with two slots per phase per pole.

two slots per phase per pole. The e.m.f. generated in a conductor in slot A is represented at any instant by curve A, and that in a conductor in slot B by curve B, which is out of phase with curve A by the angle corresponding to one slot-pitch, or 30 degrees.

When the conductors in slots A and B are connected in series so that their e.m.fs. add up, the resultant e.m.f. at any instant is given by curve C, which is got by adding together the ordinates of curves A and B. C is more nearly a sine curve than either A or B.

If the fundamental and harmonics that go to make up curves

A and B are known, then those which go to make up curve C can be readily found as follows: The fundamental of the resultant wave C is the vector sum of the fundamentals of A and B, which are θ electrical degrees apart, and the nth harmonic of C is the vector sum of the nth harmonics of A and B which are $(n \times \theta)$

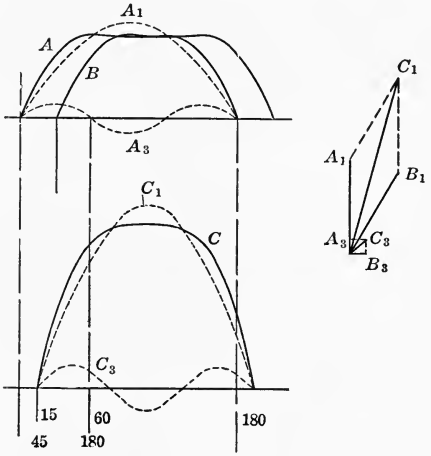


FIG. 128.—The vector diagram for the fundamental and the harmonics.

electrical degrees apart. For example, curves A and B, Fig. 128 are 30 degrees out of phase with one another and each consists of a fundamental and a third harmonic as shown. C_1 is the resultant of the fundamentals A_1 and B_1 and C_3, the third harmonic of curve C, is the resultant of the two third harmonics A_3 and B_3.

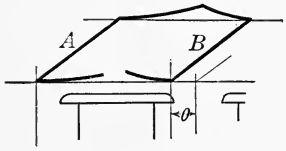


FIG. 129.—Short-pitch coil.

142. Use of Short-pitch Windings.—Fig. 129 shows a short-pitch coil. The e.m.f. waves in the conductors A and B are out of phase with one another by θ degrees, but each has the same shape as the curve of flux distribution. The problem of finding the resultant e.m.f. wave is therefore the same as that discussed in the last article.

If $n\theta$, the phase angle between the nth harmonics of curves A and B, becomes equal to 180 degrees, then the corresponding harmonic is eliminated from the resultant curve C since the two

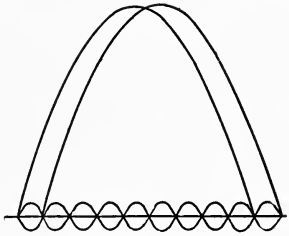


FIG. 130.—Elimination of harmonics from the e. m. f. wave.

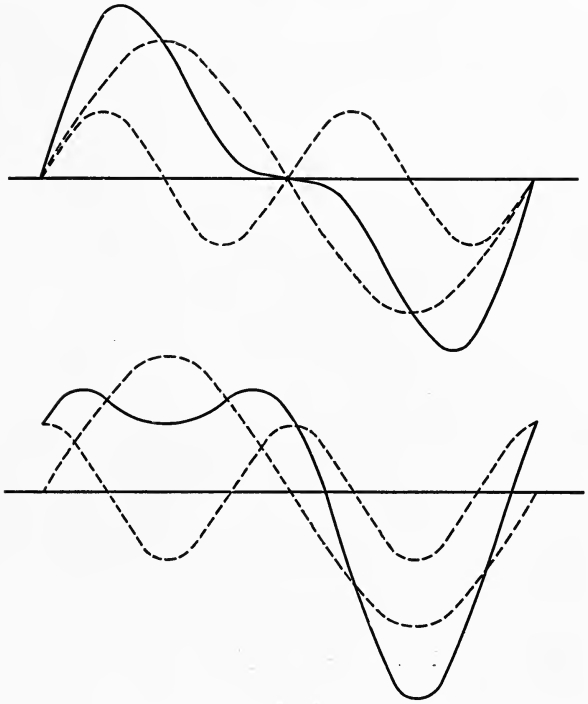


FIG. 131.—Unsymmetrical waves due to even harmonics.

harmonics which go to make it up are equal and opposite. If, for example, there are 9 slots per pole and the coil is one slot short, then the angle θ is 20 electrical degrees, and the 9th harmonic is eliminated from the voltage wave of the coil; in general, if the

pitch of the coil be shortened by $\frac{1}{n}$ of the pole-pitch then, as shown in Fig. 130, the nth harmonic will be eliminated

It may be pointed out here that an even harmonic is seldom found in the e.m.f. wave of an alternator, because the resultant of a fundamental and an even harmonic gives an unsymmetrical curve, as shown in Fig. 131, where the resultant curve is made up

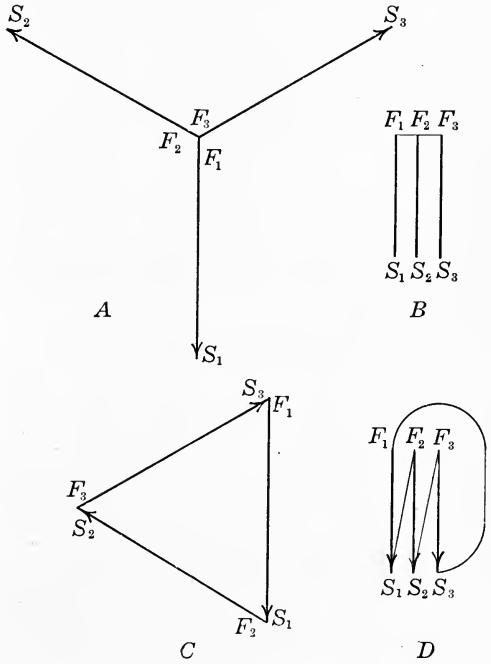


FIG. 132.—Effect of the Y- and Δ-connection on the third harmonic.

of a fundamental and a second harmonic. If then the e.m.f. wave is symmetrical it may be assumed that no even harmonics are present.

143. Effect of the Y- and Δ-Connection on the Harmonics. —The fundamentals of the three e.m.fs. are 120 degrees out of phase with one another and are represented by vectors in diagram *A*, Fig. 132. The nth harmonics are $(n \times 120)$ degrees out of phase with one another.

When the phases are Y-connected the terminal F_2 is brought to the potential of terminal F_1 and the resultant fundamental

between S_1 and S_2 is represented by the vector S_1S_2 and $=1.73$ times the fundamental in one phase. In the case of the third harmonic the e.m.fs. are $(3\times120)=360$ degrees out of phase with one another and are represented by vectors in diagram *B;* the resultant third harmonic between S_1 and S_2 is zero so that, in a Y-connected alternator, no third harmonic, nor any harmonic which is a multiple of three, is found in the terminal voltage wave.

When the phases are Δ -connected, any harmonic in the voltage wave of one phase will also be found in that of the terminal voltage; a greater objection to the use of this connection for alternators is that the harmonics cause circulating currents to flow in the closed circuit produced by the Δ -connection. Diagram *C* shows the voltage vector diagram for the fundamentals in the e.m.f. wave of each phase; the three vectors are 120 degrees out of phase with one another and the resultant voltage in the closed circuit due to the fundamentals is zero.

Diagram *D* shows the voltage vector diagram for the third harmonic in the e.m.f. wave of each phase, the three vectors are 360 degrees out of phase with one another and the resultant voltage in the closed circuit due to the third harmonics is three times the value of the third harmonic in one phase. A circulating current will flow in the closed circuit, of triple frequency and of a value $=\frac{3E_3}{3z_3}$, where E_3 is the effective value of the third harmonic in each phase and z_3 is the impedence per phase to the third harmonic.

144. Harmonics Produced by Armature Slots.—Fig. 133 shows two positions of the pole of an alternator relative to the armature. In position *A* the air gap reluctance is a minimum and in position *B* is a maximum. The flux per pole pulsates, due to this change in reluctance, once in the distance of a slot-pitch, or $2a$ times in the distance of two pole-pitches, where $a=$ slots per pole, and the e.m.f. generated in each coil by the main field goes through one cycle while the pole moves, relative to the armature, through the distance of two pole-pitches, therefore the frequency of the flux pulsation $=2af$.

The flux per pole then consists of a constant value ϕ_a, and a superimposed alternating flux which has a frequency of $2a$ times the fundamental frequency of the machine, or at any instant the flux per pole

$$=\phi_p=\phi_a+\phi_1 \cos 2a\theta$$

where $\phi_a+\phi_1$ is the maximum value of the pulsating flux and θ is the angle moved through from position A, Fig. 133, in electrical degrees; therefore the flux threading coil C, which is a full-pitch coil,

$=\phi_a+\phi_1$ when the coil is in position A

$=[\phi_a+\phi_1 \cos 2a\theta]\cos\theta$ when the coil has moved through θ electric degrees relative to the pole,

and the e.m.f. in coil C at any instant

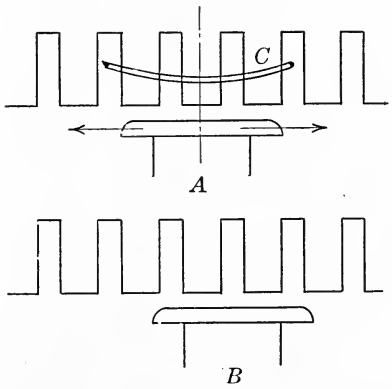


FIG. 133.—Variation of the air gap reluctance.

$$=-T\frac{d(\phi_a+\phi_1 \cos 2a\theta)\cos\theta}{dt}$$

$$=-T\frac{d(\phi_a+\phi_1 \cos 2a\theta)\cos\theta}{d\theta}\times\frac{d\theta}{dt}$$

$$=-2\pi f[-\sin\theta(\phi_a+\phi_1\cos 2a\theta)-\cos\theta(2a\phi_1 \sin 2a\theta)]T$$

$$=2\pi f[\phi_a \sin\theta+\phi_1\sin\theta \cos 2a\theta+2a\phi_1 \cos\theta \sin 2a\theta]T$$

$$=2\pi f[\phi_a \sin\theta+\frac{\phi_1}{2}\{\sin(2a+1)\theta-\sin(2a-1)\theta\}+\frac{2a\phi_1}{2}\{\sin(2a+1)\theta+\sin(2a-1)\theta\}]T$$

$$=2\pi fT[\phi_a \sin\theta+\frac{\phi_1+2a\phi_1}{2}(\sin(2a+1)\theta)+\frac{\phi_1-2a\phi_1}{2}(\sin(2a-1)\theta)]$$

so that, due to the variation of the air gap reluctance as the poles move past the armature slots, two harmonics are produced which have frequencies of $(2a+1)$ and of $(2a-1)$ times that of the fundamental respectively.

A convenient physical interpretation of the above result is as follows: A and B, Fig. 134, are two equal trains of waves which are constant in magnitude, and move in opposite directions at the same speed. The resultant of two such wave trains superimposed on one another is a series of stationary waves as shown at C.

A stationary wave such as that of the alternating magnetic field in the air gap of an alternator due to a variation in the air gap reluctance, can therefore be exactly represented by two

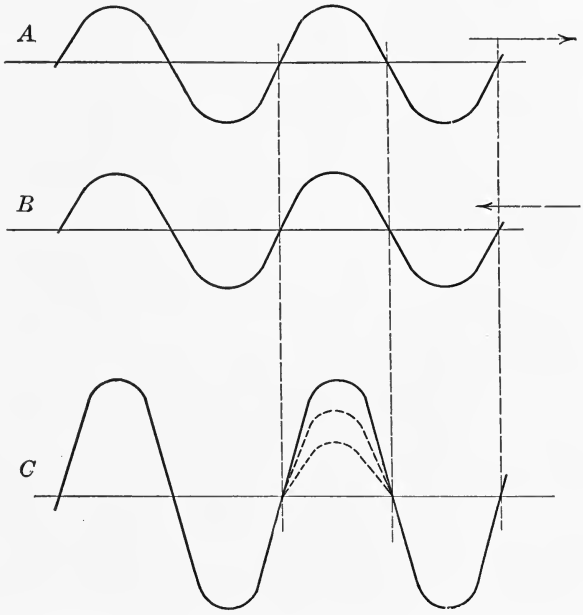


FIG. 134.—Resolution of a stationary wave into two progressive waves.

progressive waves of constant value which move in opposite directions through the distance of two pole-pitches while the alternating wave goes through one cycle.

Consider both of these waves or fields to exist separately from the main field then, when the pole moves with the constant flux ϕ_a, through a distance y relative to the armature, one of these constant progressive fields moves through a distance $2ay$ relative to the pole, or through a distance $(2a + 1)y$ relative to the armature, while the other moves through a distance $-2\,ay$ relative to the poles, or through a distance $-(2a-1)y$

relative to the armature. If then the fundamental frequency of the generated e.m.f. is f, the two other fields will generate e.m.fs. of frequencies $= (2a + 1)f$ and $(2a - 1)f$ respectively.

To keep down the value of these harmonics the reluctance of the air gap under the poles should be made as nearly constant as possible for all positions of the pole relative to the armature.

145. Effect of the Number of Slots on the Terminal E.M.F.—Fig. 135 shows the winding diagrams for an alternator with

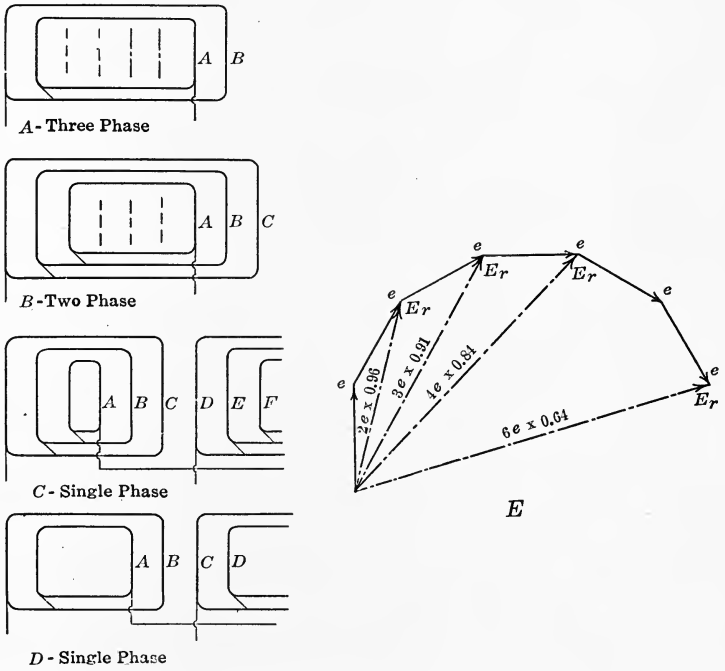


FIG. 135.—Effect of the distribution of the winding on the terminal voltage.

six slots per pole and wound for single, two- and three-phase respectively, the same punching being used in each case. The e.m.fs. in the conductors in adjacent slots are out of phase with one another by the angle corresponding to one slot-pitch $= \frac{180}{6}$ $= 30$ electrical degrees.

In the three-phase winding the conductors in slots A and B are connected in series so that their voltages act in the same direction; the resultant voltage E_r, diagram E, is not equal to

$2e$, where e is the voltage per conductor, but is the resultant of two e.m.fs. e which are 30 degrees out of phase with one another and $= 2e \times 0.96$.

In the case of the two-phase winding the conductors A, B and C are connected so that their voltages add up and the resultant voltage $E_r = 3e \times 0.91$.

In the case of the single-phase winding where all the conductors are used the resultant voltage $E_r = 6e \times 0.64$, while if only four of the six slots per pole are used, as shown in diagram D, the resultant voltage $E_r = 4e \times 0.84$, which is only 10 per cent. lower than that obtained when all six slots are used. A gain of 10 per cent. in voltage is not worth the cost of the 50 per cent. increase in armature copper that is required, so that single-phase machines are generally wound as shown in diagram D.

146. Rating of Alternators.—The maximum voltage that an alternator can give continuously is limited by the permissible value of the flux per pole, and the maximum current is limited by the armature copper loss which, along with the core loss, heats the machine. When the value of volts and amperes is fixed, the kilowatt rating depends only on the power factor of the load. The power factor is a variable quantity, and one over which the builder of the machine has no control, so that an alternator is generally rated by giving the product of volts and amperes, which is called the volt ampere rating, and this quantity divided by 1000 gives the rating in k.v.a. (kilovolt amperes).

147. Effect of the Number of Phases on the Rating.—Consider the four machines whose winding diagrams are shown in Fig. 135, and let the number of conductors per slot be the same in each, then the voltage per phase is given in the following table:

Number of phases	Voltage per phase
Single-phase (all slots used)	constant $\times 6e \times 0.64$.
Single-phase (2/3 of slots used)	constant $\times 4e \times 0.84$.
Two-phase	constant $\times$ $3e \times 0.91$.
Three-phase	constant $\times$ $2e \times 0.96$.

Since there are the same number of conductors per slot, these conductors have the same section and therefore carry the same current I_c; the volt ampere rating, which equals volts per phase $\times$ current per phase $\times$ number of phases, is given in the following table:

Number of phases	Volt ampere rating
Single-phase (all slots used)	constant $\times 6e \times 0.64 \times 1 \times I_c$.
Single-phase (2/3 of slots used)	constant $\times 4e \times 0.84 \times 1 \times I_c$.
Two-phase	constant $\times 3e \times 0.91 \times 2 \times I_c$.
Three-phase	constant $\times 2e \times 0.96 \times 3 \times I_c$.
	= a constant $\times 0.64$.
	= a constant $\times 0.56$.
	= a constant $\times 0.91$.
	= a constant $\times 0.96$.

In practice the machine is given the same rating for both two- and three-phase windings, although the three-phase machine is the better, and is given 65 per cent. of this rating when wound for single-phase operation.

148. The General E.M.F. Equation.—It is shown in Art. 145 that, when the winding of an alternator is distributed, the terminal voltage is less than $Z \times e$

where Z = conductors in series per phase

e = volts per conductor

and is equal to kZe where k is the distribution factor and is found from the following table, which is worked up by the method explained in Art. 145:

Slots per phase per pole	Distribution factor	
	Two-phase	Three-phase
1	1.0	1.0
2	0.924	0.966
3	0.911	0.96
4	0.906	0.958
6	0.903	0.956

The single phase results are not tabulated since they depend on the number of slots that are used by the winding.

When a short-pitch is used, as is often done with double layer windings, then, as shown in Fig. 136, which shows part of a three-phase double layer winding with three slots per phase per pole, the two adjacent belts A and B are out of phase with one another by θ degrees, and, as shown in Fig. 137, the resultant voltage, when these two belts are put in series, is equal to twice the voltage in one belt multiplied by $\cos \frac{\theta}{2}$.

It was shown in Art. 137 that the effective e.m.f. per conductor $= 2.22\ \phi_a f\ 10^{-8}$ volts.

If the winding is full-pitch and is not distributed then the voltage per phase $= 2.22\ Z\ \phi_a f\ 10^{-8}$ volts.

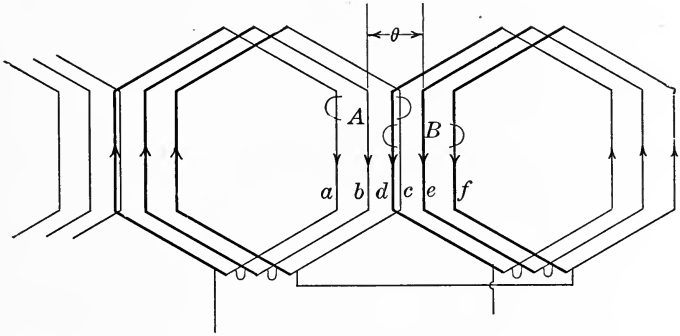


FIG. 136.—Short-pitch winding.

If the winding is full-pitch and is distributed, the voltage per phase

$$= 2.22\ kZ\phi_a f\ 10^{-8} \tag{25}$$

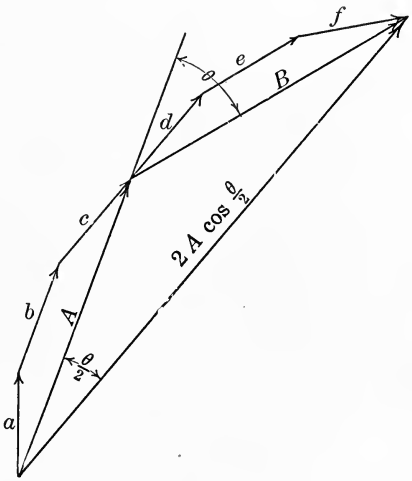


FIG. 137.—Vector diagram for a short-pitch winding.

and finally, if the winding is short-pitch, so that the winding belts are out of phase with one another by θ degrees, and is also distributed, then the voltage per phase

$$= 2.22\ kZ\phi_a f\ 10^{-8} \cos \frac{\theta}{2} \tag{26}$$

CHAPTER XIX

CONSTRUCTION OF ALTERNATORS

Figure 138 shows the type of construction that is generally used for alternators; it is known as the revolving field type.

149. The Stator.—In the revolving field type of machine the stator is the armature. *B*, the stator core, is built up of lamina-

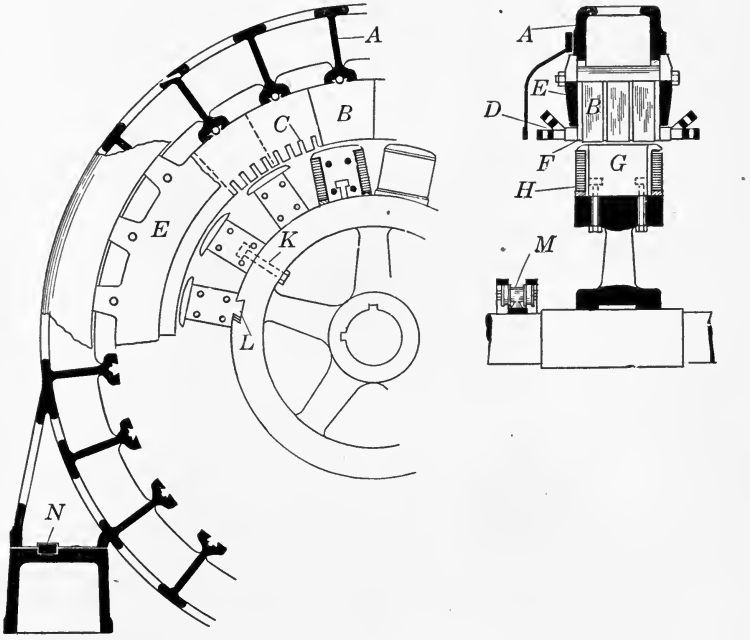


FIG. 138.—Revolving field alternator.

tions of sheet steel 0.014 in. thick, which are insulated from one another by layers of varnish and are then mounted in a self-supporting cast-iron yoke *A*. These laminations are punched on the inner periphery with slots *C* which carry the stator coils *D*. The type of slot shown is the open slot; it has the advantage over the closed slot that the coils can be fully insulated be-

fore being put into the machine and can also be more easily repaired.

The stator core is divided into blocks by means of vent segments of cast brass, and the ducts thereby provided allow air to circulate freely through the machine to keep it cool. The vent ducts are spaced about 3 in. apart and are half an inch wide.

The stator laminations and vent segments are clamped between two cast-steel end heads *E*. When the teeth are long they are supported by strong finger supports placed at *F*, between the end heads and the end punchings of the core.

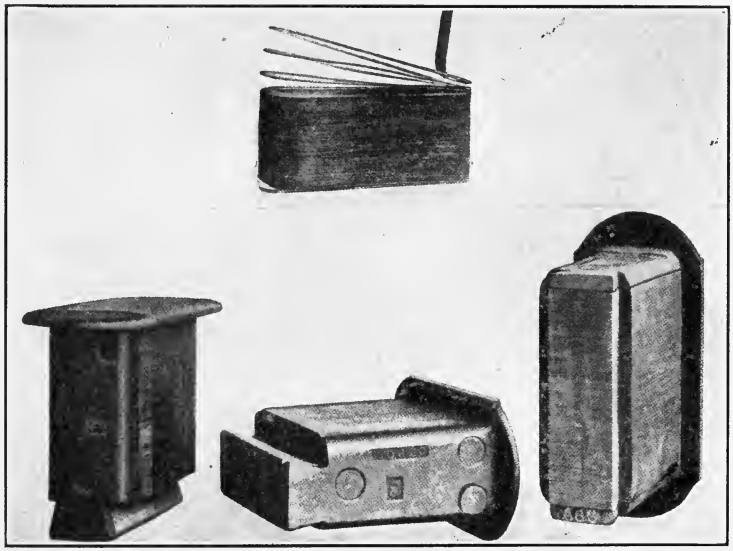


Fig. 139.—Poles and field coil.

When the external diameter of the stator core is less than 30 in. the core punching is generally made in a complete ring; when this diameter is greater than 30 in. the core is generally built up in segments which, as shown in Fig. 138, are fixed to the yoke by means of dovetails; the segments of adjacent layers of laminations break joint with one another so as to overlap and produce a solid core.

150. Poles and Field Ring.—Inside of the stator revolves the rotor or revolving field system. The poles *G* carry the exciting

coils H and are excited by direct current from some external source.

The excitation voltage is independent of the terminal voltage of the machine and is generally chosen low, so that for a given excitation the field current will be comparatively large and the field coils will have few turns.

For the usual excitation voltage of 120 it will generally be possible, except on the smaller machines, to use the type of field winding shown in Fig. 139, which is made by bending strip copper on edge; the layers of strip copper are insulated from one another by layers of paper about 0.01 in. thick, and the whole field coil is supported by and insulated from the poles and field ring as shown at A, Fig. 140. For small machines the excitation loss is com-

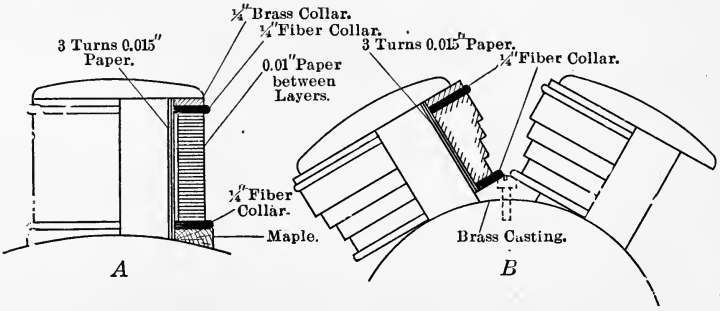


FIG. 140.—Alternator field coils.

paratively low, and with an excitation voltage of 120 the section of the wire is too small and the number of turns required too large to allow the use of a strip copper coil; in such cases double cotton-covered square wire is used as shown at B, Fig. 140; the coils are tapered so as to allow free circulation of air around them.

Since the number of poles in an alternator is fixed by the speed and the frequency, it rarely happens that this number is such as to allow the use of a pole of circular section; the pole is generally rectangular in section and, as shown in Fig. 139, is built up of punchings of sheet steel 0.025 in. thick which are riveted together between two cast-steel end plates.

The poles are generally attached to the field ring by means of bolts as shown at K, Fig. 138, or by means of dovetails and tapered keys as shown at L; two tapered keys are used which are driven in from opposite ends.

The exciting current is led into the revolving field system by means of brushes which bear on cast-iron or cast-brass slip rings; the slip rings are carried by and insulated from the shaft as shown at *M;* the brushes are generally of soft self-lubricating carbon and carry about 75 amperes per square inch.

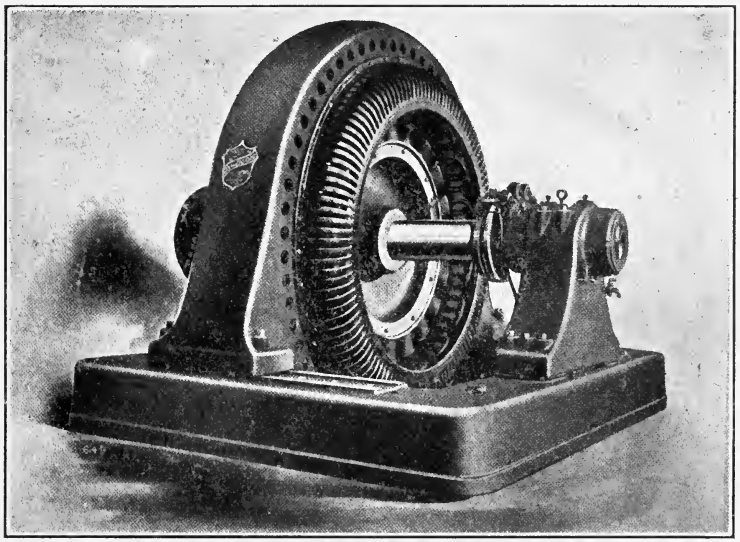


Fig. 141.—Revolving field alternator.

The stator of an alternator is seldom split except in the case of very large machines where it is done for convenience in shipment. In order that the stator windings can be examined and easily repaired it is advisable to arrange that the whole stator slide axially on the base, and the key shown at *N* is for the purpose of keeping the alignment correct.

Fig. 141 shows a 1000-k.v.a. water-wheel driven alternator of the type described in this chapter.

CHAPTER XX

INSULATION

The insulation of low-voltage machines has been discussed in Chapter IV and presents no particular difficulty, since insulation which is strong enough mechanically is generally ample for electrical purposes up to 600 volts. For higher voltages, however, the thickness of insulation required to prevent breakdown is great compared with that required for mechanical strength and, unless such insulation is carefully designed, trouble is liable to develop.

151. Definitions.—If a difference of electric potential be

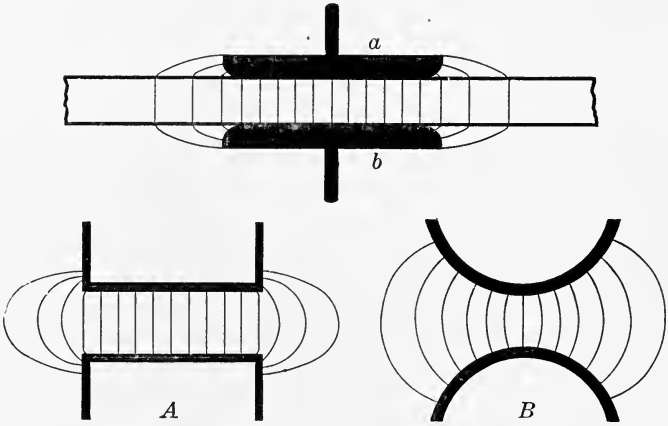


FIG. 142.—Distribution of the dielectric flux.

established between two electrodes *a* and *b*, Fig. 142, which are separated by an insulating material or dielectric, a molecular strain will be set up in the dielectric.

This molecular strain is conveniently represented by lines of dielectric flux, and the number of lines per unit area, which is called the *Dielectric Flux Density*, is taken as a measure of the strain.

When the dielectric flux density reaches a certain critical value

the material is disrupted and loses its insulating properties; this critical value depends on the nature, thickness and condition of the material.

The distribution of dielectric flux depends largely on the shape of the electrodes, as shown in diagrams *A* and *B*, Fig. 142. When an insulating material of uniform thickness t is placed between two parallel plates and subjected to a voltage E, the dielectric flux density or molecular strain is uniform through the total thickness of the material and is conveniently represented by the ratio $\frac{E}{t}$, the volts per unit thickness. Under such conditions of test, the highest effective alternating voltage that 1 mil (0.001 in.) thickness of the material will withstand for 1 minute is generally called its *Dielectric Strength.*

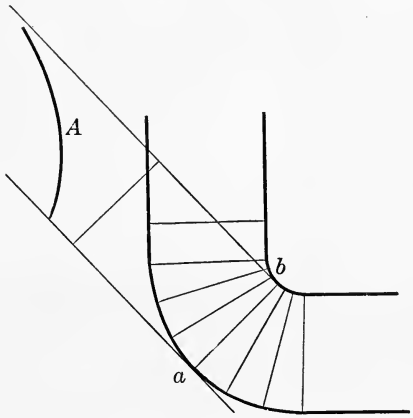


Fig. 143.—Potential gradient at a slot corner.

When the dielectric flux density is not uniform throughout the total thickness of the material, the ratio $\frac{E}{t}$, the volts per unit thickness, has little meaning. In Fig. 143 for example, which shows the dielectric flux distribution at the corner of a slot, it will be seen that the dielectric flux density, or molecular strain, is greatest at the surface of the conductor and decreases as the lines spread out. Under such conditions the strain at any point is conveniently expressed by what is known as the *Potential Gradient* at the point, where this quantity is the volts per unit thickness that would be required to set up the same dielectric

flux density as that at the point in question if the material were of uniform thickness and tested between two parallel plates. The potential gradient across *ab* is given by curve *A*.

152. Insulators in Series.—If an air film be placed between two electrodes and subjected to a difference of potential, a certain dielectric flux density will be produced in the air. If now the air be replaced by mica, a greater dielectric flux density will be produced for the same difference of potential. The *Specific Inductive Capacity* of an insulating material is defined as

$$\frac{\text{the dielectric flux density in the material}}{\text{the dielectric flux density in air}}$$

for the same value of volts per mil.

Figure 144 shows two cases of dielectric subjected to a difference of electric potential between electrodes of the same size and the same distance apart. In *A* the dielectric is air, and in *B* is made up of air and mica in series. Since the specific inductive capacity of mica is greater than that of the air which it replaces,

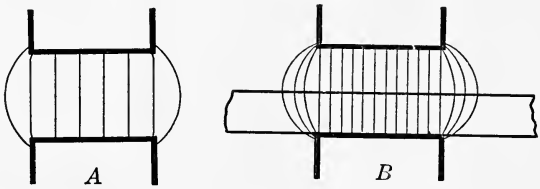


FIG. 144.—Effect of the specific inductive capacity of the dielectric on the dielectric flux density.

being about 6, the dielectric flux density is greater in *B* than in *A* for the same difference of potential between the electrodes, and the air in *B* is subjected to a greater strain than that in *A*, so that it will break down at a lower value of voltage between the terminals, although at the same value of volts per mil.

Since in *B* the two materials, air and mica, are in series, the dielectric flux density is the same in each and therefore, from the definition of specific inductive capacity,

$$\frac{\text{the volts per mil thickness in the mica}}{\text{the volts per mil thickness in the air}} = \frac{1}{\text{sp. ind. cap of mica}}$$

$$= \frac{1}{6} \text{ approximately}$$

The greater the thickness of mica in the total thickness between electrodes the larger will be the dielectric flux density

and the greater therefore the value of volts per mil thickness in the air for a given voltage between the electrodes.

153. Effect of Air Films in Insulation.—From the above discussion it will be seen that, should there be an air film in the thickness of a solid dielectric, then, for a perfectly conservative value of volts per mil of total thickness of the dielectric, the volts per mil across the air film may be sufficient to disrupt it if the solid dielectric have a specific inductive capacity greater than one.

When air is disrupted ozone and oxides of nitrogen are formed, these oxidize nearly all the insulators used for electrical machinery except mica and thereby seriously impair their insulating properties.

154. The Design of Insulation.—The voltage that a given insulation will stand depends on

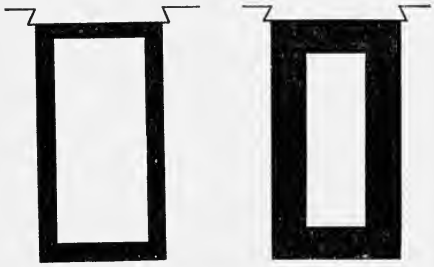


FIG. 145.—Effect of the voltage on the thickness of the slot insulation.

The thickness of the insulation;
The dielectric strength of the material;
The potential gradient across the material;
The length of time that the voltage is applied.

155. The Thickness of the Insulation.—Fig. 145 shows the slot of an alternator insulated in the one case for high voltage and in the other case for low voltage. If the space occupied by insulation could be filled with copper the output of the machine could be considerably increased, so that the solution of high voltage insulation problems is not solved economically by indefinitely increasing the thickness of the dielectric with increasing voltage, but by the selection and proper use of the most suitable materials.

156. The Potential Gradient.—Insulating materials break down wherever they are overstressed, and if the stress is not uniform

they break down first at the point of highest stress; it is therefore necessary to make a study of the distribution of stress, or of the potential gradient in the material.

Consider the case of the slot corner shown in Fig. 143, the lines of dielectric flux pass radially from the conductor to the side of the slot, so that the dielectric flux density is a maximum at the surface of the conductor and a minimum at the surface of the slot and the potential gradient curve, if the dielectric is of the same material throughout, is as shown in diagram *A*. The inner layers of the insulation therefore carry more than their share of the voltage, and these layers break down long before the stress in the outer layers reaches the breakdown point.

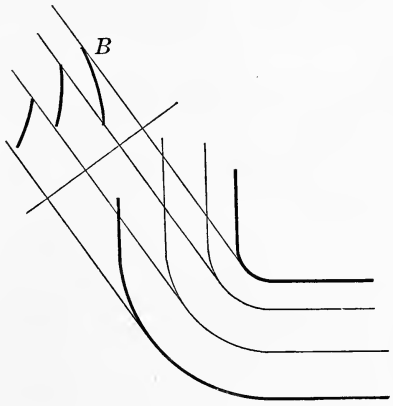


FIG. 146.—Potential gradient with graded insulation.

It is possible to make the outer layers carry their share of the total voltage by grading the insulation in the following way: Materials having different specific inductive capacities are used and put on in layers in such a way that the lower the specific inductive capacity the further away is the material from the conductor. This relieves the strain on the inner layers because, as pointed out in the discussion of insulators in series, the voltage required to send a given dielectric flux through a layer of insulating material is inversely as the specific inductive capacity of the material, so that the higher the specific inductive capacity of the inner layers the lower the voltage drop across these layers and the higher therefore the voltage drop across the outer layers.

The potential gradient for such insulation, made up in three layers, is shown in diagram *B*, Fig. 146.

The potential gradient can be controlled in many cases by a slight alteration in the shape of the surfaces to be insulated from one another. Fig. 147 shows three cases of slot insulation, and it is evident that the potential gradient is more uniform in case *B* than in case *A*, while case *C* is the best of the three because of the extra thickness of the dielectric at the corner; in this last case the insulation generally punctures between the parallel sides of

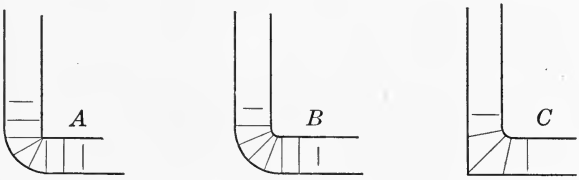


FIG. 147.—Effect of the shape of the surfaces to be insulated on the distribution of dielectric flux.

the slot and conductor, and not at the corner. Since the stress is uniform through the thickness of the insulation when it is between two parallel surfaces, grading of the insulation has no advantages for machines whose slots have square corners.

157. Time of Application of Electric Strain.[1]—That the voltage at which an insulating material will puncture depends on the

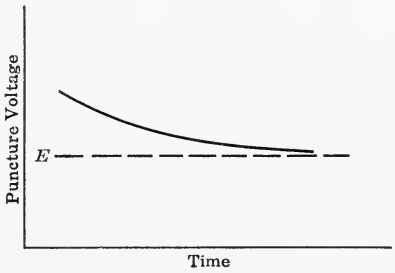


FIG. 148.—Effect of time on the puncture voltage.

length of time that this voltage is applied is shown by the curve in Fig. 148. *E* is the maximum voltage that the material will withstand for an infinite length of time without deterioration due to heating and consequent puncture.

If air films are present in the insulation then a lower voltage than *E* will cause puncture if applied for some time, but the action in this case is a secondary one.

[1]Fleming and Johnson, *Journal of the Institution of Elect. Eng.*, Vol. 47, page 530.

In Art. 152, page 197, it was pointed out that the stress on an air film bedded in a material of specific inductive capacity greater than one is very high, and that the film may become ruptured and ozone and oxides of nitrogen be produced which attack the other insulation causing deterioration and consequent puncture.

The amount of these gases produced by the rupture of a thin air film is not enough to do much harm unless there is a constant supply of air to the film. When an electrical machine is started up its coils become heated and, since the gases in the film expand, some of them are expelled. When the machine is shut down the coils cool off, the gases in the film contract, and a fresh supply of air is drawn in. This action is known as the breathing action of the coils.

Trouble due to this breathing action takes months to develop and usually shows up as a breakdown between adjacent turns; the insulation between these turns having become brittle due to oxidization, readily pulverizes due to vibration. The trouble can be eliminated by constructing the coils so that they contain no air pockets, and in the endeavor to do this various methods have been adopted for impregnating the coils and sealing their ends. The compounds generally used for impregnating purposes are made fluid by heating to a temperature of about 100° C., and in cooling to normal temperatures most of them contract about 10 per cent. and this 10 per cent. becomes filled with air.

Since, with the present methods of insulating, it may be considered impossible to eliminate all the air pockets from a coil, it is necessary to keep the stress in the air films below that value at which they will rupture. This can be done by increasing the total thickness of the insulation and also by the use of material which has a low specific inductive capacity. It is unfortunate that mica, which is one of the most reliable of insulators, has a high specific inductive capacity; a good composite insulator can be made up of mica paper, which consists of a layer of mica backed by a layer of paper; the specific inductive capacity of the former is about six, and of the latter is about two, while the combination has a value between these two figures.

Such an insulation is described fully on page 205 and may be expected to withstand 45 volts per mil indefinitely without trouble due to the breakdown of air films. For such insulation then the minimum thickness in mils between the conductor and

the side of the slot $= \frac{\text{volts from cond. to ground}}{45}$. If the insulation were made up entirely of mica a greater total thickness would be required on account of the high specific inductive capacity of the mica, while if made up entirely of paper a smaller total thickness would be required so far as the breakdown of the air film is concerned, but paper is not reliable as an insulator when used alone.

158. Design of Slot Insulation.—The following points are taken up fully in Chapter IV:—

The materials in general use are; micanite and empire cloth, which are used principally on account of their dielectric strength;

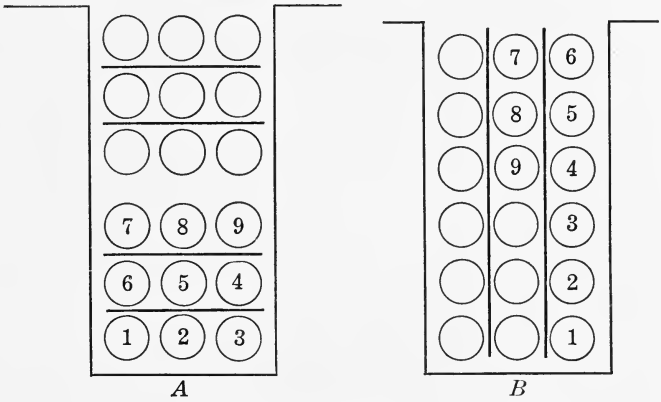


Fig. 149.—Insulation between layers of conductors.

tape, which is used principally to bind the conductors together; paper, which is used to protect the other insulation.

The puncture test recommended by the American Institute of Electrical Engineers is given in the table on page 34.

The end connections should be insulated for the full voltage between the terminals.

To minimize surface leakage the slot insulation should be carried beyond the core for a distance which depends on the voltage and which is found from the table on page 35.

159. Insulation between Conductors in the Same Slot.—In high-voltage alternators the number of turns per coil is large and the size of the wire comparatively small. Fig. 149 shows sections through two alternator slots; in *A* the winding is double layer and in *B* is chain; the conductors are numbered in the order in which they are wound.

Between conductors 1 and 2, 2 and 3, 3 and 4, etc., there is the voltage of only one turn or of two conductors, between 2 and 5 there is the voltage of six conductors and between 1 and 6 the voltage of ten conductors. These conductors are usually of double cotton-covered wire, and it has been found advisable to increase this insulation, by putting in layers of empire cloth as shown, when the voltage between adjacent conductors exceeds 25 volts, because the impregnation of the cotton may not be thorough, and the cotton covering may become damaged when the conductors are squeezed together.

160. Examples of Alternator and Induction Motor Insulation.—

Example 1.—Insulation for a 440-volt induction motor with a wire-wound coil and a double layer winding.

A section through the slot and insulation is shown in Fig. 150, and the insulation consists of:

(a) Double cotton covering on the conductors.

(b) A layer of empire cloth 0.006 in. thick between horizontal layers of conductors.

(c) One turn of paper 0.01 in. thick on the slot part of the coil to hold the conductors in layers.

(d) One layer of half-lapped empire cloth tape 0.006 in. thick all round the coil.

(e) One turn of paper 0.01 in. thick on the slot part of the coil to protect the empire cloth.

(f) One layer of half-lapped cotton tape 0.006 in. thick on the end connections to protect the empire cloth.

The coil is baked and impregnated before the paper and cotton tape are put on, and is dipped in finishing varnish after they are put on to make it water- and oil-proof.

The thickness of the slot insulation and the apparent dielectric strength are given in the table below:

	Width	Depth	Voltage
D. C. C. on wires	.....	.....	600
Paper	0.02	0.03	2,500
Empire cloth	0.024	0.024	4,500
Paper	0.02	0.03	2,500
	0.064	0.084	10,100

In the above table, under heading of width, is given the space taken up in the width of the slot by the different layers of insulation. The insulation on the conductors has not been added since it varies with the number of conductors per slot.

Under the heading of depth is given the space taken up in the depth of half a slot by the different layers of insulation.

Under the heading of voltage is given the apparent dielectric strength of the insulation; the figures used for the different materials are taken from Chapter IV.

The puncture test voltage is 2000; therefore the factor of safety is 5.0.

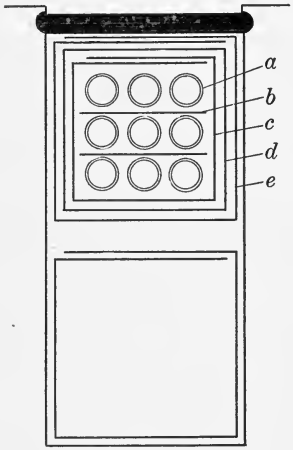


FIG. 150.—440 volt slot insulation.

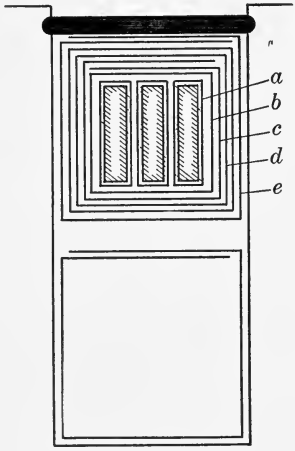


FIG. 151.—2200 volt slot insulation.

Example 2.—Insulation for a 2200-volt induction motor with a strip copper coil and a double-layer winding.

A section through the slot and insulation is shown in Fig. 151 and the insulation consists of:

(a) One layer of half-lapped cotton tape 0.006 in. thick on each conductor to form the insulation between conductors.

(b) One layer of half-lapped cotton tape 0.006 in. thick all round the coil to bind the conductors together.

(c) One turn of micanite 0.02 in. thick on the slot part of the coil.

(d) Two layers of half-lapped empire cloth 0.006 in. thick all round the coil.

(e) One turn of paper 0.01 in. thick on the slot part of the coil to protect the empire cloth.

(f) One layer of half-lapped cotton tape 0.006 in. thick on the end connections to protect the empire cloth.

The coil is baked and impregnated before the paper and last taping of cotton tape are put on. After they are on, the slot part of the coil is hot pressed and then allowed to cool under pressure, after which the coil is dipped in finishing varnish to make it water- and oil-proof.

The thickness of the insulation and the apparent dielectric strength are given in the table below.

	Width	Depth	Voltage
Tape on conductor	.....	.....	1,000
Tape on coil	0.024	0.024	1,000
Micanite	0.04	0.06	16,000
Empire cloth	0.048	0.048	9,000
Paper	0.02	0.03	2,500
	0.132	0.162	29,500

The puncture test voltage is 5000; therefore the factor of safety is 5.8.

Example 3.—Insulation for a 11,000-volt alternator with a strip copper coil and a chain winding.

A section through the slot and insulation is shown in Fig. 152 and the insulation consists of :

(a) One layer of half-lapped cotton tape 0.006 in. thick on each conductor to form the insulation between adjacent conductors.

(b) One layer of micanite 0.02 in. thick between vertical layers of conductors, all round the coil.

(c) Two layers of half-lapped empire cloth 0.006 in. thick on each of the two sections of the coil, this empire cloth to go on both slot part and end connections of the coil.

(d) One layer of cotton tape 0.006 in. thick, half-lapped on the ends and taped with a butt joint on the slot part of the coil. This tape is to bind the two sections of the coil firmly together.

(e) Three turns of mica paper, made of 5 mil paper and 7 mil mica, on the slot part of the coil.

(f) One layer of cotton tape 0.006 in. thick, half lapped on the ends and butt joint on the slot part of the coil.

(g) Bake and impregnate the coil.

(h) Three layers of half-lapped empire cloth 0.006 in. thick all round the coil.

(j) Three turns of mica paper on the slot part of the coil.

(k) One layer of cotton tape 0.006 in. thick, half-lapped on the ends and butt joint on the slot part of the coil.

(l) Two layers of half-lapped empire cloth 0.006 in. thick all round the coil.

(m) Three turns of mica paper on the slot part of the coil.

(n) Three layers of half-lapped empire cloth 0.006 in. thick all round the coil.

(o) One layer of cotton tape 0.006 in. thick, half-lapped on the ends and butt joint on the slot part of the coil.

(p) Bake and impregnate the coil.

(q) One turn of paper 0.015 in. thick on the slot part of the coil.

(r) Hot press the slot part of the coil and allow it to cool while under pressure.

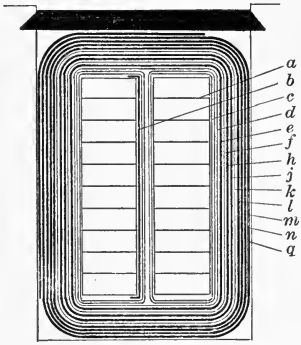


FIG. 152.—11,000 volt slot insulation.

The thickness of the insulation and its apparent dielectric strength are given in the table below.

	Width	Depth	Voltage
(a) Tape on conductor			1,000
(b) Micanite between layers	0.02		
(c) Empire cloth on each section	0.096	0.048	9,000
(d) Cotton tape	0.012	0.012	
(e) Mica paper	0.084	0.072	20,550
(f) Cotton tape	0.012	0.012	
(h) Empire cloth	0.072	0.072	13,500
(j) Mica paper	0.084	0.072	20,550
(k) Cotton tape	0.012	0.012	
(l) Empire cloth	0.048	0.048	9,000
(m) Mica paper	0.084	0.072	20,550
(n) Empire cloth	0.072	0.072	13,500
(o) Cotton tape	0.012	0.012	
(q) Paper	0.045	0.045	3,750
	0.653	0.549	111,400

The minimum thickness of insulation between copper and iron = 279 mils, therefore the volts per mil at normal voltage = 40.

The puncture test voltage for this insulation is 22,000 volts, therefore the factor of safety is 5.1.

The insulation on each end connection consists of:

10 layers of half-lapped empire cloth

5 layers of half-lapped cotton tape;

this insulation, along with the air space between coils, is ample.

CHAPTER XXI

ARMATURE REACTIONS IN ALTERNATORS

POLYPHASE MACHINES

161. The Armature Fields.—a and b, Fig. 153, are two conductors of one phase of a polyphase alternator. When current flows in these conductors they become encircled by lines of force. These lines may be divided into two groups; ϕ_r, the lines which pass through the magnetic circuit and whose effect is called armature reaction, and ϕ_x, called leakage lines, which do not pass through the magnetic circuit.

162. Armature Reaction.—Diagram A, Fig. 154, shows an end view of part of a three-phase alternator which has six slots per pole. The starts of the three windings are spaced 120 electrical

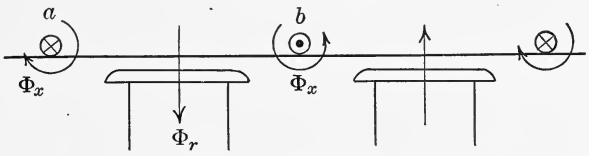


FIG. 153.—The armature fields.

degrees apart and are marked S_1, S_2, S_3. The armature moves relative to the poles in the direction of the arrow and the e.m.f. in each phase at any instant is given by the curves in diagram F. In diagram A is shown the relative position of the armature and poles, and also the direction of the e.m.f. in each conductor, at instant 1, diagram F, at which instant the e.m.f. in phase 1 is a maximum.

Let the current be in phase with the generated e.m.f., then diagram G shows the current in each phase at any instant, and the three diagrams, B, C and D, show the direction of the current in each conductor and also the relative position of the poles and armature at the three instants 1, 2 and 3. It may be seen from these diagrams that the currents in the three phases produce a resultant armature m.m.f. which moves in the same direction as the poles and at the same speed. Since the armature resultant

m.m.f. is added to that of the main field at one pole tip, and subtracted from it at the other tip of the same pole, the resultant effect is cross-magnetizing.

Let the current lag the generated e.m.f. by 90 degrees, then diagram *H* shows the current in each phase at any instant, and

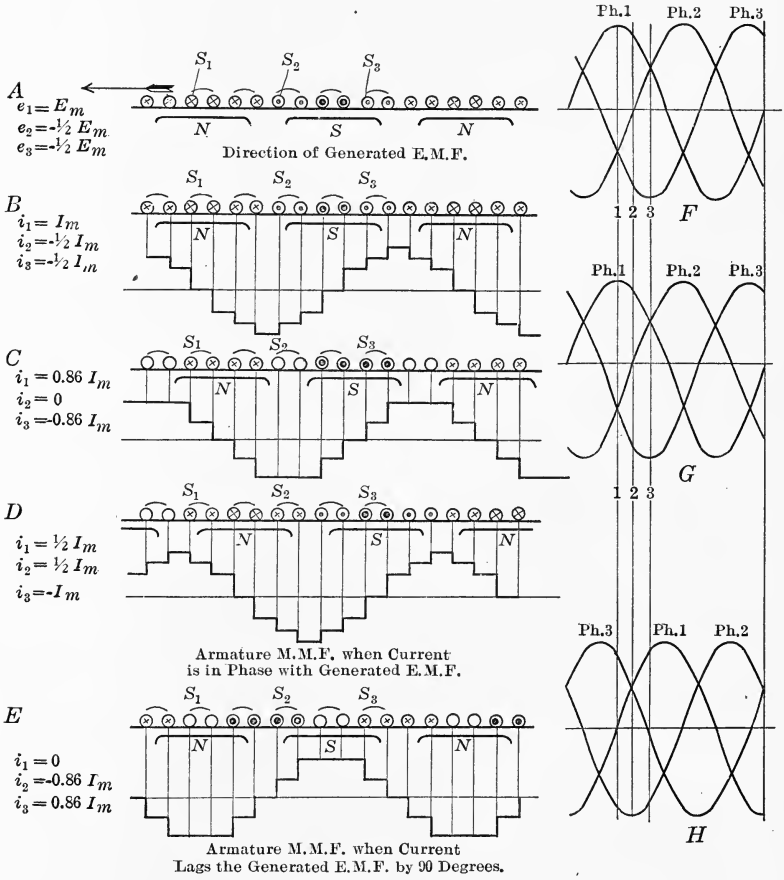


FIG. 154.—The armature m.m.f. of a three phase alternator.

diagram *E* shows the direction of the current in each conductor and also the relative position of the poles and armature at instant 1. The resultant m.m.f. of the armature has the same value as before and moves at the same speed and in the same direction, but has now a different position relative to the poles and is demagnetizing in effect.

If the current lead the generated e.m.f. by 90 degrees; then it can be shown in a similar way that the resultant armature m.m.f. has the same magnitude, speed and direction as before, but is magnetizing in effect.

163. The Alternator Vector Diagram.—On no-load the current in the windings of an alternator is zero and the only m.m.f. which is acting across the air gap is F_0, that due to the main field; F_0 sends a constant flux ϕ_a across the gap. This flux moves relative to the armature surface so that the flux which, due to the m.m.f. F_0, threads the windings of one phase of the armature is alternating and has a maximum value $=\phi_a$.

The voltage generated in a coil lags the flux which threads the coil, and whose change produces the voltage, by 90 degrees; thus, in Fig. 155, the flux threading coil a is a maximum but the voltage

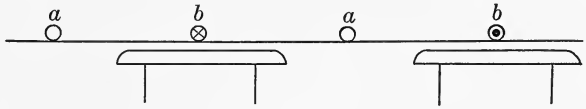


FIG. 155.—The generated e.m.f. in a coil.

in that coil is zero, while the total flux threading the coil b is zero and the voltage in that coil is a maximum; that is, the voltage is in phase with the flux which the coil cuts, but lags the flux which threads the coil by 90 degrees.

The vector diagram for one phase of a polyphase alternator is shown in Fig. 156. On no-load, F_0, the m.m.f. of the main field referred to the armature, produces an alternating flux in the windings of each phase, and E_0, the voltage generated in each phase by that flux, lags F_0 by 90 degrees.

When the alternator is loaded the currents in the armature produce a m.m.f. F_a of armature reaction which, if acting alone, produces a field ϕ_r of constant strength which moves in the same direction and at the same speed as the main field.

In Fig. 154, diagram B, it may be seen that when the current in phase 1 is a maximum the flux which threads the windings of that phase is also a maximum; in diagram C the current in phase 2 is zero and the flux which threads the windings of that phase is also zero. In general it may be shown that the flux which, due to the revolving field ϕ_r, threads the winding of one phase of the armature is an alternating flux which has a maximum value $=\phi_r$ and is in phase with the current in that winding.

If the alternator is loaded and I, Fig. 156, is the current per phase, then F_g, the resultant of the m.m.fs. F_0 and F_a, will produce a resultant magnetic flux ϕ_g which is alternating with respect to the armature windings, and E_g, the voltage per phase due to the flux ϕ_g, will lag it by 90 degrees.

In addition to the m.m.f. of armature reaction the current in the windings of one phase sets up an alternating magnetic flux ϕ_x which, as shown in Fig. 153, circles these windings but does not link the magnetic circuit; this flux is in phase with the current in the windings and is proportional to that current since its magnetic circuit is not saturated at normal tooth densities.

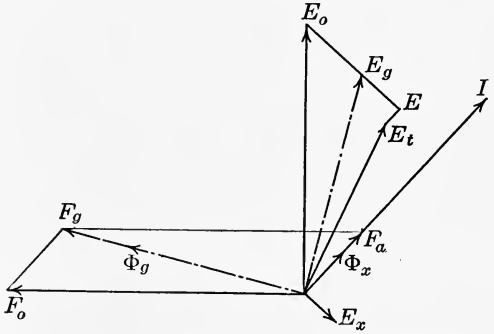


FIG. 156.—The vector diagram for an alternator.

In Fig. 156 F_o is the m.m.f. due to the field excitation referred to one phase of the armature.

E_o is the voltage per phase which would be generated by the flux produced by F_o.

I is the current per phase.

F_a is the m.m.f. of armature reaction referred to one phase of the armature and, as pointed out above, is in phase with I.

F_g is the resultant of the two m.m.fs. F_o and F_a.

ϕ_g is the flux that threads the windings of one phase due to F_g.

E_g is the voltage generated in that phase by ϕ_g.

ϕ_x is the armature leakage flux per phase produced by I.

E_x is the voltage per phase generated by the flux ϕ_x and is called the leakage reactance voltage.

$E_x = IX$ where X is the leakage reactance per phase.

E is the resultant of E_g and E_x.

E_t is the terminal voltage per phase and $= E - IR$ taken as vectors, where R is the effective resistance per phase.

Figure 157 shows the above diagram for the particular case where the power factor is approximately zero and the current lags the generated voltage by 90 degrees. In this case the m.m.f. of armature reaction is subtracted directly from that of

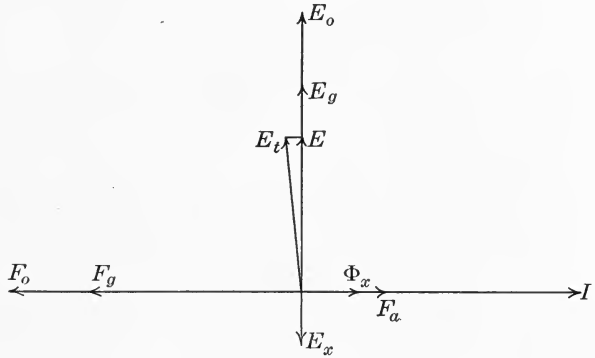


FIG. 157.—The vector diagram for an alternator on zero power factor.

the main field to give the resultant m.m.f., and the leakage reactance voltage is subtracted directly from the generated voltage E_g to give the terminal voltage. The resistance drop

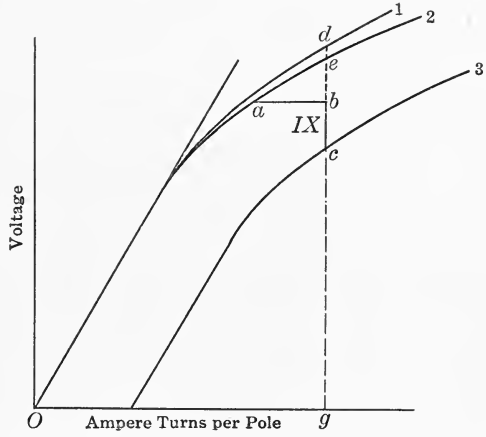


FIG. 158.—The saturation curves of an alternator.

can be neglected in this case since its phase relation is such that it has little effect on the value of the terminal voltage.

164. Full-load Saturation Curve at Zero Power Factor with Lagging Current.—Curve 1, Fig. 158, shows the no-load satura-

tion curve of an alternator. When the machine is loaded, the power factor of the load zero, and the current lagging, the m.m.f. of armature reaction is directly demagnetizing, and to overcome its effect and maintain the flux ϕ_a which crosses the air gap constant, a number of ampere-turns per pole equal to the armature demagnetizing ampere-turns per pole must be added to the main field excitation. Under these conditions the increase in the field excitation causes the leakage flux ϕ_e, Fig. 159, to increase to the value ϕ_{fe}, where

$$\phi_{fe} = \phi_e \left(\frac{AT._{g+t} + \text{demagnetizing } AT. \text{ per pole}}{AT._{g+t}} \right)$$

without increasing the value of ϕ_a, the flux crossing the air gap.

The leakage factor at no-load $= \frac{\phi_a + \phi_e}{\phi_a}$ while that for the above load conditions $= \frac{\phi_a + \phi_{fe}}{\phi_a}$.

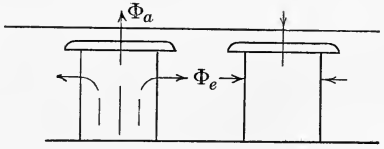


FIG. 159.—The main field and the pole leakage.

Curve 2, Fig. 158, is a new no-load saturation curve which is calculated with the value of the leakage factor corresponding to full-load, zero power factor and lagging current.

To maintain the flux crossing the air gap constant and $= \phi_a$ a number of ampere-turns per pole, *ab*, equal to the armature demagnetizing ampere-turns per pole, must be added to the value obtained from curve 2.

The terminal voltage is less than that generated due to the flux ϕ_a by IX, the leakage reactance voltage; the resistance drop being neglected on zero power factor since, as shown in Fig. 157, its phase relation is such that it has little effect on the terminal voltage.

The locus of point *c* so found is the full-load saturation curve at zero power factor with lagging current.

165. Synchronous Reactance.—In Fig. 158, *de*, the drop in voltage due to the increase in leakage factor, depends on the demagnetizing effect of the armature, which is proportional to

the current and, as shown in Fig. 154, varies with the power factor, being zero for unity power factor and a maximum for zero power factor lagging; this drop may, therefore, be considered as part of that due to armature reaction. In Fig. 158, then, the total voltage drop is made up of two parts one, db, due to armature reaction, which varies with the slope of curve 1 and therefore with the excitation, and the other, bc, the armature reactance drop, which is practically constant for all excitations.

Since, for a given excitation, each of these voltage drops is proportional to the current and since, as shown in Fig. 156, they are always in phase with one another, their sum, namely dc, Fig. 158 or E_oE, Fig. 156, may be represented by a fictitious reactance voltage called the synchronous reactance voltage and $=I\overline{X}$, where $\overline{X}$ is the synchronous reactance per phase. It may be seen from Fig. 158 that dc, and therefore $\overline{X}$, is not constant but decreases as the field excitation and, therefore, the saturation of the magnetic circuit increases.

166. The Demagnetizing Ampere-turns per Pole at Zero Power Factor.—The distribution of the m.m.f. of armature

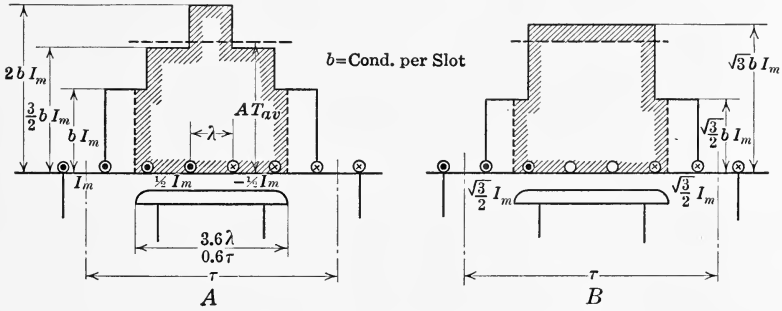


FIG. 160.—Armature m.m.f. on zero power factor.

reaction at two different instants is shown in Fig. 160 for a machine with six slots per pole and b conductors per slot. These diagrams are taken directly from Fig. 154, and the relative position of poles and armature shown is that corresponding to zero power factor and lagging current.

The portion of this m.m.f. which is effective in demagnetizing the poles is shown cross hatched, it is required to find AT_{av}, the average value of this cross hatched part of the curve, for the case where $\phi=0.6$ and therefore the pole covers 3.6 slots.

$AT_{av} \times 3.6\ \lambda$ = area of cross hatched curve, diagram A
$= 2\ bI_m\lambda + 1.5\ bI_m \times 2\lambda + 0.6\ bI_m\lambda$
$= 5.6\ bI_m\lambda$
= area of cross hatched curve, diagram B
$= 1.73\ bI_m \times 3\lambda + 0.866\ bI_m \times 0.6\lambda$
$= 5.7\ bI_m\lambda$
$= 5.65 bI_m\lambda$; an average value from diagrams A and B

therefore $AT_{av} = 1.57\ bI_m$
$= 2.22\ bI_c$ where I_c is the effective current
$= 2.22 bI_c \times \left(\frac{\text{slots per pole}}{6}\right)$, since there are 6 slots per pole
$= 0.37 \times$ cond. per pole $\times I_c$. (27)

If the pole enclosure be increased to 0.7 the value of AT_{av} will be reduced to $0.33 \times$ cond. per pole $\times I_c$.

The value can be found in a similar way to the above for any particular value of pole enclosure, number of phases and slots per pole, but the value of AT_{av} is generally taken as $0.35 \times$ cond. per pole $\times I_c$ for all polyphase machines.

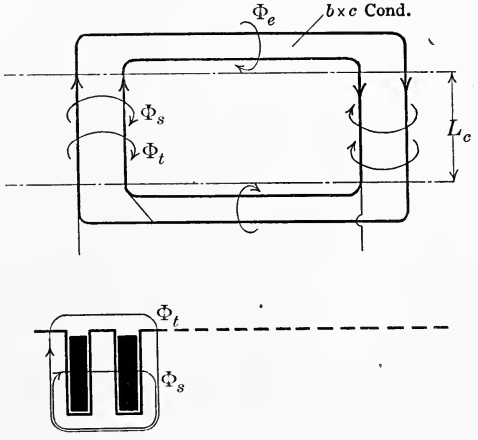


FIG. 161.—The armature leakage fields with a chain winding.

167. Calculation of the Leakage Reactance.—Fig. 161 shows part of the winding of one phase of a polyphase alternator which has a chain winding.

If ϕ_e = the lines of force that circle 1 in. length of the belt of end connections for each ampere conductor in that belt,

ϕ_s = the lines of force that cross the slots and circle 1 in. length of the phase belt of conductors for each ampere conductor in that belt,

ϕ_t = the lines of force that cross the tooth tips and circle 1 in. length of the phase belt of conductors for each ampere conductor in that belt,

b = conductors per slot,

c = slots per phase per pole,

then the total flux that links the coils shown

$$= \phi_c$$
$$= [\phi_e \times 2L_e + (\phi_s + \phi_t) \times 2\ L_c] b \times c \times i$$

the coefficient of self-induction of these coils

$$= \frac{b \times c \times \phi_c}{i} 10^{-8} \text{ henry}$$
$$= b^2 c^2\ [\phi_e \times 2L_e + (\phi_s + \phi_t) \times 2L_c] \times 10^{-8} \text{ henry}$$

the reactance of these coils in ohms

$$= 2\pi f b^2 c^2 [\phi_e \times 2L_e + (\phi_s + \phi_t) \times 2L_c] \times 10^{-8}$$

since there are p/2 of these groups of coils per phase the reactance of one phase in ohms

$$= 2\pi f p b^2 c^2 [\phi_e L_e + (\phi_s + \phi_t) \times L_c] \times 10^{-8} \qquad (28)$$

For a double layer winding a slight modification is required in the above formula.

Figure 162 shows part of the winding of one phase of a polyphase alternator which has a double layer winding.

The number of turns in the coils shown $= \frac{bc}{2}$

the total flux that links these coils

$$= \phi_c$$
$$= \phi_e \times 2L_e \times \frac{b \times c}{2} \times i + (\phi_s + \phi_t) \times 2L_c \times b \times c \times i$$

the coefficient of self-induction of these coils

$$= \frac{b \times c}{2 \times i} \times \phi_c \times 10^{-8} \text{ henry}$$

$$= \frac{b^2 \times c^2}{2} [\phi_e L_e + (\phi_s + \phi_t) 2L_c] 10^{-8} \text{ henry}$$

the reactance of these coils in ohms

$$= 2\pi f b^2 c^2 \left[\frac{\phi_e L_e}{2} + (\phi_s + \phi_t) L_c \right] 10^{-8}$$

since there are p of these groups of coils per phase, the reactance of one phase

$$= 2\pi f p b^2 c^2 \left[\frac{\phi_e L_e}{2} + (\phi_s + \phi_t) L_c \right] \times 10^{-8} \qquad (29)$$

which differs from the formula for the chain winding in that the end connection reactance is reduced to half the value.

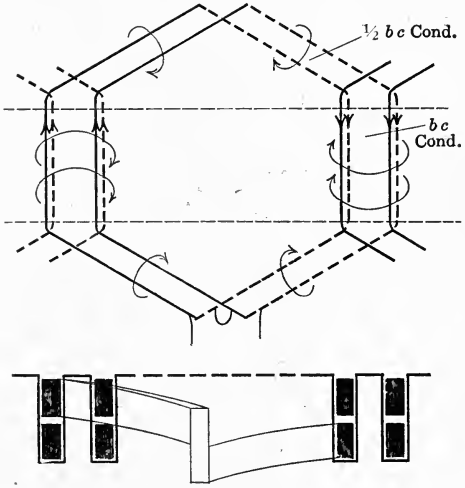


FIG. 162.—The armature leakage fields with a double layer winding.

168. End-connection Reactance.—ϕ_e depends principally on

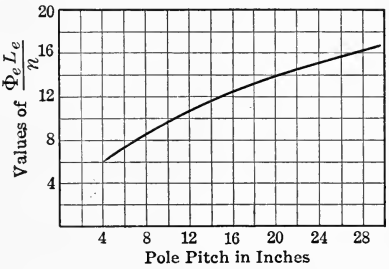


FIG. 163.—The end connection leakage flux.

the length of the end-connection leakage path, namely, the length around the belt of end connections, which, as may be seen

from Figs. 161 and 162, is directly proportional to the pole-pitch and inversely proportional to the number of the phases, and ϕ_e decreases as this length increases, so that $\frac{\phi_e}{n}$ is approximately proportional to $\frac{1}{\text{pole pitch}}$ for any number of phases.

L_e, the length of the end connections, increases with the pole-pitch as may be seen from Figs. 161 and 162, and in Fig. 163 $\frac{\phi_e L_e}{n}$ is plotted against pole-pitch from test results.

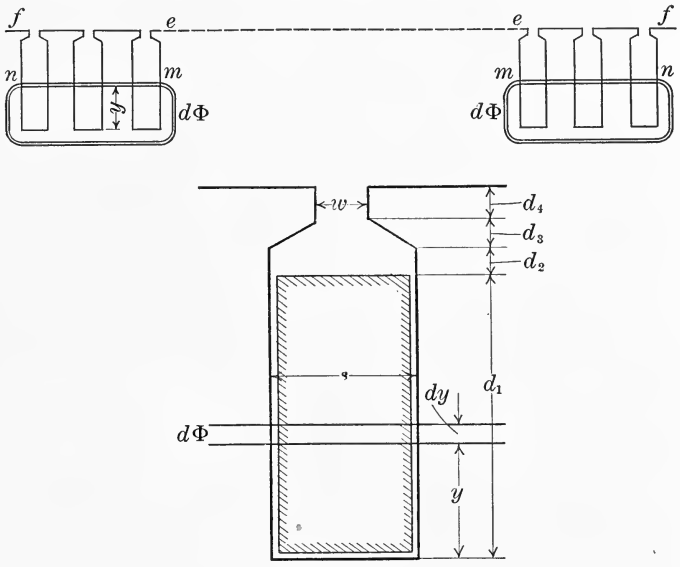


FIG. 164.—The slot leakage flux.

169. Slot Reactance.—Fig. 164 shows part of one phase of an alternator which has a chain winding. The m.m.f. between m and $n = b \times c \times i \times \frac{y}{d_1}$ ampere-turns, therefore the leakage flux $d\phi$

$$= \frac{4\pi}{10} \times b \times c \times \frac{y}{d_1} \times i \times \frac{L_c \times dy}{cs} \times 2.54 \text{ all in inch units}$$

$$= 3.2 \times b \times c \times \frac{y}{d_1} \times i \times \frac{L_c dy}{cs}$$

the reluctance of the iron part of the leakage path is neglected since it is small compared with that of the air path across the slots.

This flux $d\phi$ links each side of the $bc\,\frac{y}{d_1}$ turns of the coils shown and the interlinkages per unit current

$$=b\times c\times\frac{y}{d_1}\times\frac{2\,d\phi}{i}$$

The coefficient of self-induction of the coils shown, due to the flux which crosses the slots, between the limits $y=o$ and $y=d_1$

$$=\int_o^{d_1} bc\frac{y}{d_1}\times\frac{2\,d\phi}{i}\times10^{-8}\text{ henry}$$

$$=\int_o^{d_1} b^2c^2\left(\frac{y}{d_1}\right)^2\times\frac{3.2\times2\times L_c\times dy}{c\times s}\times10^{-8}\text{ henry}$$

$$=\frac{b^2c^2\times3.2\times2L_c}{c\times s}\times\frac{d_1}{3}\times10^{-8}\text{ henry}$$

The m.m.f. between e and f, Fig. 164, $=cbi$ ampere-turns, therefore the leakage flux crossing the slots above the conductors

$$=3.2cbi\times L_c\left(\frac{d_2}{cs}+\frac{2d_3}{c(s+w)}+\frac{d_4}{cw}\right)$$

the coefficient of self-induction of the coils shown due to this flux

$$=3.2c^2b^2\times2L_c\left(\frac{d_2}{cs}+\frac{2d_3}{c(s+w)}+\frac{d_4}{cw}\right)10^{-8}\text{ henry}$$

therefore the total coefficient of self-induction of the coils shown due to the total slot leakage

$$=3.2cb^2\times2L_c\left(\frac{d_1}{3s}+\frac{d_2}{s}+\frac{2d_3}{(s+w)}+\frac{d_4}{w}\right)10^{-8}\text{ henry}$$

and the slot reactance of these coils in ohms

$$=2\pi f\times3.2cb^2\times2L_c\left(\frac{d_1}{3s}+\frac{d_2}{s}+\frac{2d_3}{s+w}+\frac{d_4}{w}\right)10^{-8}$$

since there are $p/2$ of these groups of coils per phase the slot reactance per phase

$$= 2\pi f p b^2 c L_c \times 3.2 \left(\frac{d_1}{3s} + \frac{d_2}{s} + \frac{2d_3}{s+w} + \frac{d_4}{w}\right) 10^{-8}$$

$$= 2\pi f p b^2 c^2 L_c \times \phi_s \times 10^{-8}$$

where $\phi_s = \frac{3.2}{c}\left(\frac{d_1}{3s} + \frac{d_2}{s} + \frac{2d_3}{s+w} + \frac{d_4}{w}\right)$

170. Tooth-tip Reactance.—*A*, Fig. 165, shows the tooth-tip leakage path round one side of the coils of one phase of a machine which has 1 slot per phase per pole, when the winding of that phase lies between the poles.

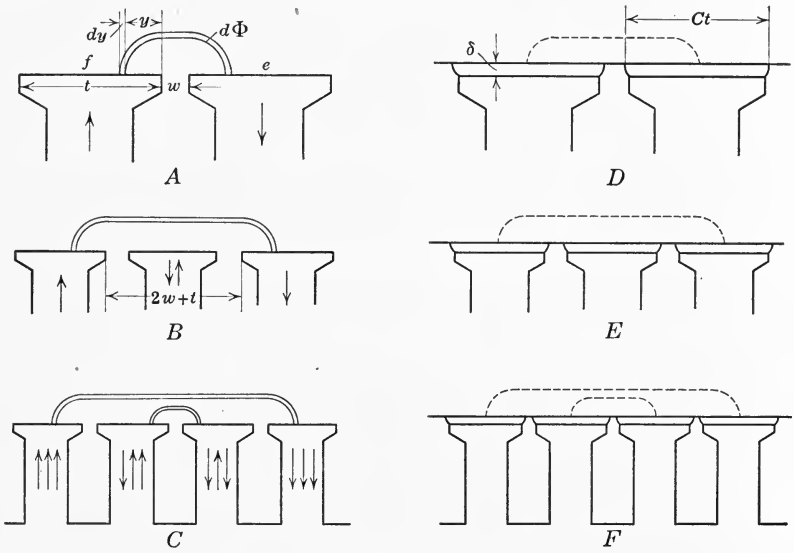


FIG. 165.—Tooth-tip leakage flux.

The m.m.f. between e and $f = bci$ ampere-turns
therefore the leakage flux $d\phi$ per 1 in. length of core

$$= \frac{4\pi}{10} bci \frac{dy}{w + \pi y} \times 2.54 \text{ all in inch units}$$

$$= 3.2 bci \frac{dy}{w + \pi y}$$

and the total tooth-tip leakage flux that links one phase belt per 1 in. length of core

$$= 3.2 bci \int_o^t \frac{dy}{w + \pi y}$$

$$=2.35\ bci\ \log_{10}\left(1+\frac{\pi t}{w}\right)$$

therefore ϕ_{ta}, the lines of force that cross the tooth tips and circle 1 in. length of the phase belt of conductors for each ampere conductor in that belt, when the belt lies between the poles,

$$=2.35\ \log_{10}\left(1+\frac{\pi t}{w}\right)$$

B shows the case where there are two slots per phase per pole and it may be seen that in such a case

$$\phi_{ta}=2.35\ \log_{10}\left(1+\frac{\pi t}{2w+t}\right)$$

C shows the case where there are three slots per phase per pole and it may be seen that in such a case

the flux $2.35\ \log_{10}\left(1+\frac{\pi t}{3w+2t}\right)$ circles the total belt

while the flux $\frac{2.35}{3}\log_{10}\left(1+\frac{\pi t}{w}\right)$ is produced by, and circles 1/3 of the total belt.

This latter flux is equivalent to a flux $\frac{2.35}{9}\log_{10}\left(1+\frac{\pi t}{w}\right)$ circling the whole belt,

therefore $\phi_{ta}=2.35\ \log_{10}\left(1+\frac{\pi t}{3w+2t}\right)+\frac{2.35}{9}\log_{10}\left(1+\frac{\pi t}{w}\right)$

D shows the tooth-tip leakage path round the coils of one phase of a machine with 1 slot per phase per pole, when the winding of that phase lies under the poles. In this case ϕ_{tp}, the lines of force that cross the tooth tips and circle 1 in. length of the phase belt of conductors for each ampere conductor in that belt, when the belt lies under the poles,

$$=3.2\frac{t\times C}{2\delta}$$

In the case where there are two slots per phase per pole as shown in *E*

$$\phi_{tp}=3.2\frac{t\times C}{2\delta}$$

In the case where there are three slots per phase per pole as shown in F

$$\phi_{tp}=3.2\frac{t\times C}{2\delta}+\frac{3.2}{9}\times\frac{t\times C}{2\delta}$$

In Fig. 166 the tooth-tip flux per ampere conductor per inch is approximately

$=\phi_{ta}$ while the conductors are in the belt hk
$=\phi_{tp}$ while the conductors are in the belt kl

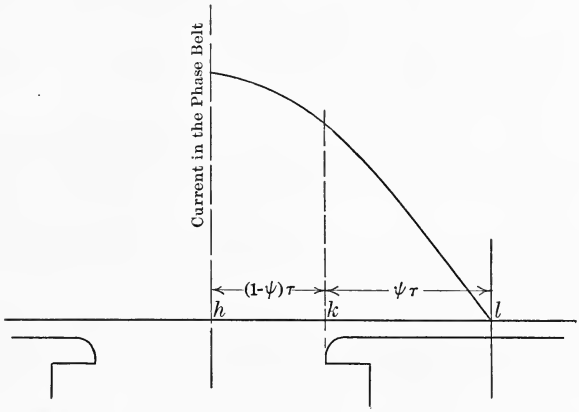


Fig. 166.—Variation of the current in the phase belt at zero power factor with the position of the belt relative to the poles.

On zero power factor the current in a conductor is a maximum when the e.m.f. generated in that conductor is zero, that is when the conductor is between the poles. As the conductor moves relative to the poles the current in the conductor varies according to a sine law as shown in Fig. 166, therefore, since the tooth-tip reactance per phase

$=2\pi fb^2c^2p\phi_{ta}L_c10^{-8}$ when the cond. are in the belt hk
and $=2\pi fb^2c^2p\phi_{tp}L_c10^{-8}$ when the cond. are in the belt kl
the effective voltage per phase on zero power factor due to tooth-tip leakage

$=2\pi fb^2c^2p\phi_{ta}L_c10^{-8}\times$effective current between h and k, while they are in this belt
and $=2\pi fb^2c^2p\phi_{tp}L_c10^{-8}\times$effective current between k and l, while they are in this latter belt

and from formula 28, page 216, the effective voltage per phase due to tooth-tip leakage

$$=2\pi f b^2 c^2 p \phi_t L_c 10^{-8} \times I_{eff}$$

therefore ϕ_t is approximately

$$= \left(\frac{1-\psi}{1} \times \frac{\text{effect. current in belt } hk}{I_{eff}} \times \phi_{ta} + \psi \frac{\text{effect. current in belt } kl}{I_{eff}} \times \phi_{tp} \right)$$

As a general rule, ψ, the pole enclosure $=0.6$
and for this value $1-\psi=0.4$
effective current in cond. in belt hk $=0.93\ I_{max}$
effective current in cond. in belt kl $=0.5\ I_{max}$

$$\text{therefore } \phi_t = \phi_{ta} \times 0.4 \times \frac{0.93\ I_{max}}{0.71\ I_{max}} + \phi_{tp} \times 0.6 \times \frac{0.5\ I_{max}}{0.71\ I_{max}}$$

$$=0.52\,\phi_{ta}+0.42\,\phi_{tp}$$

171 Final Reactance Formula.—The reactance of one phase of a polyphase machine with a pole enclosure of approximately 60 per cent. is

$$=2\pi f b^2 c^2 p\ (\phi_e L_e + (\phi_s + \phi_t) L_c) 10^{-8} \qquad 28$$

for chain windings

$$=2\pi f b^2 c^2 p \left(\frac{\phi_e L_e}{2} + [\phi_s + \phi_t] L_c \right) 10^{-8} \qquad 29$$

for double layer windings

where $\phi_e L_e$ may be found from Fig. 163

$$\phi_s = \frac{3.2}{c} \left(\frac{d_1}{3s} + \frac{d_2}{s} + \frac{2d_3}{s+w} + \frac{d_4}{w} \right)$$

$$\phi_t = 0.52\ \phi_{ta} + 0.42\ \phi_{tp}$$

$$\phi_{ta} = 2.35\ \log_{10} \left(1 + \frac{\pi t}{w} \right) \text{ for 1 slot per phase per pole}$$

$$= 2.35\ \log_{10} \left(1 + \frac{\pi t}{2w+t} \right) \text{ for 2 slots per phase per pole}$$

$$= 2.35\ \log_{10} \left(1 + \frac{\pi t}{3w+2t} \right) + \frac{2.35}{9} \log_{10} \left(1 + \frac{\pi t}{w} \right)$$

for 3 slots per phase per pole

$\phi_{tp} = 3.2 \frac{t \times C}{2 \times \delta}$ for 1 or 2 slots per phase per pole

$= 3.2 \times \frac{10}{9} \times \frac{t \times C}{2 \times \delta}$ for 3 slots per phase per pole

f = frequency in cycles per second
b = cond. per slot
c = slots per phase per pole
p = poles
n = phases
L_c = frame length in inches
t = width of tooth at the tip
C = Carter coefficient found from Fig. 40, page 44.

slot dimensions are given in Figs. 164 and 165.

Example of Calculation.—The armature reaction and armature reactance can be checked approximately by the no-load saturation and the short-circuit curves of an alternator. Fig. 167 shows the actual test curves on a small alternator which was built as follows:

Poles,	6
Pole enclosure,	0.6
Pole pitch,	10.5 in.
Air gap clearance,	0.2 in.
Slots per pole,	6
Conductors per slot,	12
Size of slot,	0.75×1.75 in., open
Tooth width,	1.0 in.
Carter coefficient,	1.2
Frame length,	6.5 in.
Winding,	double layer Y-connected
Turns per field coil,	420
Rating,	65 k.v.a., 600 volts, three-phase, 60 cycles.

To send full-load current through the machine on short-circuit requires an excitation of 4.8 amperes. Now the power factor during this test is zero since the machine is carrying no-load, and the terminal voltage is zero since the machine is short-circuited, therefore m is a point on the full-load saturation curve at zero power factor, the resistance drop being neglected.

The demagnetizing ampere-turns per pole

$= 0.35 \times$ cond. per pole $\times I_c$

$= 0.35 \times 12 \times 6 \times 62.5$

$= 1580$ ampere-turns per pole

the corresponding field current

$$= \frac{\text{demagnetizing ampere-turns per pole}}{\text{field-turns per pole}}$$

$$= \frac{1580}{420}$$

$$= 3.75 \text{ amperes}$$

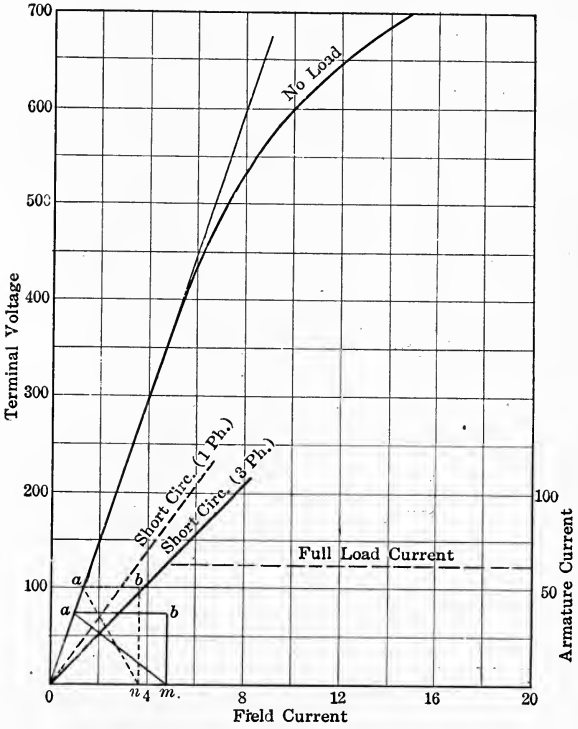


FIG. 167.—No-load saturation and short circuit curves on a 65 k.v.a three-phase alternator.

The reactance per phase

$$= 2\pi f b^2 c^2 p \left(\frac{\phi_e L_e}{2} + (\phi_s + \phi_t) L_c \right) 10^{-8} \text{ ohms.}$$

where $\frac{\phi_e L_e}{2} = 5 \times n$ from Fig. 163, since the pole-pitch $= 10.5$ in.

$$= 15$$

$$\phi_s = \frac{3.2}{2} \left(\frac{1.5}{3 \times .75} + \frac{.25}{.75} \right) = 1.6$$

$$\phi_{ta} = 2.35 \log \left(1 + \frac{\pi \times 1}{(2 \times .75) + 1} \right) = 0.82$$

$$\phi_{tp} = 3.2 \times \frac{1 \times 1.2}{2 \times .2} = 9.6$$

$$\phi_t = 0.52 \times 0.82 + 0.42 \times 9.6 = 4.4$$

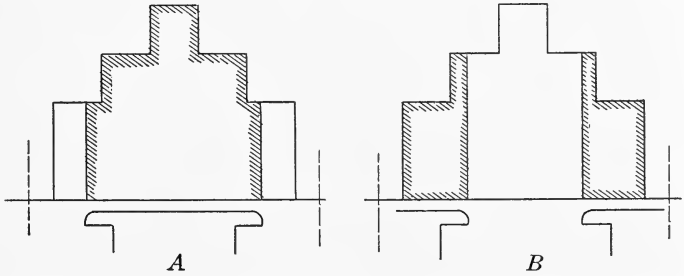
and reactance per phase

$$= 2 \times \pi \times 60 \times 12^2 \times 2^2 \times 6(15. + (1.6 + 4.4)6.5) \times 10^{-8}$$
$$= 0.7 \text{ ohms.}$$

The voltage drop per phase $= 0.7 \times 62.5 = 44$ volts
and the voltage drop at the terminals $= 1.73 \times 44$
$= 76$ volts

since the winding is Y-connected.
These two figures, 3.75 field amperes
76 terminal volts
are the only ones required for the construction of triangle *abm*, Fig. 167, and it may be seen that the calculated results check the test results very closely.

172. Variation of Armature Reaction and Armature Reactance with Power Factor.—When an alternator is carrying a load whose power factor is zero, the current in the conductors reaches



Current lags by 90 degrees. Current is in phase.

FIG. 168.—Effect on the armature reaction of the phase relation between the current and the generated e.m.f.

a maximum when the conductors are between the poles; the tooth-tip leakage is then a minimum. When, however, the current is in phase with the generated voltage, it reaches a maximum when the conductors are under the center of the poles; the tooth-tip leakage is then a maximum. The voltage drop due to armature reactance, therefore, varies with the power factor and is a maximum when the current is in phase with the generated voltage and a minimum when the current is out of phase with that voltage by 90 degrees.

The distribution of the m.m.f. of armature reaction for the same two cases is shown in Fig. 168; diagram *A* shows the distribution when the current lags the generated voltage by 90 degrees and the armature m.m.f. is demagnetizing; diagram *B* shows the distribution when the current is in phase with the generated voltage and the armature m.m.f. is cross magnetizing. It may be seen from these diagrams that AT_{av}, the average value of the part of this m.m.f. which is effective, is a minimum when the current is in phase with the generated voltage and a maximum when the current is out of phase by 90 degrees.

The variation of armature reactance and armature reaction with power factor is difficult to calculate and it is usual to assume that any decrease in one of them with a change in power factor is counterbalanced by an increase in the other so that the resultant effect, which is represented by a synchronous reactance, is constant at all power factors, for a given field excitation.

173. Full-Load Saturation Curve at Any Power Factor.—Given the no-load saturation curve and also that at full-load and zero

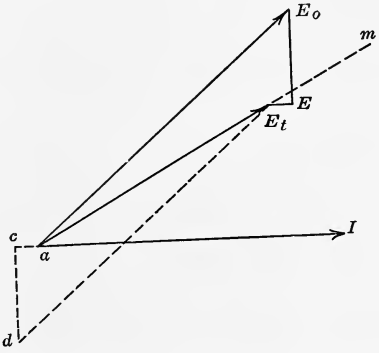


FIG. 169.—Vector diagram for an alternator.

power factor, it is required to draw in the full-load saturation curves for other power factors. The method adopted to determine different points on these curves is shown in Fig. 169, where the diagram shown by heavy lines is taken from Fig. 156.

E_o = the no-load voltage per phase corresponding to the field excitation *og*, Fig. 176, page 237 and = *dg*,

E_oE = the synchronous reactance drop = *dc*, Fig. 176,

E_tE = the resistance drop per phase,

θ = the phase angle between E_t and I,
$\cos\theta$ = the power factor of the load.

The construction generally adopted to find E_t is shown by the dotted lines in Fig. 169.

Triangle *acd* is drawn to scale in the proper phase relation with I and equal to triangle EE_oE_t.

Line *am* is set off at an angle θ such that cos θ is the power factor at which it is desired to find the terminal voltage.

With *d* as center and E_o as radius the line *am* is cut at E_t and the value E_t is then plotted along the ordinate of field excitation for which E_o is the no-load voltage and $= gf$, Fig. 176, for 85 per cent. power factor.

174. Regulation.—The regulation of an alternator is defined as the per cent. increase in voltage when full load is thrown off the machine, the speed and excitation being kept constant, and is equal to

$\frac{ab}{be}$, Fig. 176, at unity power factor,
$\frac{df}{fg}$, Fig. 176, at 85 per cent. power factor,
$\frac{hk}{kl}$, Fig. 176, at zero power factor.

175. Effect of Pole Saturation on the Regulation.—A_1 and B_1, Fig. 170, are the no-load saturation curves of two alternators which have armatures that are exactly alike; the field systems differ in that machine *A* has a smaller air gap and a greater pole density than has machine *B*; the excitation is the same for each machine at normal voltage and no-load.

A_2 and B_2 are the no-load saturation curves with the full-load leakage factor, the effect of the increase in the leakage factor is the greater in the machine which has the higher pole density.

A_3 and B_3 are the full-load saturation curves at zero power factor; the demagnetizing ampere-turns per pole are the same in each case; the armature reactance drop is slightly greater in the machine with the smaller air gap clearance.

It may be seen that the machine with the saturated poles is that which has the best regulation; but it is also the machine which takes the largest field excitation on load.

Of two machines that are built to meet the same regulation guarantee, that with the higher pole density is generally the

cheaper, because, as shown in Fig. 170, for the same armature reaction and reactance it gives the better regulation or for the same regulation it can have the greater armature reaction and reactance. If these quantities are increased by increasing the number of armature turns then, for constant output, the flux per pole must be decreased in the same ratio and the whole machine may be made smaller. In order to carry an increased number of conductors on a smaller diameter it is necessary to increase the slot depth.

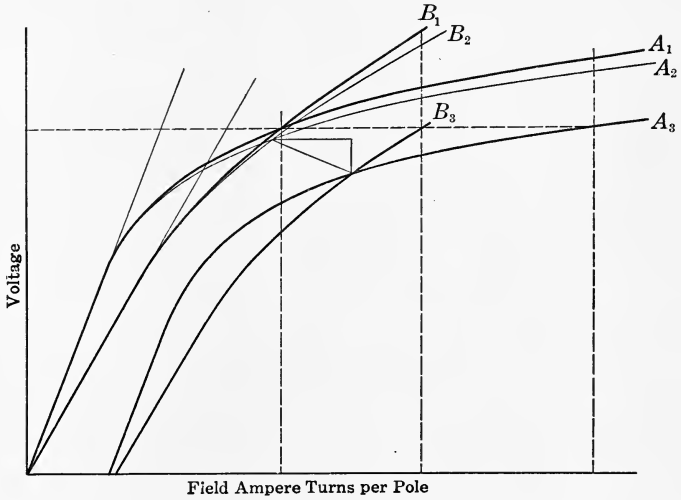


FIG. 170.—Effect of pole saturation on the regulation.

The pole density is seldom carried above 95,000 lines per square inch at no-load and normal voltage, because for higher densities it is difficult to predetermine the saturation curves with sufficient accuracy to ensure that there is enough field excitation at the high densities corresponding to normal voltage, full-load and zero power factor, since the permeability of the iron used may be lower than was expected.

176.—Relation between the M.M.Fs. of Field and Armature.—In Fig. 171, A_1 and B_1 are the no-load saturation curves, A_2 and B_2 the no-load saturation curves with the full-load leakage factor, A_3 and B_3 the full-load saturation curves at zero power factor, of two machines which are alike in every respect except that the air gap of machine A is smaller than that of machine B.

It may be seen from these curves that, the larger the air gap

and therefore the larger the ratio of $\frac{\text{field } AT. \text{ per pole}}{\text{armature } AT. \text{ per pole}}$, the better is the regulation, and that the armature ampere-turns per pole must be a small fraction of the field ampere-turns per pole if the regulation is to be at all reasonable. To consider an extreme case, suppose that the demagnetizing ampere-turns per pole at full-load were equal to the no-load ampere-turns on the field,

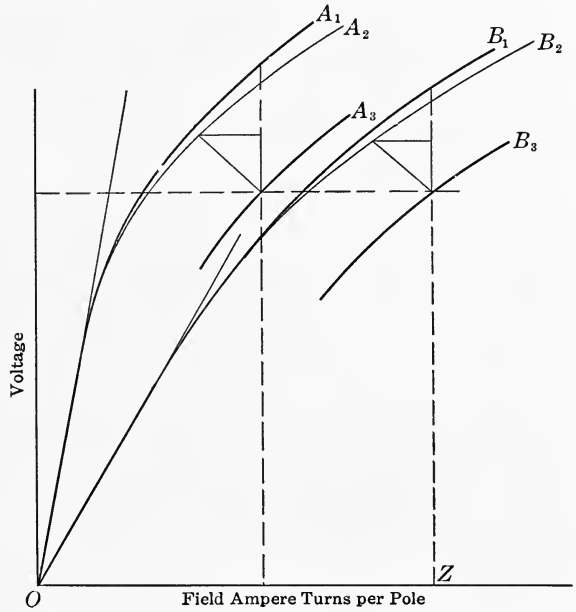


FIG. 171.—Effect of the relative strengths of the field and armature m.m.f's. on the regulation.

then on zero power factor with full-load current the armature field would wipe out the main field and the terminal voltage would be zero.

In practice, the maximum field ampere-turns per pole, namely *OZ*, Fig. 171, is generally made greater than three times the armature ampere-turns per pole; the greater this ratio the better will be the regulation of the machine, other things being unchanged, but the greater will be the cost of the field copper required and the greater the difficulty in cooling the field coils.

It must be understood that the regulation of an alternator will seldom be better than 8 per cent. at full-load and unity power factor and 18 per cent. at full-load and 85 per cent. power factor

because of the cost of the machine, which increases rapidly with the decrease in the per cent. regulation required. If worse regulation is permissible, as in the case where a regulator or a suitable compounding device is used, then a given machine can have the current rating considerably increased, but in order to carry this current deeper slots must be used and the reactance thereby increased.

SINGLE PHASE MACHINES

177. Armature Reaction in Single-phase Machines.—Consider the conditions at zero power factor; the current lags the e.m.f. by 90 degrees and so reaches a maximum in the conductors when these conductors lie between the poles.

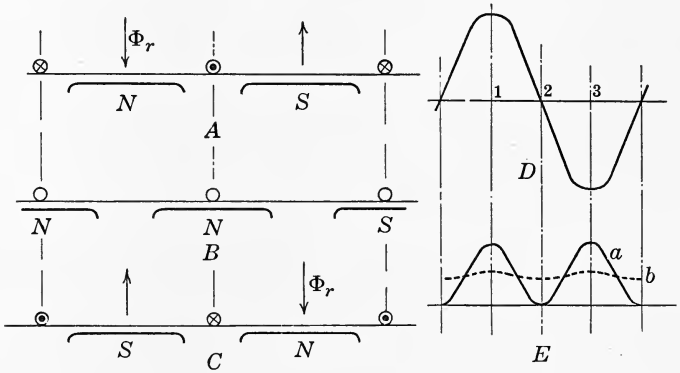


FIG. 172.—Pulsation of the armature field in a single-phase alternator.

A, *B* and *C*, Fig. 172, show the position of the poles relative to the armature when the current has the values given at instants 1, 2 and 3 diagram *D*. It may be seen that while the current passes through half a cycle and the poles move through one pole pitch the flux ϕ_r, due to the armature m.m.f., goes through half a cycle relative to the armature coils, while relative to the poles it changes from a maximum in diagram *A* to zero in diagram *B* and back to a maximum in diagram *C*, and so, as shown in curve *a*, diagram *E*, is pulsating relative to the poles with double normal frequency.

The poles are surrounded by the field coils, which are short-circuited through the exciter, and any pulsating flux in the poles induces a current in these coils which, according to Lenz's law, tends to wipe out the flux; thus the pulsation becomes

dampened out, as shown in curve *b*, diagram *E*. The armature field is therefore equivalent in effect to a constant field, which is fixed relative to the poles and has a value of half the max mum value which it would have if the pulsation were not dampened out.

Another way to look at the above subject is as follows: The armature field is stationary in space and is alternating. Such a field, as pointed out in Art. 144, page 184, can be exactly represented by two progressive fields of constant value which move in opposite directions through the distance of two pole-pitches while the alternating field goes through one cycle. Consider these two fields to exist separately, then one moves in the same direction and at the same speed as the poles while the other moves at the same speed but in the opposite direction to the poles. The former is therefore stationary with regard to the poles and may be treated in exactly the same way as the armature field in a polyphase machine; the latter field revolves at the same speed as the main poles but in the opposite direction and so causes a double frequency pulsation of flux in these poles which pulsation induces currents in the field windings that tend to wipe out the flux, so that the effect of this latter field can be neglected in a discussion of armature reaction.

A flux of double frequency in the poles will produce a third harmonic of e.m.f. in the armature, as shown in Art. 144, page 184; a third harmonic of current in the armature will produce a fourth harmonic of flux in the poles and so on; however, high frequency harmonics in the poles are so well dampened out that those higher than the third can be neglected.

The demagnetizing ampere-turns per pole and the leakage reactance are worked out below for a particular case; the method is quite general, but since the result depends on the per cent. of the pole-pitch that is covered by the winding, no general formula is deduced.

178. The Demagnetizing Ampere-turns per Pole at Zero Power Factor.—Fig. 173 shows the distribution of the m.m.f. of armature reaction for a machine with *b* conductors per slot, and six slots per pole, of which four are used, at the instant when the current has its maximum value.

If the pole arc = 0.6 times the pole-pitch then
the average value of that part of the maximum m.m.f. which is effective $\times 3.6\lambda$

$= \text{area of cross-hatched curve in Fig. 173}$
$= 2\,bI_m \times 3\lambda + bI_m \times 0.6\lambda$
$= 6.6\,bI_m\lambda$

The constant field which is fixed relative to the poles, and which is the only one that need be considered in calculations, has a value of half the above maximum value, therefore, the average value of the part of this constant field which is effective $= AT_{av}$ is such that $AT_{av} \times 3.6\lambda = \dfrac{6.6\,bI_m\lambda}{2}$

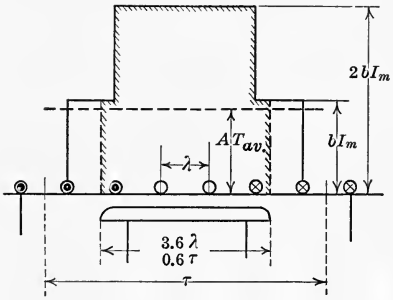


FIG. 173.—The maximum m.m.f. of a single-phase alternator on zero power factor.

and $AT_{av} = 0.92\ bI_m$
$= 1.3\ bI_c$ where I_c is the effective current

$$= 1.3\ bI_c \times \frac{\text{effective slots per pole}}{4}$$

$$= 0.32 \times \text{cond. per pole} \times I_c. \qquad (30)$$

179. The Leakage Reactance.—As in the case of the polyphase machine this consists of end connection, slot and tooth-tip reactance.

The end-connection reactance $= 2\pi fpb^2c^2\ (\phi_e L_e)\ 10^{-8}$ for a machine with a chain winding, and half of this value for a machine with a double-layer winding; where c is the effective number of slots per pole or the number which carry conductors.

$\phi_e L_e$ is found from Fig. 163, but it must be noted that for the type of winding shown in Fig. 174, ϕ_e links the coils of four slots and the length of the end-connection leakage path is two-thirds of the value which it would have if all the slots were used; the value of $\phi_e L_e$ is therefore 3/2 times that found from Fig. 163.

The slot reactance $= 2\pi fpb^2c^2L_c \times \phi_s \times 10^{-8}$

where $\phi_s = \frac{3.2}{c}\left(\frac{d_1}{3s} + \frac{d_2}{s} + \frac{2d_3}{s+w} + \frac{d_4}{w}\right)$; the slot dimensions are given in Fig. 164, page 218, and c is the effective number of slots per pole.

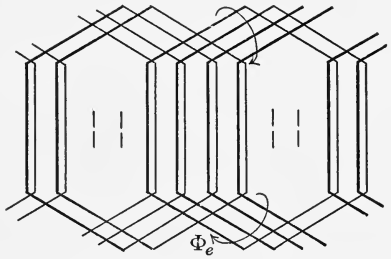


FIG. 174.—The end connection leakage path in single-phase alternators.

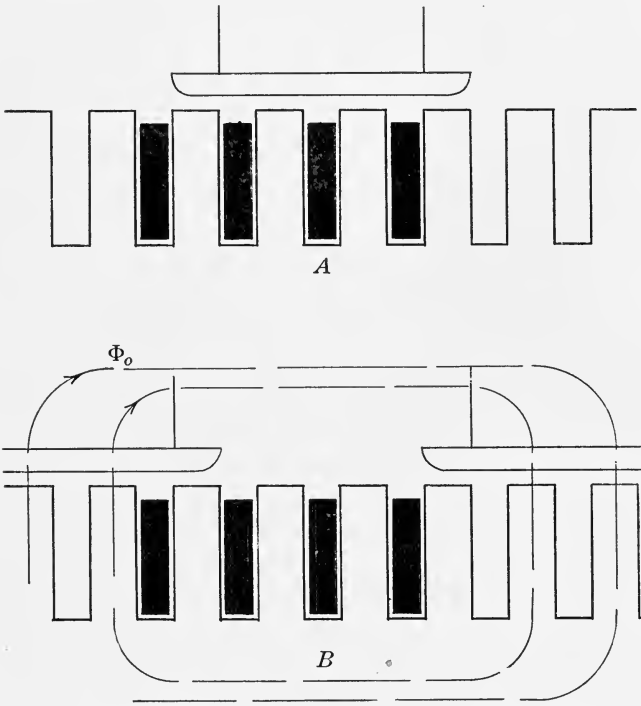


FIG. 175.—The positions of maximum and minimum tooth tip leakage.

The tooth-tip reactance varies from a maximum, when the conductors are in position A, Fig. 175, to a minimum, when they are in position B. In the former position it may be seen

that three only of the slots lie under the pole at any time, while in the latter position it may be assumed that three only of the slots are effective since the leakage lines ϕ_o which link all the slots have to pass through the field windings and are therefore dampened out.

An approximate solution for the case of a machine with six slots per pole of which four are used, and which has a pole arc = 0.6 times the pole-pitch, is

tooth-tip reactance $= 2\pi f b^2 c^2 p L_c \times \phi_t \times 10^{-8}$

where ϕ_t has the value given in Art. 176, page 223, for 3 slots per phase per pole and c for the assumed conditions = 3.

Example of Calculation.—Fig. 167 shows the actual test curves on a small alternator which has six slots per pole of which four are used; the single-phase short-circuit curve is shown dotted.

The demagnetizing ampere-turns per pole

$$= 0.32 \times 12 \times 4 \times 62.5$$
$$= 960 \text{ ampere-turns per pole}$$

the corresponding field current

$$= \frac{\text{demagnetizing ampere-turns per pole}}{\text{field-turns per pole}}$$
$$= \frac{960}{420}$$
$$= 2.3 \text{ amperes}$$

The reactance per phase

$$= 2\pi f b^2 c^2 p \left(\frac{\phi_e L_e}{2} + (\phi_s + \phi_t) L_c \right) 10^{-8} \text{ ohms}$$

where these symbols have to be interpreted in the light of the last article

$\frac{\phi_e L_e}{2}$ = (5 × 1 from Fig. 163) × $\frac{3}{2}$ since, as pointed out in the last Art. the value found from Fig. 163 has to be increased 50 per cent. because four only of the six slots per pole are used

$$\phi_s = \frac{3.2}{4} \left(\frac{1.5}{3 \times 0.75} + \frac{0.25}{0.75} \right) = 0.8$$

$$\phi_{ta} = 2.35 \log_{10} \left(1 + \frac{\pi \times 1}{(3 \times 0.75) + (2 \times 1)} \right) + \frac{2.35}{9} \log_{10} \left(1 + \frac{\pi \times 1}{0.75} \right)$$
$$= 0.75$$

$$\phi_{tp} = 3.2 \times \frac{10}{9} \times \frac{1 \times 1.2}{2 \times 0.2} = 10.7$$

$$\phi_t = 0.52 \times 0.75 + 0.42 \times 10.7 = 4.9$$

reactance per phase

$$= \{2 \times \pi \times 60 \times 12^2 \times 4^2 \times 6 \ (7.5 + 0.8 \times 6.5) \ 10^{-8}\}$$
$$+ \{2 \times \pi \times 60 \times 12^2 \times 3^2 \times 6 \ (4.9 \times 6.5) \ 10^{-8}\}$$
$$= 1.6 \text{ ohms}$$

the voltage drop per phase $= 1.6 \times 62.5$
$= 100$ volts.

with this value and with the demagnetizing effect $= 2.3$ amp. the triangle *abn*, shown dotted in Fig. 167, is drawn in and it may be seen that the calculated results check the test results very closely.

180. Regulation of Single-Phase and Three-Phase Alternators.—In the machine whose test results are shown in Fig. 167 the output at full-load

$= 1.73 \times 600 \times 62.5 = 65$ *k.v.a.* for the three-phase rating
$= 600 \times 62.5 \quad = 37.5$ *k.v.a.* for the single-phase rating

or the single-phase output is about 60 per cent. of the three-phase output. For this ratio, *n* is a point on the full-load saturation curve of the single-phase machine and *m* a point on that of the three-phase machine, and, as may be seen, the former machine has the better regulation. For the same regulation in each case the single phase machine may be given 65 per cent. of the three-phase rating.

CHAPTER XXII

DESIGN OF A REVOLVING FIELD SYSTEM

The problem to be solved in this chapter is, given the armature of an alternator and also its rating to design the whole revolving field system.

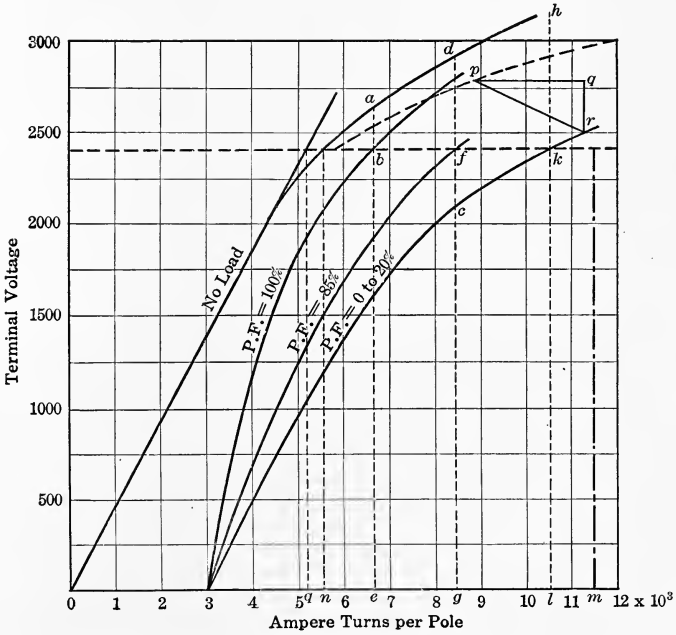


FIG. 176.—Saturation curves for a 400 k.v.a. 2400-volt, 3-phase, 60-cycle, 600 r.p.m. alternator.

181. Field Excitation.—Fig. 176 shows several of the saturation curves of an alternator:
the excitation required for normal voltage

$= on$ at no-load
$= oe$ at full-load and unity power factor
$= og$ at full-load and 85 per cent. power factor
$= ol$ at full-load and zero power factor

$$\text{the maximum exciting current} = \frac{\text{exciter voltage}}{\text{hot resistance of field coils}} \text{ and}$$

$$\text{the maximum excitation} = \text{max. exciting current} \times \text{field-turns per pole}$$

$$= om$$

The radiating surface of the field coils should be large enough to keep the temperature rise with the excitation *og* below that guaranteed, which is generally 40° C.

The maximum excitation *om* should be large enough to enable the alternator to give normal voltage on all loads and power factors to be met in practice. If the maximum excitation is equal to *ol*, and, therefore, large enough to maintain normal voltage at full-load and zero power factor, it will generally be found ample for all ordinary overloads and power factors.

The field resistance should have such a value that the maximum excitation *om* may be obtained with the normal exciter voltage.

The ratio $\frac{om}{og}$ is about 1.25 for normal machines, so that if the temperature rise with excitation *og* is 40° C. that with excitation *om* will be 40×1.25^2 or 62° C. It is therefore usual to design the field coils for an excitation *om* and for a temperature rise of 65° C.

182. Procedure in the Design of a Revolving Field System.—(*a.*) Find AT_{max}, the maximum excitation = 3(armature *AT.* per pole) for a first approximation, see Art. 176, page 230.

(*b.*) Find M, the section of the field coil wire from the formula

$$M = \frac{AT_{max} \times \text{mean turn}}{\text{volts per coil}} \quad \text{Formula 7, page 65.}$$

where the volts per coil $= \frac{\text{exciter voltage}}{\text{poles}}$ and the mean turn is found as follows:

$$\phi_a = \text{the flux per pole crossing the gap} = \frac{E \times 10^8}{2.22\, kZf}$$

Formula 25, page 190.

lf., the leakage factor, is assumed to be 1.2 for a first approximation.

$$\text{Pole area} = \frac{\phi_a \times lf}{\text{pole density}}$$ where the pole density at normal voltage and frequency and at no-load is taken as 95,000 lines per square inch, which is about the point of saturation. The pole area also $= 0.95 \times L_p \times W_p$.

L_p, the axial length of the pole, is made 0.5 in. shorter than the frame length so that the rotor can oscillate freely. The value of W_p, the pole waist, and of MT, the mean turn of the coil, can then be found approximately.

(*c*) Find L_f, the radial length of the field coil, from the formula

$$L_f = \frac{AT_{max}}{1000}\sqrt{\frac{\text{mean turn}}{\text{ext. periphery} \times \text{watts per sq. in.} \times d_f \times sf \times 1.27}}$$

Formula 8, page 66, where:—

external periphery is found approximately from the pole dimensions;

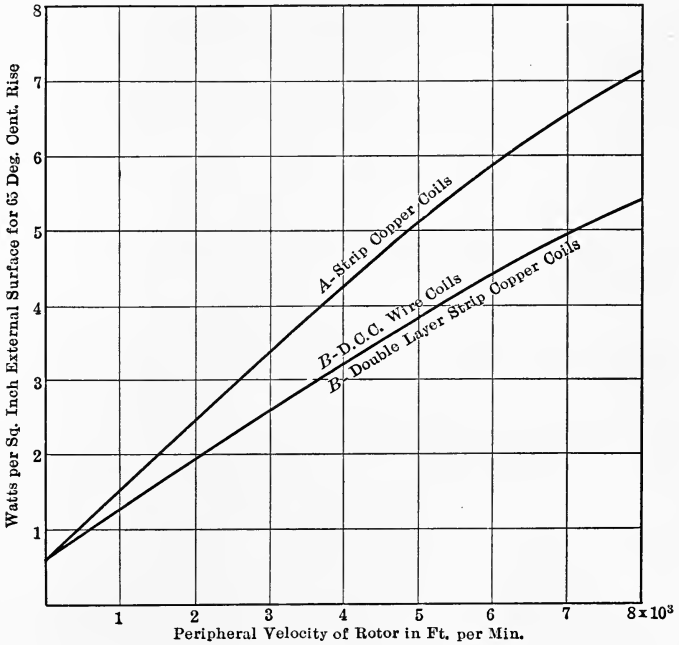


FIG. 177.—Heating curves for the field coils of revolving field alternators.

watts per square inch is found from Fig. 177 which gives the results of tests on machines similar in construction to that shown in Fig. 138, and with the type of field coil shown in Fig. 140;

d_f, the winding depth, is chosen so as to give the most economical field structure. It was shown in Art. 56, page 66, that the smaller the value of d_f the lower the cost of the field copper but the longer and more expensive the poles. The most

economical depth can be found by trial but the following values may be used as a first approximation:

Pole pitch	Depth of field coil	
	60-cycle	25-cycle
5 in.	0.6 in.	
10 in.	0.75 in.	1.0 in.
15 in.	1.0 in.	1.25 in.
20 in.		1.5 in.
30 in.		2.0 in.
40 in.		2.5 in.

For a given pole-pitch the 60-cycle machine runs at a higher peripheral velocity than the 25-cycle machine, it therefore requires less radiating surface for the same excitation and the poles become very short unless the value of d_f is decreased below that which is found best for 25-cycle machines. The above figures are for strip copper field windings; when d.c.c. wire is used the above depth should be increased about 20 per cent. because of the poorer space factor of this type of winding, and the coils should be tapered, if necessary, as shown in Fig. 140. (*d*) T_f is the number of field-turns that will fill up the space $d_f L_f$, the size of wire being fixed.

183. Calculation of the Saturation Curves.—It is necessary first of all to find the air gap clearance, which is done as follows: The approximate number of ampere-turns per pole required for the air gap at no-load and normal voltage, namely *oq*, Fig. 176 $= ol - lq$ and is approximately

$$= AT_{max} - 1.5(\text{armature } AT. \text{ per pole}).$$

Since the flux per pole and also the dimensions of the machine are given, the apparent gap density and, therefore, the air gap clearance required, can be found from the formulæ

$$\text{Apparent gap density} = \frac{\text{flux per pole crossing the gap}}{\tau \times \psi \times L_c}$$

$$\text{Air gap ampere-turns} = \frac{\text{apparent gap density}}{3.2} \times \delta \times C$$

Formula 3, page 48.

The magnetic circuit is now drawn in to scale and the leakage factor and the no-load saturation curves calculated by the method explained in Arts. 46 and 47, page 46.

The leakage factor at full-load and zero power factor

$$= 1 + 0.\text{m}\left(\frac{AT_{g+t} + \text{demagnetizing } AT. \text{ per pole}}{AT_{g+t}}\right) \text{ Art. 164, page}$$

212, where l.m is the no-load leakage factor, and
the demagnetizing $AT.$ per pole $= 0.35 \times$ cond. per pole $\times I_c$.

A new no-load saturation curve is calculated using the above full-load leakage factor and plotted as shown in the dotted curve in Fig. 176.

The reactance per phase is determined from the formulæ on page 223 and the triangle *pqr* and the full-load saturation curve at zero power factor are then drawn in.

The full-load saturation curves at other power factors are calculated by the use of the diagram shown in Fig. 169, and from these curves the regulation at the different power factors is determined.

If the regulation found from the curves is equal to or slightly better than the regulation required, then the field design is complete.

If the regulation found is considerably better than that required, then the machine is unnecessarily expensive and the following changes may be made:

a. The air gap may be made smaller so that less excitation is required and the cost of the field copper is reduced; the effect of such a reduction in air gap is explained in Art. 176, page 229.

b. The armature can be redesigned and the value of q, the ampere conductors per inch, increased. If the total number of conductors is increased, the flux per pole is decreased, and for the same tooth density and a reduced value of flux per pole the diameter or frame length or both must be reduced; in order to carry the increased number of ampere conductors on each inch of the periphery, deeper slots must be supplied. These changes will increase both the armature reaction and the armature reactance and tend to make the regulation worse, but will make the machine cheaper.

If the regulation found from the curves is worse than that required, then the following changes may be made on the machine to improve it:

c. The air gap may be increased so that more excitation is required and the cost of the field copper is increased, the effect of such an increase in air gap is explained in Art. 176, page 229.

d. The air gap may be decreased and the pole section reduced so as to increase the pole density and bend over the saturation curve as shown in Fig. 170; this, however, is a risky expedient unless all the material that goes into the machine is carefully tested for permeability and rejected if not up to standard. If the permeability of the material put into the machine is lower than was expected then the chances are that the machine will not be able to give its voltage on low power factor loads.

e. The armature can be redesigned and the value of q, the ampere conductors per inch, decreased, this will have an effect opposite to that discussed in section *b* and will require a more expensive machine but will give better regulation.

Example.—The armature of a 400-kv.a., 2400-volt, 96-amp., 3-phase, 60-cycle, 600-r.p.m. revolving field alternator is built as follows:

Poles,	12
Internal diameter,	43 in.
Frame length,	12.25 in.
Center vent ducts,	$3-0.5$ in.
Slots per pole, number,	6 open type
Slots per pole, size,	0.75×2.0 in.
Conductors per slot,	12
Connection,	Y
Exciter voltage,	120

It is required to design the revolving field system:

The armature ampere-turns per pole

$$= \text{slots per pole} \times \frac{\text{cond. per slot}}{2} \times I_c$$

$$= 6\times6\times96$$

$$= 3450 \text{ ampere-turns.}$$

AT_{max} = the maximum excitation $= 3\times3450$

$= 10{,}400$ ampere-turns, approximately.

E = the voltage per phase $= \frac{2400}{1.73}$ since the connection is Y

$= 1380$

ϕ_a = the flux per pole $= \frac{1380\times10^8}{2.22\times0.96\times\frac{72}{3}\times12\times60}$

$= 3.8\times10^6$

lf = the leakage factor, is assumed to be $= 1.2$.

$$\text{pole area} = \frac{3.8\times10^6\times1.2}{95{,}000}$$
$$= 48 \text{ sq. in.}$$

L_p = the axial length of the pole, is made 0.5 in. shorter than the frame length and $= 11.75$ in.

$$W_p = \text{the pole waist} = \frac{48 \text{ sq. in.}}{0.95\times11.75}$$
$$= 4.25 \text{ in.}$$

d_f, the depth of the field coil $= 0.75$ in. from the table on page 240.

$$MT = \text{the mean turn} = 2(11.75+4.25)+\pi\times1.25$$
$$= 36 \text{ in.}$$

External periphery of field coil $= 38$ in.

$$M, \text{ the section of field wire} = \frac{10{,}400\times36}{10}$$
$$= 37{,}500 \text{ circular mils}$$
$$= 0.03 \text{ sq. in.}$$
$$= 0.04\times0.75 \text{ (strip copper wound on edge)}$$

Watts per square inch for 65° C. rise $= 6.3$ from Fig. 177, page 239.

$$sf. \text{ the space factor} = \frac{0.04}{0.04+\text{thickness of insulation}}$$
$$= 0.8 \text{ using paper which is 0.01 in. thick}$$

$$L_f \text{ the radial length of field coil} = \frac{10{,}400}{1000}\sqrt{\frac{36}{38\times6.3\times0.75\times0.8\times1.27}}$$
$$= 4.75 \text{ in.}$$

$$T_f, \text{ the number of turns per pole} = \frac{4.75}{0.05}$$
$$= 95 \text{ turns.}$$

$$\text{The maximum exciting current} = \frac{AT_{max}}{T_f}$$
$$= \frac{10{,}400}{95}$$
$$= 110 \text{ amperes}$$

$$\text{Maximum output from exciter} = 120\times110$$
$$= 13 \text{ kw.}$$
$$= 3.25 \text{ per cent. of the volt ampere rating.}$$

AT_g, the gap ampere-turns per pole at normal voltage and no-load,
$= AT_{max} - 1.5\times$ armature AT. per pole, approximately
$= 10{,}400 - 1.5\times3450$
$= 5200$ ampere-turns.

ϕ, the pole enclosure, is taken as 0.65 which is an average value; if the pole arc be made too large, the pole leakage becomes excessive and the leakage factor high.

$$B_g, \text{ the apparent gap density} = \frac{3.8\times10^6}{11.25\times0.65\times12.25}$$
$$= 42{,}500 \text{ lines per square inch.}$$

$$\delta \times C = \frac{5200\times3.2}{42500}$$
$$= 0.39 \text{ in.}$$

C = the Carter coefficient = 1.12 from Fig. 40, page 44
δ = the air gap in inches = 0.35

Calculation of the leakage factor:

In Fig. 178 h_s = 1. in.
L_s = 11.75 in.
l_1 = 3.8 in.
W_s = 7.0 in.
h_p = 5.25 in.
L_p = 11.75 in.
l_3 = 5.2 in.
W_p = 4.25 in.

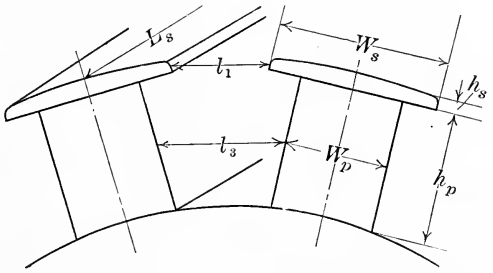


FIG. 178.—The pole dimensions.

$$\text{and } \phi_{e_1} = 13 \times AT_{g+t} \times \frac{1 \times 11.75}{3.8} = 40\ AT_{g+t}$$

$$\phi_{e_2} = 19 \times AT_{g+t} \times 1 \times \log_{10}\left(1 + \frac{\pi \times 7}{2 \times 3.8}\right) = 11\ AT_{g+t}$$

$$\phi_{e_3} = 6.5 \times AT_{g+t} \times \frac{5.25 \times 11.75}{5.2} = 77\ AT_{g+t}$$

$$\phi_{e_4} = 9.5 \times AT_{g+t} \times 5.25 \times \log_{10}\left(1 + \frac{\pi \times 4.25}{2 \times 5.2}\right) = 18\ AT_{g+t}$$

and ϕ_e = the total leakage flux per pole $= 146\ AT_{g+t}$

The value of AT_{g+t} = 5200 ampere-turns approximately, since the tooth densities in alternators are so low that the ampere-turns for the teeth can be neglected here,
therefore $\phi_e = 146 \times 5200 = 760{,}000$

$$\text{and the no-load leakage factor} = \frac{3.8 \times 10^6 + 760000}{3.8 \times 10^6} = 1.20$$

The full-load leakage factor, at zero power factor,

$$= 1 + \left(0.2 \times \frac{5200 + 0.35 \times 6 \times 12 \times 96}{5200}\right)$$

$$= 1.3$$

The magnetic areas:

τ = the pole-pitch	=11.25 in.
ψ = the per cent. enclosure	=0.65
L_g = the gross iron	=10.75 in.
L_n = the net iron	=9.6 in.
A_g = the apparent gap area	=90 sq. in.
C = the Carter coefficient	=1.12, already found.
A_t = minimum tooth area per pole	$=6\times0.65\times1.13\times9.6$.
	=42.3 sq. in.

No-load voltage Flux per pole Leakage factor			2400 3.8×10^6 1.2		2700 4.28×10^6 1.2		3000 4.75×10^6 1.2	
	Length	Area	Density	AT.	Density	AT.	Density	AT.
Air gap	0.35	$\frac{90}{1.12}$	42,500	5200	.....	5850	.....	6500
Teeth	2.0	42.3	90,000	50	101,000	100	112,000	220
Pole	6.0	47.5	96,000	325	108,000	960	120,000	2280
Total amp.-turns per pole				5575	.....	6910	.....	9000

To get the figures for the no-load saturation curve with the full-load leakage factor it is necessary to recalculate the excitation for the poles, using a leakage factor of 1.3; the air gap and teeth are not affected.

Pole	6.0	47.5	105,000	770	119,000	2220	132,000	5200
Total amp.-turns per pole				6020	.....	8170	.....	11,920

From the former set of values the no-load saturation curve in Fig. 176 is plotted and from the latter set the dotted curve, which is the no-load saturation curve with the full-load leakage factor, is plotted.

In the above calculation the core and revolving field ring have been neglected since the flux density is very low in the case of the core to keep down the temperature of the iron and in the case of the field ring to give the necessary mechanical strength and rigidity.

The reactance per phase is found by the method shown on page 224, as follows:

$\frac{\phi_e L_e}{2} = 5.3 \times 3$ from Fig. 163 $= 16$

$\phi_s = \frac{3.2}{2}\left(\frac{1.75}{3 \times .75} + \frac{0.25}{0.75}\right) = 1.7$

$\phi_{ta} = 2.35 \log_{10}\left(1 + \frac{\pi \times 1.13}{2 \times 0.75 + 1.13}\right) = 0.87$

$\phi_{tp} = 3.2 \times \frac{1.13 \times 1.12}{2 \times 0.35} = 5.8$

$\phi_t = 0.52 \times 0.87 + 0.42 \times 5.8 = 2.9$

Reactance per phase

$$= 2 \times \pi \times 60 \times 12^2 \times 2^2 \times 12(16 + (1.7 + 2.9) \times 12.25)\, 10^{-8}$$
$$= 1.88 \text{ ohms}$$

The voltage drop per phase $= 1.88 \times 96$
$= 180$ volts

The voltage drop at the terminals $= 1.73 \times 180$
$= 310$ volts since the winding is Y-connected
$= 12.9$ per cent. of normal voltage

The demagnetizing ampere-turns per pole $= 0.35 \times 6 \times 12 \times 96$
$= 2420$

and the corresponding field current $= \frac{2420}{\text{field-turns per pole}}$
$= 25.5$ amperes.

The full-load saturation curve at zero power factor and lagging current can now be drawn in and is shown in Fig. 176.

The curves at unity power factor and at 85 per cent. power factor are also drawn in and the regulation as determined from these curves $= 9$ per cent. at unity power factor
$= 21$ per cent. at 85 per cent. power factor
$= 26$ per cent. at zero power factor.

CHAPTER XXIII

LOSSES, EFFICIENCY AND HEATING

Many of the losses in an alternator are similar to and are figured out in the same way as those in a D.-C. machine.

184. Bearing Friction Loss $= 0.81dl\left(\frac{V_b}{100}\right)^{\frac{3}{2}}$ watts

where d = the bearing diameter in inches,
l = the bearing length in inches,
V_b = the rubbing velocity of the bearing in feet per minute.

185. Brush Friction.—This loss is small since there are few brushes and the rubbing velocity is low;

the loss $= 1.25\ A\ \frac{V_r}{100}$ watts

where A is the total brush rubbing surface in square inches,
V_r is the rubbing velocity in feet per minute.

186. Windage Loss.—This loss cannot be separated out from the bearing friction loss so that its value is not known and, except in the case of turbo generators, it can be neglected since it is comparatively small.

187. Iron or Core Losses.—As in the case of the D.-C. machine these include the hysteresis and eddy current losses; additional core loss due to filing, to leakage flux in the yoke and end heads, and to non-uniform distribution of flux in the core; losses in the pole face.

The total core loss is figured out by the use of the curves in Fig. 81, page 102, which curves are found to apply to alternators as well as to D.-C. machines.

188. Excitation Loss.—The field excitation and therefore the field excitation loss vary with the power factor, as shown in Fig. 176. The loss = amperes in the field × voltage at the field terminals; for separately excited machines the rheostat is not considered as part of the machine.

189. Armature Copper Loss.—

The resistance of one conductor $= \frac{L_b}{M}$ ohms

and the loss in one conductor $= \frac{L_b I_c^2}{M}$ watts

where L_b is the length of one conductor in inches,

I_c is the current in each conductor,

M is the section of each conductor in cir. mils.

The total armature copper loss $= Z_c \frac{L_b I_c^2}{M}$ watts (31)

where Z_c is the total number of conductors.

In addition to the above copper loss there are eddy current losses in the conductors which losses may be large and cause trouble due to heating unless care is taken to properly laminate the conductors.

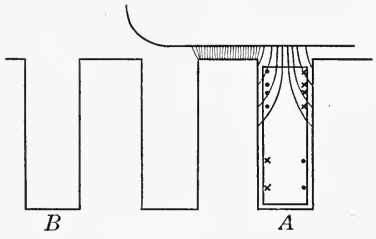


FIG. 179.—Flux in an armature slot.

Figure 179 shows the flux distribution in the slots and air gap of an alternator. The flux entering a slot changes from a maximum in position A to zero in position B, and this change of flux causes eddy currents to flow in the conductors in the direction indicated by crosses and dots. To prevent these eddy currents from having a large value it is necessary to laminate the conductors both vertically and horizontally. This flux seldom gets beyond half the depth of the slot, so that it is not important to laminate conductors which lie in the bottom of the slot.

The loss due to these eddy currents is a constant loss, being present at no-load as well as at full-load, and is therefore measured with the core losses.

Of greater importance are the eddy currents produced by the armature leakage flux. Fig. 180 shows the leakage flux crossing an alternator slot due to the current in the conductors. This flux is alternating and therefore sets up eddy currents, the direc-

tion of which is indicated at one instant by crosses and dots. To prevent these eddy currents from having a large value it is necessary to laminate the conductors horizontally.

The m.m.f. tending to send this leakage flux across the slot at mn is due to the current in the conductors which lie below mn, therefore the leakage flux density is zero at the bottom of the slot and a maximum at the top of the slot; it is therefore more important to laminate the conductors in the top of the slot than those in the bottom.

The loss due to leakage flux is a load loss since the leakage flux which produces it is proportional to the current in the conductors. The following example will show how large it may become.

Consider the case of the double layer winding shown in Fig. 180; the flux $d\phi = 3.2\left(bI_c \times \frac{y}{d}\right) \times \frac{L_c dy}{s}$

and ϕ, the flux crossing the upper layer of conductors

$$= 3.2\, bI_c L_c \int_{d/2}^{d} \frac{y \times dy}{d \times s}$$

$$= 3.2\, bI_c L_c \times \frac{3d}{8s}$$

$$= 1.2\, bI_c L_c \frac{d}{s}$$

ϕ_m, the maximum flux crossing the upper layer of conductors

$$= 1.2\, bI_m L_c \times \frac{d}{s}$$

$$= 1.7\, bI_c L_c \times \frac{d}{s}$$

Consider a loop kl of width g and thickness dx, the effective e.m.f. in this loop due to the flux which crosses the conductors

$$= 4.44\, \phi_m\, f\, 10^{-8} \text{ volts}$$

the resistance of this loop

$$= \frac{1 \times 2L_c}{g \times dx \times 1.27 \times 10^6} \text{ ohms}$$

the eddy current in the loop

$$= \frac{\text{effective e.m.f.}}{\text{resistance}}$$

$$= \frac{4.44 \times \left(1.7 \times bI_c L_c \times \frac{d}{s}\right) \times f \times 10^{-8}}{\frac{2L_c}{g \times dx \times 1.27 \times 10^6}}$$

$$= 4.8 \times 10^{-2} \times bI_c \times \frac{d}{s} \times f \times g \times dx \text{ amperes}$$

and the current density in the loop

$$= \frac{\text{current}}{g \times dx}$$

$$= 4.8 \times 10^{-2} \times b I_c \times \frac{d}{s} \times f \text{ amp. per square inch}$$

the normal current density in one of the top conductors

$$= \frac{I_c \times 2}{g \times d} \text{ amp. per square inch approximately}$$

and the ratio

$$\frac{\text{current density at the edge of conductor due to slot leakage}}{\text{normal current density}}$$

$$= 2.4 \times 10^{-2} \times d^2 \times f \times \frac{b \times g}{s}$$

To take a particular example

$$f = 60 \text{ cycles}$$

$$\frac{b}{2} g = 0.7s$$

$$d = 2 \text{ in.}$$

and the above ratio $= 8$

The eddy currents however tend to wipe out the flux which produces them so that the above result is greatly exaggerated.[1]

For 60-cycle machines the depth of conductor should not exceed 0.5 in. and for 25-cycle machines should not exceed 0.75 in., otherwise trouble is liable to develop due to heating caused by the eddy current loss in the conductors. These figures represent standard practice for machines up to 2400 volts; for higher voltages the insulation is thick and the difference in temperature between the copper and the iron quite large; due to additional eddy current losses, this temperature difference may become excessive and cause local deterioration of the insulation, unless the lamination of the conductors is carried much further than in machines for low voltages.

[1] For an accurate solution of this problem see Field, *Proc.* Amer. Inst. Elec. Engrs., Vol. 24, page 765.

190. The Efficiency.—For an alternating current generator

$$\text{the efficiency} = \frac{\text{output in kw.}}{\text{output in kw.} + \text{losses in kw.}}$$

where the losses are:
Windage, bearing and brush friction;
Excitation loss;
Iron losses;
Armature copper loss (neglecting eddy current loss);
Load losses, which are principally the eddy current losses in the conductors.

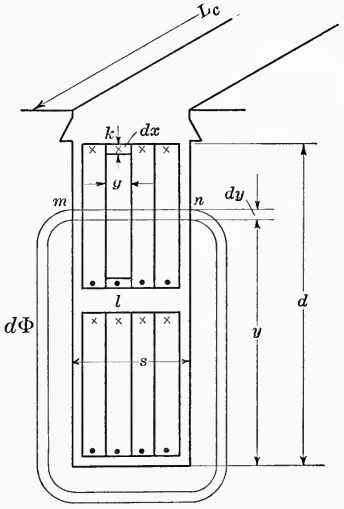


FIG. 180.—Eddy currents due to armature slot leakage.

These load losses are determined in the following way:
The alternator is run at normal speed with the armature short-circuited through an ammeter, it is then excited with sufficient field to circulate full-load current through the armature and under these conditions the input into the driving motor is measured; the field is then taken off the machine, the armature open-circuited, and the power taken by the driving motor again measured; the difference between the two readings is called the short-circuit core loss and includes:
The armature I^2R loss;
The eddy current loss in the armature conductors due to leakage flux;
The small iron loss due to the flux in the core.

It is recommended in the standardization rules of the American Institute of Electrical Engineers that the load losses be taken as 1/3 (short-circuit core loss − the I^2R loss) in the absence of accurate data on the subject.

191. Heating.—The temperature rise of the stator of a revolving field alternator is limited in the same way as that of the armature of a D.-C. machine.

For stator cores, built with iron of the same grade as used in D.-C. machines and of a thickness = 0.014 in., the following flux densities may be used for a machine whose temperature rise at normal load must not exceed 40° C.

Frequency	Maximum tooth density in lines per sq. in.	Maximum core density in lines per sq. in.
60 cycles	90,000	45,000
25 cycles	110,000	65,000

The above represent standard practice for open slot machines. When the slots are partially closed, so that the tufting of the flux in the air gap is eliminated, the pole face losses and also the eddy current losses in the conductors at no-load are negligible and the densities may safely be increased 15 per cent.

The end connection heating is limited by keeping the value of the ratio $\frac{\text{amp. cond. per in.}}{\text{cir. mils per amp.}}$ below that given in Fig. 181; the curve applies to revolving field open type machines.

The same curve is used for both chain and double layer windings; the chain winding has a slightly smaller radiating surface than the double-layer winding but it is more open and therefore allows freer circulation of air between the coils.

The stator heating is affected by that of the rotor because the air which cools the stator passes over the rotor surface, during which time it is heated; the hotter the rotor the hotter the air which is blown on to the stator and the greater the temperature rise of the stator above that of the air in the power-house.

The ventilation of the power-house has a good deal to do with the temperature rise of the machines operating therein. In the case of belt driven units the belt keeps the air in the power-house circulating; the flywheel of a direct connected engine type of machine has the same effect. In the case of water-wheel units, however, the circulation of the air in the power-house has generally to be brought about by the machines themselves and, unless they are built with fans, so as to prevent

the air which has passed through the machines and has thereby become heated from being sucked in again, they will often get hot even although liberally designed. Such machines generally stand over a pit and it will often be found that, if this pit is connected by a duct to the external air, the temperature rise of the machine above that of the air in the power-house will be reduced.

192. Internal Temperature in High Voltage Machines.—It was pointed out in Art. 94, page 109, that the difference in

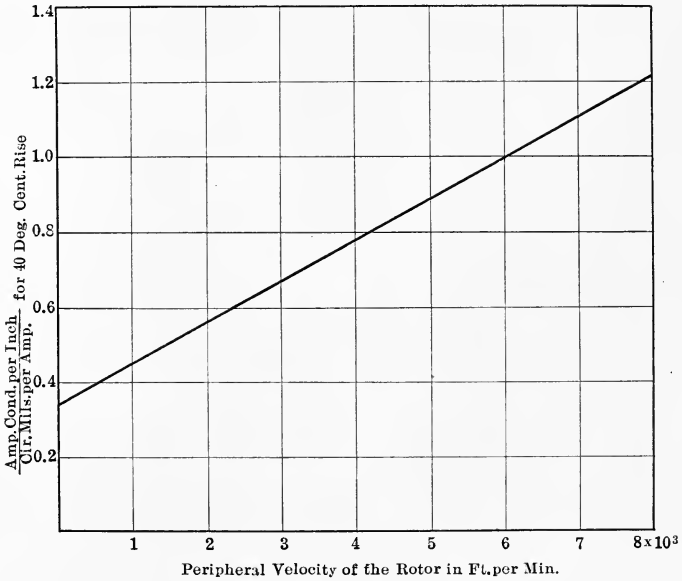


FIG. 181.—Heating curve for stator end connections of revolving field alternators.

temperature between the copper in the center of a machine and that on the end connections, if all the heat in the copper is conducted axially along the conductors

$$= \frac{5.7 \times 10^4 \times \left(\frac{L_c}{2}\right)^2}{(\text{cir. mils per amp.})^2} \text{ deg. cent.}$$

while if all the heat is conducted through the slot insulation the difference in temperature between the copper and the iron

$$= \frac{\text{amp. cond. per slot}}{\text{cir. mils per amp.}} \times \frac{\text{thickness of insulation}}{2d+s} \times \frac{1}{0.003} \text{ deg. cent.}$$

That the results of these formulæ require careful consideration

in the case of high voltage machines, is shown by the following example.

In Art. 202 is given the preliminary design of a 2750 k.v.a. revolving field generator for direct connection to a water wheel which must run safely at an overspeed of 75 per cent. This overspeed requirement limits the diameter of the machine so that the frame has to be long in order to give the output.

The frame length	=28.5 in.
Insulation thickness	=0.25 in. for 11,000 volts.
Slot pitch	=2.8 in.
Slot width	=1.0 in.
Slot depth	=3.5 in.
Ampere conductors per inch	=700 assumed
Circular mils per ampere	=700 assumed.

The difference in temperature between the copper in the center of the machine and that on the end connections if all the heat is conducted along the conductor

$$= \frac{5.7 \times 10^4 \times (14.25)^2}{700^2}$$

$$= 24° \text{ C.}$$

the difference in temperature between the copper and the iron if all the heat is conducted through the slot insulation

$$= \frac{700 \times 2.8}{700} \times \frac{0.25}{8} \times \frac{1}{0.003}$$

$$= 29° \text{ C.}$$

In such a case a large part of the copper loss will be conducted to and dissipated by the end connections, and in order that these may remain cool the value of the ratio $\frac{\text{amp.cond.per inch}}{\text{cir.mils per amp.}}$ must be taken lower than that given in Fig. 181, and the temperature rise on the end connections must be limited to about 30° C. at normal load in order that the copper in the center of the machine may not get too hot and char the insulation.

CHAPTER XXIV

PROCEDURE IN DESIGN

193. The Output Equation.

$E = 2.22\ kZ\phi_a f \times 10^{-8}$ formula 25, page 190.

$$= 2.12 \times Z \times (B_g \psi\tau \times L_c) \times \frac{p \times \text{r.p.m.}}{120} \times 10^{-8} \text{ taking k} = 0.96$$

and $q = \dfrac{nZI_c}{\pi D_a}$

therefore $nEI =$ the output in watts

$$= n \times 2.12\ ZB_g \psi\tau L_c \times \frac{p \times \text{r.p.m.}}{120} \times 10^{-8} \times \frac{\pi D_a q}{nZ}$$

$$= \frac{2.12 \times \pi^2 \times 10^{-8}}{120} \times B_g \psi L_c \text{ r.p.m. } qD_a{}^2$$

$$\text{and } D^2{}_a L_c = \frac{\text{volt amperes}}{\text{r.p.m.}} \times \frac{5.7 \times 10^8}{B_g \psi q} \tag{32}$$

The value of B_g, the apparent gap density, is limited by the permissible value of B_t, the maximum stator tooth density which, as pointed out in Art. 191, page 252, is approximately

90,000 lines per square inch at 60 cycles
110,000 lines per square inch at 25 cycles

for open slot machines.

Now $\phi_a = B_t \psi\tau L_n \times \dfrac{t}{\lambda}$

and is also $= B_g \psi\tau L_c$

therefore $B_g = B_t \times \dfrac{t}{\lambda} \times \dfrac{L_n}{L_c}$

The ratio $\dfrac{L_n}{L_c}$ is approximately equal to 0.75 taking the vent ducts and the insulation between the laminations into account. The ratio $\dfrac{\lambda}{t}$ varies with the slot pitch. Suppose, for example,

that in a given alternator the number of slots is halved; since the same total number of conductors is required in each case, the total space required for copper remains constant, but the space required for slot insulation is approximately proportional to the number of slots since its thickness is unchanged, so that when the number of slots is halved the slot may be less than twice the width of the original one; the ratio $\frac{\text{tooth}}{\text{slot}}$ therefore increases as the slot pitch increases.

The slot pitch is seldom made greater than 2.75 in. because the section of the copper in a coil for such a slot is large compared with the radiating surface of the coil and it becomes difficult to keep the windings cool.

Figure 182, which shows the relation between $\frac{\text{tooth}}{\text{slot}}$ and slot pitch found in practice for open slot machines, may be used in preliminary design.

B_g is found from the formula given above and is plotted against pole-pitch in Fig. 183, a reasonable value for the slot pitch being assumed.

The value of B_g for alternators is considerably less than that found for D.-C. machines because of the lower tooth density in the former type of machine due to the fact that its armature is stationary while that of the D.-C. machine is revolving. For a given output and speed, therefore, the value of $D^2{}_a L_c$ is greater for alternators than for D.-C. machines.

The value of q is limited partly by heating but principally by the regulation expected. Suppose for example that for a given rating the value of q is increased, which can be done by increasing the number of conductors in the machine or decreasing the diameter. In the former case the armature reaction will be increased since it is proportional to the number of conductors per pole and the armature reactance will be increased since it is proportional to the square of the number of conductors per slot. In the latter case the frame length must increase as the diameter is decreased in order to carry the flux per pole and the slots must be made deeper in order to carry the larger number of ampere conductors on each inch of the periphery, both of which changes increase the armature reactance; the armature reaction, since it depends on the conductors per pole, being equal to 0.35 $\times$ cond. per pole $\times$ I_c, is unchanged.

The value of q given in Fig. 184 may be used for a first approximation in preliminary design and will give reasonably good regulation if the ratio of field excitation at full-load and zero power factor to armature ampere-turns per pole be not less than three, and the pole density at no-load and normal voltage be about 95,000 lines per square inch.

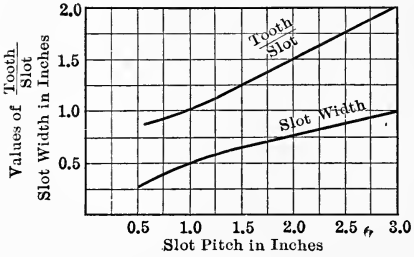


FIG. 182.

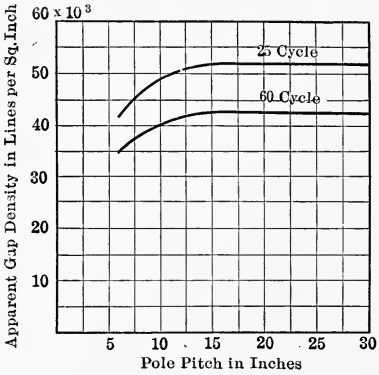


FIG. 183.

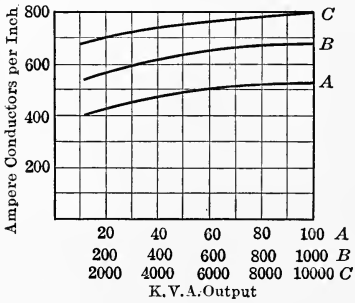


FIG. 184.

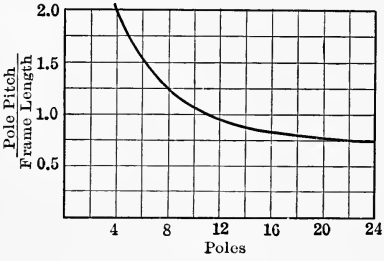


FIG. 185.

Curves used in preliminary design.

194. The Relation between D_a and L_c.—There is no simple method whereby $D_a{}^2L_c$ can be separated into its two components so as to give the best machine. In the case of the D.-C. machine the ratio between the magnetic and the electric loading was used in order to determine D_a and L_c and then the number of poles was chosen so as to give an economical shape of coil.

The number of poles in an alternator is fixed by the speed and the frequency, and the diameter and length of the machine has to be chosen so as to give an economical shape of coil. If for

example the number of poles on a given diameter be increased, the pole-pitch will be reduced, the field coil will become more and more flattened out, and a point will finally be reached at which it would be more economical to increase the diameter and shorten the length of the machine than to keep the diameter constant.

195. Effect of the Number of Poles on the Ratio $\frac{\text{Pole-pitch}}{\text{Frame length}}$.

Figure 186 shows part of two machines which are duplicates of one another so far as the pole unit is concerned; that is, they have

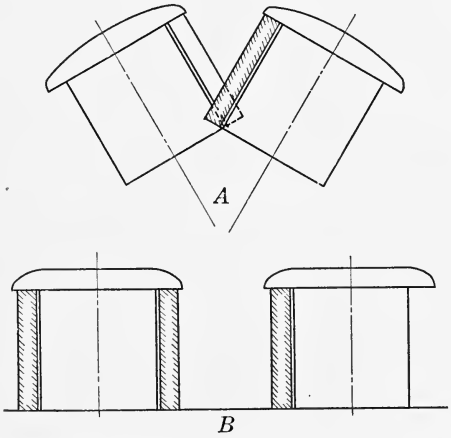


FIG. 186.—Effect of the number of poles on the length of field coils.

the same pole-pitch, air gap, armature ampere-turns per pole and field ampere-turns per pole; but the total number of poles is small in machine *A* and large in machine *B*. It may be seen that in the former machine there is not room for the field coils because of the large angle between the poles. In order to get the field coils on to the poles it is necessary to increase the diameter of the machine without increasing the radial length of the field coil, that is without increasing the armature ampere-turns per pole, on which this length principally depends, so that the same number of armature ampere-turns per pole must now be put on a larger pole-pitch and the value of q, the ampere conductors per inch, thereby reduced. Although the diameter of the machine is increased, the flux per pole must be kept constant otherwise the pole waist will increase; the pole arc will generally

be unchanged and the ratio $\frac{\text{pole arc}}{\text{pole-pitch}}$ will decrease as the diameter is increased, and in four- and six-pole machines will have a value of about 0.6.

The above difficulty, due to a small number of poles, is more apparent in machines of low than in those of high frequency, because in the former the output per pole is generally larger, for example

a 400-k.v.a., 514-r.p.m., 60-cycle machine has 14 poles and an output of 28.5 k.v.a. per pole;

a 400-k.v.a., 500-r.p.m., 25-cycle machine has 6 poles and an output of 67 k.v.a. per pole;

a 6 pole, 60-cycle alternator with an output of 67 k.v.a. per pole would have a rating of 400 k.v.a. at 1200 r.p.m., which would be as difficult to build as the above 25-cycle machine but is such an unusual rating that the difficulty seldom occurs.

The value of q in Fig. 184 applies to machines which have more than 10 poles; for machines with four poles this value should be reduced 30 per cent. for a first approximation, and for a machine with six poles should be reduced about 20 per cent.

Figure 185 shows the value of the ratio $\frac{\text{pole-pitch}}{\text{frame length}}$ generally found in revolving field machines when the diameter is not limited by peripheral velocity, and may be used in preliminary design; the reason for the increase in the ratio as the number of poles decreases has been pointed out above.

196. Procedure in Design.

$$D_a^2 L_c = \frac{\text{volt amperes}}{\text{r.p.m.}} \quad \frac{5.7 \times 10^8}{B_g \psi q} \text{ formula 32, page 255}$$

and $\frac{\text{pole-pitch}}{\text{frame length}} =$ a constant, found from Fig. 185, page 257,

therefore $L_c = \frac{\text{pole-pitch}}{\text{a constant}}$

$$= \frac{\pi \times D_a}{p \times \text{a constant}}$$

$$\text{and } D_a^3 = \frac{\text{volt amperes}}{\text{r.p.m.}} \times \frac{5.7 \times 10^8 \times p \times \text{a constant}}{\pi \times B_g \times \psi \times q} \tag{33}$$

from which D_a may be found approximately since

B_g can be found approximately from Fig. 183,

q can be found from Fig. 184,

ψ is assumed to be $=0.65$ for a first approximation,

the constant = the ratio $\frac{\text{pole-pitch}}{\text{frame length}}$ from Fig. 185.

Tabulate three preliminary designs, one for a diameter 20 per cent. larger than that already found and the other 20 per cent. smaller.

Find the probable total number of conductors $= \frac{q \times \pi D_a}{I_c}$ and assume that the winding to be used is single circuit, and Y connected if three-phase, so that I_c = the full-load line current.

Find the number of slots; there should be at least two slots per pole, if possible, in order to get the advantages of the distributed winding, see Art. 141, page 180, but the slot pitch should not exceed 2.75 in.; Art. 193, page 256.

Find the conductors per slot $= \frac{\text{probable total conductors}}{\text{number of slots}}$; take the nearest number that will give a suitable winding.

Find the corresponding total number of conductors.

Find ϕ_a from the formula $E = 2.22\, kZ\phi_a\, f\, 10^{-8}$ formula 25, page 190.

Find the actual frame length as follows:

slot pitch $= \frac{\pi \times D_a}{\text{total slots}}$;

divide this into $s+t$ by the use of Fig. 182, page 257;

the minimum tooth area required $= \frac{\phi_a}{\text{max. tooth density}}$

where the max. tooth density = 90,000 lines per square inch for 60 cycles
110,000 lines per square inch for 25 cycles

for open slot machines, and is 15 per cent. larger for machines with partially closed slots;

the tooth area per pole = slots per pole $\times \psi \times t \times L_n$, from which L_n can be found

$$L_g = \frac{L_n}{0.9}$$

$L_c = L_g +$ (the center vent ducts), where these ducts are 0.5 in. wide and are spaced 3 in. apart.

Find δ, the air gap clearance, as follows:

$$B_g = \frac{\phi_a}{\tau \times \psi \times L_c}.$$

$$\delta C = \frac{AT_g \times 3.2}{B_g}$$

where AT_g is taken as 1.5 (armature ampere-turns per pole) for a first approximation.

The field is now designed roughly as explained in Art. 182, page 238, and the machine drawn out to scale, after which the saturation curves are calculated, drawn in, and the regulation determined.

If the regulation is better or worse than that desired from the machine then the design must be changed as explained in Art. 183, page 241.

Example.—Determine approximately the dimensions of an alternator of the following rating:

400 k.v.a., 2400 volts, 3-phase, 60 cycle, 96 amperes, 600 r.p.m. The work is carried out in tabular form as follows:

Apparent gap density,	B_g = 42,000, from Fig. 183.		
Amp. cond. per inch,	q = 614, from Fig. 184.		
Per cent. enclosure,	ϕ = 0.65 assumed.		
$\frac{\text{Pole pitch}}{\text{Frame length}}$ = a constant	= 0.95, from Fig. 185.		
Poles,	p = 12 for 600 r.p.m. at 60 cycles.		
Armature diameter,	D_a = 43 in., from formula 33, page 259.		
Take a larger and a smaller diameter so that			
Armature diameter,	D_a = 43 in.	36 in.	52 in.
Total conductors, probable,	Z_c = 865	725	1050
Pole-pitch,	τ = 11.3 in.	9.4 in.	13.6 in.
Slots per pole	= 6	6	6
Total slots	= 72	72	72
Cond. per slot	= 12	10	14
Connection	= Y	Y	Y
Total conductors, actual,	Z_c = 864	720	1008
Flux per pole,	$\phi_a = 3.8 \times 10^6$	4.56×10^6	3.25×10^6
Slot pitch,	λ = 1.88 in.	1.57 in.	2.27 in.
Slot width,	s = 0.75 in.	0.67 in.	0.85 in., Fig. 182
Minimum tooth width,	t = 1.13 in.	0.9 in.	1.42 in.
Tooth area per pole required	= 42.3 sq. in.	51 sq. in.	36 sq. in.
Net axial length of iron,	L_n = 9.6 in.	14.5 in.	6.5 in.
Gross length,	L_g = 10.6 in.	16.1 in.	7.2 in.
Center vent ducts	= 3–05 in.	5–0.5 in.	2–0.5 in.
Frame length,	L_c = 12.10 in.	18.6 in.	8.2 in.
Apparent gap density,	B_g = 42,800	40,200	44,800
Armature AT. per pole	= 3450	2880	4050
AT_g assumed	= 5200	4300	6100
δC	= 0.39	0.34	0.44
Field system:			
Maximum AT. per pole,	AT_{max} = 10,400	8,600	12,200
Leakage factor, assumed	= 1.2	1.2	1.2
Pole area required	= 48 sq. in.	58 sq. in.	41 sq. in.
Axial length of pole,	L_p = 11.6 in.	18 in.	7.7 in.
Pole waist,	W_p = 4.25 in.	3.4 in.	5.6 in.
Depth of field coil,	d_f = 0.75 in.	0.75 in.	0.85 in.
Mean turn,	MT = 37 in.	47 in.	32 in.
External periphery	= 39 in.	49 in.	34 in.
Exciter voltage	= 120	120	120
Section of wire,	M = 38,500	40,500	39,000
	= 0.04 × 0.75 in.	0.04 × 0.8 in.	0.04 × 0.8 in.

Watts per square inch	=6.0	5.5	6.5 page 239
Space factor,	sf=0.8	0.8	0.8
Radial length of field coil,	L_f=4.7 in.	4.0 in.	5.2 in.

The three machines are now drawn in to scale as shown in Fig. 187.

The 36-in. machine is expensive in core assembly while the 52-in. machine is expensive in inactive material, such as the material in the yoke.

So far as operation is concerned there is little to choose between the machines. The armature ampere-turns per pole is the same fraction of the field ampere-turns per pole in each case, the voltage drop due to armature reactance is also about the same for each design. In the 36-in. machine the air gap is small and the length L_c is large which tend to make the reactance large, but there are only 10 conductors per slot, whereas in the 52-in. machine, while the air gap is larger and the frame length shorter, the number of conductors per slot is 14; the reactance is proportional to the square of the number of conductors per slot.

The 36-in. machine is rather long for the diameter and would be difficult to ventilate properly, in fact it would probably require fans.

Assume that, after careful consideration of the weights of the three machines and of the probable cost of the labor, which latter quantity varies with the size of the factory and its organization, the 43-in. machine is chosen as the most satisfactory; it is now necessary to complete the design of this machine.

Winding

Amp. cond. per inch $= \frac{96 \times 864}{\pi \times 43} = 614$

$\frac{\text{Amp. cond. per in.}}{\text{Cir. mils per amp.}}$ =1.05 from Fig. 181

Cir. mils per amp. =600

Amp. per cond. at full load =96

Section of conductor =58,000 cir. mils

=0.046 sq. in.

0.75 slot width

0.132 width of slot insulation, see page 205.

0.04 clearance between coil and core

0.578 availaible width for copper and insulation on conductor.

Use strip copper 0.07 in. wide and arrange the conductors six wide in the slot, each conductor to be taped with half-lapped cotton tape 0.006 in. thick.

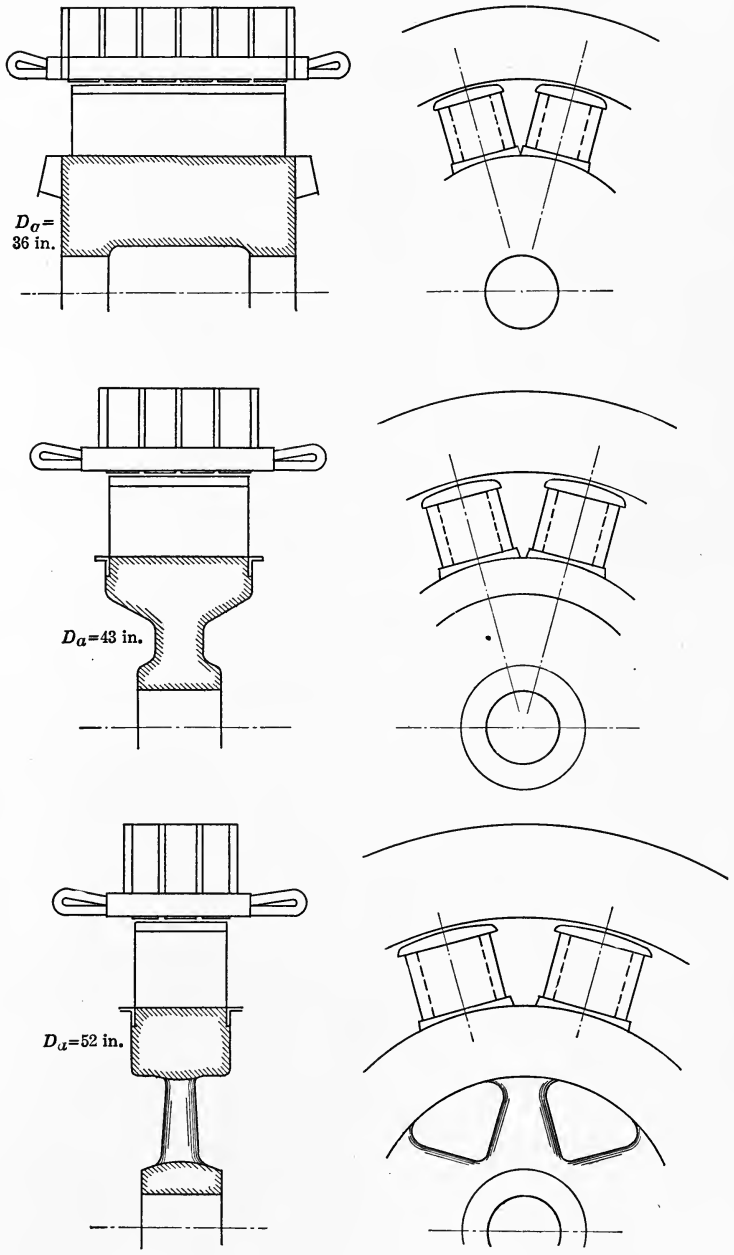


FIG. 187.—Comparative designs for a 400 k.v.a., 60-cycle, 600 r.p.m. alternator.

$$\text{Depth of conductor} = \frac{\text{section of cond.}}{\text{width of cond.}} = \frac{0.046}{0.07}$$
$$= 0.65 \text{ in.}$$

In order that there may be no trouble due to eddy current loss caused by the slot leakage flux, the conductor is divided into two strips each 0.325 in. deep.

Slot depth is found as follows:

0.325	depth of each conductor
0.024	insulation thickness on each conductor
0.70	depth of two insulated conductors
0.162	depth of slot insulation on each coil
0.862	depth of each insulated coil
2	number of coils in depth of the slot
1.72	depth of coil space
0.20	thickness of stick in top of slot
2.0	depth of slot.

Flux density in the core = 45,000 assumed

$$\text{Core area} = \frac{\phi_a}{2 \times 45{,}000} = 42 \text{ sq. in.}$$

$$\text{Core depth} = \frac{\text{core area}}{\text{net iron}} = 4.5 \text{ in.}$$

External dia. of armature = 56 in.

The above data should now be filled in on a design sheet similar to that shown on page 125.

197. Field Design.—This is carried out as shown on pages 242 to 246, where the field system for the machine in question is designed completely, the saturation curves determined and drawn in and the regulation found.

198. Variation of the Length of a Machine for a Given Diameter.—When a machine for a new rating is being designed it is often possible to save considerable expense by using the same punchings as on a machine which has already been built, and adjusting the length of the machine to suit the output and the fixed diameter.

Suppose that the 400 k.v.a. machine which is designed in Art. 196 be shortened or lengthened 50 per cent., it is required to find the output that may be expected and also the probable characteristics of the machine. The essential parts of the three designs are tabulated below.

Armature diameter.....	43 in.	43 in.	43 in.
Frame length.....	12.25 in.	8 in.	18.5 in.
End ducts.....	2–0.75 in.	2–0.75 in.	2–0.75 in.
Center ducts.....	3–0.5 in.	2–0.5 in.	5–0.5 in.
Gross iron.....	10.75 in.	7.0 in.	16 in.

Net iron	9.6 in	6.3 in.	14.4 in.
Slots	72	72	72
Cond. per slot	12	18	8
Size of cond	2 (0.70×0.325 in.)	0.09×0.325 in.	2 (0.11×0.3 in.)
Connection	Y	Y	Y
Amp. cond. per inch	614	614	614
Cir. mils per ampere	600	580	580
Arm. AT. per pole	3450	3450	3450
Amperes per cond	96	64	144
Terminal voltage per ph.	2400	2400	2400
Output in k.v.a	400	267	600

The above machines are discussed under the following heads:

Conductors per Slot.—Since the intention is to use the same armature punching, the number of slots is fixed, and for the same flux density in the different machines the flux per pole must be directly proportional to the net iron. Now the voltage per conductor is proportional to the flux per pole, so that, for the same terminal voltage, the number of conductors in series per phase must be inversely as the flux per pole and therefore inversely as the net iron in the frame length.

Size of Conductor.—For the same total copper section, this is inversely proportional to the number of conductors per slot and therefore directly proportional to the net iron in the frame length.

Current Rating.—For the same current density in the conductors the current in each conductor must be proportional to the conductor section and therefore to the net iron in the frame length.

Output.—For the same voltage in each case the output in k.v.a. is proportional to the current and therefore to the net iron in the frame length.

Air Gap Clearance.—The current per conductor is inversely as the number of conductors per slot, so that the ampere conductors per slot is the same in each case, and the armature ampere-turns per pole, which $= \frac{\text{amp. cond. per slot}}{2} \times$ slots per pole, is independent of the frame length; the maximum excitation and the air gap clearance are, therefore, also independent of the length of the frame.

Regulation.—The demagnetizing ampere-turns per pole

$$= 0.35 \times \text{cond. per pole} \times I_c$$

and since I_c is inversely proportional to the number of conductors per slot the demagnetizing ampere-turns per pole is independent of the frame length.

The armature reactance drop

$$= 2\pi f p b^2 c^2 \left(\frac{\phi_e L_e}{2} + (\phi_s + \phi_t)\ L_c \right) 10^{-8} \times I_c$$

where $\frac{\phi_e L_e}{2}$ is independent of frame length since it depends on the pole-pitch, which is constant,

ϕ_s is the same in each case since the slots are unchanged,

ϕ_t is the same in each case since the teeth and air gap are unchanged.

The armature reactance drop is therefore proportional to

(cond. per slot)^2 $\times$ current $\times$ $(A + BL_c)$

or to (cond. per slot) $(A + BL_c)$

or to $\frac{(A+BL_c)}{L_c}$, where A and B are constants,

so that the longer the machine the lower is its reactance drop and the better the regulation, other things being unchanged.

The pole leakage flux consists of flank leakage which is constant and pole-face leakage which is proportional to the frame length; ϕ_a, the flux per pole, is directly proportional to the frame length; the leakage factor $= \frac{\phi_a + \phi_e}{\phi_a} = \frac{C + DL_c}{FL_c}$ where C, D and F are constants and so is smallest for the longest machine that is built on a given diameter.

A machine cannot be lengthened indefinitely, however, because a point is finally reached at which it becomes impossible to cool the center of the core without a considerable modification in the type of construction and the addition of fans, and still further, as the length increases the pole section departs more and more from the square section.

199. Windings for Different Voltages.—The armature of a 400-k.v.a., 2400-volt, 96-ampere, 3-phase, 60-cycle, 600-r.p.m. alternator is built as follows:

Internal diameter,	43 in.
Frame length,	12.25 in.
Slots, number,	72
Slots, size,	0.75 in. $\times$ 2.0 in.
Conductors per slot, number,	12
Conductors per slot, size,	2(0.07 in. $\times$ 0.325 in.)
Connection,	Y

It is required to design armature windings for the following voltages:

600 volts, 3-phase, 60 cycles,
2400 volts, 2-phase, 60 cycles.

To find the Conductors per Slot.

$$E = 2.22\, kZ\phi_a f\, 10^{-8}$$

$$= \text{a constant} \times k \times \text{cond. per slot} \times \frac{\text{slots}}{\text{phases}}$$

$$= \text{a constant} \times k \times \frac{\text{cond. per slot}}{\text{phases}} \text{ for a given frame and frequency.}$$

For the machine in question $k = 0.966$ for a 3-phase winding
$= 0.911$ for a 2-phase winding

and the constant $= \dfrac{\text{volts per phase} \times \text{phases}}{k \times \text{cond. per slot}}$

$$= \frac{\frac{2400}{1.73} \times 3}{0.966 \times 12}$$

$$= 360$$

For the 600-volt, 3-phase winding

$$\text{cond. per slot} = \frac{\text{volts per phase} \times \text{phases}}{k \times \text{a constant}}$$

$$= \frac{600 \times 3}{0.966 \times 360}$$

$= 5.2$, or say 5, if 600 is the voltage per phase, that is if the winding is Δ connected

and

$$= \frac{\frac{600}{1.73} \times 3}{0.966 \times 360}$$

$= 3.0$ if $\dfrac{600}{1.73}$ is the voltage per phase, that is if the winding is Y connected.

The Y connected winding is the better of the two because with such a connection any third harmonic in the e.m.f. wave is eliminated, whereas in the case of a Δ connected winding the third harmonics cause a circulating current to flow in the closed circuit of the delta.

Since the winding is double layer, the number of conductors per slot must be a multiple of two, so that the winding actually used will have six conductors per slot and will be connected two circuit.

For the 2400-volt, 2-phase winding

$$\text{cond. per slot} = \frac{\text{volts per phase} \times \text{phases}}{k \times \text{a constant}}$$
$$= \frac{2400 \times 2}{0.911 \times 360}$$
$$= 14.7$$

In order to use 15 conductors per slot it will be necessary, if the winding is double layer, to use 30 conductors per slot and connect the winding two circuit, and the size of each conductor will be small; it will be preferable to use 14 conductors per slot, for which number the flux density in the machine is 5 per cent. higher than for the original 2400-volt, 3-phase winding.

To find the Size of Conductor. In order that the stator loss and stator heating be the same in each case it is necessary to keep the ratio $\frac{\text{amp. cond. per inch}}{\text{cir mils per amp.}}$ constant; the work is carried out in tabular form as follows:

k.v.a.	Terminal voltage	Phases	Amperes per line	Connection	Amperes per Cond.	Cond. per slot	Size of cond.	Amp. cond. per inch	Cir. mils. per amp.
400	2400	3	96	Y	96	12	2(0.07×0.325 in.)	614	600
400	600	3	384	YY	192	6	4(0.07×0.325 in.)	614	600
400	2400	2	83	1 circuit	83	14	2(0.055×0.325 in.)	620	550

It may be seen from the value of circular mils per ampere that the two-phase machine will get about 10 per cent. hotter than the three-phase machines because it is not possible to get sufficient copper into the slot for this rating. This trouble is inherent to the two-phase machine because, for a given frame, rating and terminal voltage

$$\frac{\text{cond. per slot—2 phase}}{\text{cond. per slot—3 ph. Y.}} = \frac{E_t \times 2 \times 1.73 \times 0.966}{0.911 \times E_t \times 3} = 1.22$$

$$\text{and } \frac{\text{current per cond.—2 phase}}{\text{current per cond.—3 ph.Y.}} = \frac{\text{k.w.}}{2E_t} \times \frac{1.73E_t}{\text{k.w.}} = \frac{1}{1.16}$$

that is to say, the number of conductors per slot for the two-phase rating is 22 per cent. greater than that for the three-phase rating, while the current per conductor is reduced only 16 per cent.

200. Example of a Machine with Field Coils of D. C. C. Wire.—The following is a preliminary design for a 75-k.v.a., 2400-volt, 3-phase, 18-ampere, 60-cycle, 1200-r.p.m. alternator.

Apparent gap density,	$B_g = 42{,}000$, from Fig. 183
Amp. cond. per inch,	$q = 450$; 20 per cent. lower than the value from Fig. 184; see Art. 195, page 258.
Per cent. enclosure,	$\psi = 0.65$, assumed
$\frac{\text{Pole pitch}}{\text{Frame length}}$ = a constant	$= 1.6$, from Fig. 185
Poles,	$p = 6$ for 1200 r.p.m. at 60 cycles
Armature diameter,	$D_a = 21$ in.
Total conductors, probable,	$Z_c = 1650$
Pole pitch,	$\tau = 11$ in.
Slots per pole	$= 6$
Total slots	$= 36$
Cond. per slot	$= 46$
Connection	Y
Total conductors, actual	$= 1656$
Flux per pole,	$\phi_a = 2.0 \times 10^6$
Slot pitch,	$\lambda = 1.83$
Slot width,	$s = 0.75$, from Fig. 182
Minimum tooth width,	$t = 1.08$
Tooth area per pole required	$= 22.2$ sq. in.
Net axial length of iron,	$L_n = 5.3$ in.
Gross length,	$L_g = 5.9$ in.
Center vent ducts	$= 1$–0.5 in.
Frame length,	$L_c = 6.4$ in.
Apparent gap density,	$B_g = 43{,}600$
Armature AT. per pole	$= 2480$
AT_g assumed	$= 3700$
δC	$= 0.27$ in.

Field system.

Maximum AT per pole,	$AT_{max} = 7500$
Leakage factor assumed	$= 1.2$
Pole area	$= 25.5$ sq. in.
Axial length of pole,	$L_p = 6.0$ in.
Pole waist,	$W_p = 4.5$ in.
Depth of field coil,	$d_f = 1.0$ in., assumed
Mean turn	$= 25$ in.
External periphery	$= 29$ in.
Exciter voltage	$= 120$
Section of field wire	$= 9500$ cir. mils. Use No. 11 square B. & S. gauge which has a section of 10,500 cir. mils.
Watts per square inch	$= 4.7$, page 239.
Space factor	$= 0.86$
Radial length of field coils,	$L_f = 3.25$ in.

The field coil as designed with a uniform thickness of 1 in. is shown dotted in Fig. 188, the actual shape of coil used is shown by heavy lines.

201. Design of a 25-cycle Alternator.—Two designs are given below for an alternator of the following rating; 400 k. v. a., 2400 volts, 3 phase, 96 amperes, 25 cycle, 500 r.p.m.

In the first design the value of ampere conductors per inch is taken from Fig. 184, in the second design a value 20 per cent. lower is used and the per cent. enclosure is also reduced.

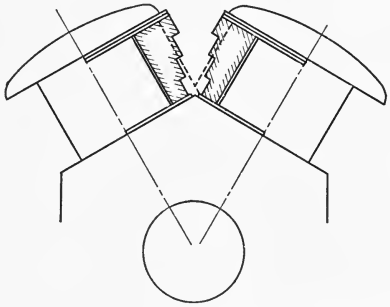


FIG. 188.—Field system of a 75 k.v.a., 60-cycle, 1200 r.p.m. alternator.

Apparent gap density,	$B_g = 52{,}000$	52,000
Amp. cond. per inch,	$q = 614$	510
Per cent. enclosure,	$\psi = 0.65$	0.6
$\frac{\text{Pole pitch}}{\text{Frame length}}$ = a constant	$= 1.6$	1.6
Poles,	$p = 6$	6
Armature diameter,	$D_a = 41$ in.	44 in.
Probable total conductors,	$Z_c = 820$	730
Pole-pitch,	$\tau = 21.5$ in.	23 in.
Slots per pole	$= 9$	9
Total slots	$= 54$	54
Cond. per slot	$= 15$	13
Connection	$= Y$	Y
Total conductors,	$Z_c = 810$	702
Flux per pole,	$\phi_a = 9.7 \times 10^6$	11.2×10^6
Slot pitch,	$\lambda = 2.39$ in.	2.56 in.
Slot width,	$s = 0.9$ in.	0.9 in.
Minimum tooth width,	$t = 1.49$ in.	1.66 in.
Tooth area required per pole	$= 88$ sq. in.	102 sq. in.
Net axial length of iron,	$L_n = 10$ in.	11.4 in.
Gross iron,	$L_g = 11$ in.	12.7 in.
Center vent ducts	$= 3$–0.5 in.	3–0.5 in.
Frame length,	$L_c = 12.5$ in.	14.2 in.
Apparent gap density,	$B_g = 55{,}000$	57,000
Armature AT per pole	$= 6500$	5600
AT_g assumed	$= 9750$	8400
C	$= 0.57$	0.47

Field system.

Maximum $AT.$ per pole	$=19{,}500$	16,800
Leakage factor assumed	$=1.2$	1.2
Pole area	$=120$ sq. in.	142 sq. in.
Axial length of pole,	$L_p=12$ in.	13.7 in.
Pole waist,	$W_p=10.5$ in.	11 in.
Depth of field coil,	$d_f=1.5$ in.	1.5 in.
Mean turn	$=50$ in.	55 in.
External periphery	$=55$ in.	60 in.
Section of wire	$=49{,}000.$	46,000.
	$=0.026$ in. $\times$ 1.5 in.	0.024 in. $\times$ 1.5 in.
This wire is difficult to bend, therefore use a double layer winding and conductors of section	0.052 in. $\times$ 0.75 in.	0.048 in. $\times$ 0.75 in.
Watts per square inch	$=4.0$.	4.2, page 239
Space factor	$=0.86$.	0.86.
Radial depth of field coil	$=7.3$ in.	6.1 in.

Part of each machine is drawn to scale in Fig. 189 and it may be seen that the first design is an impossible one unless modified; the field coils may be made shorter but then the radiating surface will be reduced and the temperature rise of the field coils be too great or they may be made of d.c.c. wire and tapered off as shown in Fig. 188.

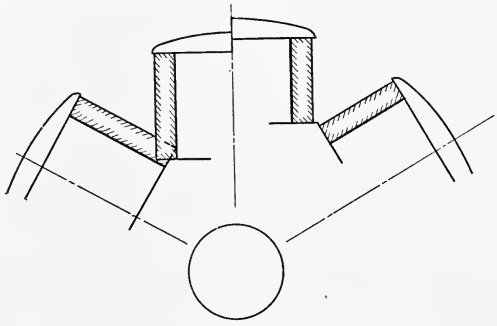


FIG. 189.—Field system of a 400 k.v.a., 25-cycle, 500 r.p.m. alternator.

It must not be imagined that the designs which have been worked out in this chapter are the only possible ones that might have been used. If the regulation expected from the machine differs from that obtained from the particular design which has been worked out then a radical change in that design will be necessary, but when one design has been worked out completely and its characteristics determined the changes necessary to meet certain requirements can readily be determined.

CHAPTER XXV

HIGH-SPEED ALTERNATORS

202. Alternators Built for an Overspeed.—A typical example of such a machine is an alternator which is direct connected to a water-wheel.

The peripheral velocity of a water-wheel is less than the velocity of the operating water; if the load on such a machine be suddenly removed, and the governor does not operate rapidly enough, then the machine will accelerate until it runs with a peripheral velocity which is approximately equal to the velocity of the water and is from 60 to 100 per cent. above normal, depending on the type of wheel used.

An alternator which is direct connected to a water-wheel must have a diameter small enough to allow the machine to run at the above overspeed without the stresses due to centrifugal force becoming dangerous. When this diameter has been reached, the output for a given speed can be increased only by increasing the length of the machine, and after a certain length has been reached it becomes impossible to keep the center of the machine cool without the use of special methods of ventilation.

Example.—Determine approximately the dimensions of an alternator of the following rating:
2750 k.v.a., 2400 volts, three phase, 660 amperes, 60 cycles, 600 r.p.m.; the machine has to run at an overspeed of 75 per cent.

If the design were carried out in the usual way then the following would be the result.

Apparent gap density,	$B_g = 43{,}000.$
Amp. cond. per inch,	$q = 720.$
Per cent. enclosure,	$\psi = 0.7.$
$\frac{\text{Pole-pitch}}{\text{Frame length}}$ = a constant	$= 0.95.$
Poles,	$p = 12.$
Armature diameter,	$D_a = 76$ in.

With this diameter the peripheral velocity at 600 r.p.m. would be 12,000 ft. per minute and the peripheral velocity at

the overspeed would be 21,000 ft. per minute. At such a speed the stresses due to centrifugal force are so large that it is difficult to build a safe rotor.

With the type of construction shown in Fig. 190, a safe and comparatively cheap machine can be built with a peripheral velocity of 17,500 ft. per minute, which corresponds to a peripheral velocity of 10,000 ft. per minute under normal running conditions, and a maximum diameter of 64 in. for the machine in question.

The design can now be continued as follows:

Armature diameter,	$D_a = 64$ in.
Total conductors (probable)	$Z_c = 220$.
Pole-pitch,	$\tau = 16.8$ in.
Slots per pole	$= 6$.
Total slots	$= 72$.
Conductors per slot	$= 3.0$; use 6 cond. per slot and connect the winding two circuit Y.
Total conductors (actual),	$Z_c = 216$.
Flux per pole,	$\phi_a = 15.2 \times 10^6$.
Slot pitch,	$\lambda = 2.8$ in.
Slot width,	$s = 0.95$ in.
Tooth width,	$t = 1.85$ in.
Tooth area required per pole	$= 169$ sq. in.
Net axial length of iron,	$L_n = 21.7$ in.
Gross length of iron,	$L_g = 24.25$ in.
Center vent ducts	$= 9$–0.5 in.
Frame length,	$L_c = 28.75$ in.

The remainder of the design is carried out in the usual way and the machine is then drawn to scale as shown in Fig. 190, which shows the kind of ventilation required to keep the center of the machine cool. Fans are placed at the ends of the rotor to create an air pressure and so force air out across the back of the punchings and also through the vent ducts. Coil retainers are put at A to prevent the rotor coils from bulging out due to centrifugal force.

203. Turbo Alternators.—Alternators which are direct connected to steam tubines run at a high speed and have therefore few poles, even for large ratings. It was pointed out in Art. 195, page 258, that when the number of poles is small it becomes difficult to find space for the necessary field copper, but that this difficulty could be overcome by increasing the diameter of the machine and lowering the value of q, the ampere conductors per inch.

In the case of the turbo alternator the diameter cannot be increased beyond the value at which the stresses in the machine due to centrifiugal force reach their safe limit, and some other method of solving the difficulty must be found.

The number of ampere-turns per pole on the field may be reduced below that desired, without changing the armature design, but this, as pointed out in Art. 176, page 229, will cause the regulation of the machine to be poor.

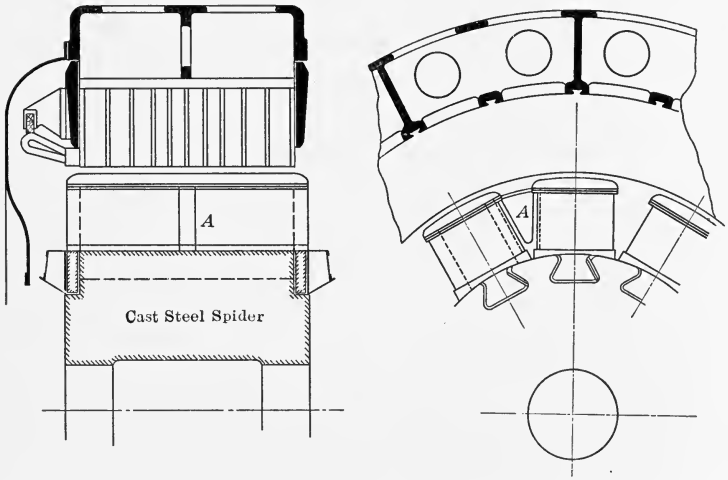


FIG. 190.—Outline of a 2750 k.v.a., 600 r.p.m., 60-cycle, water wheel driven alternator, to run at 75 per cent. overspeed.

The number of ampere-turns per pole may be reduced below the value desired, on both field and armature; then the regulation may be good, but, since the number of conductors is reduced, the flux per pole will be increased for the same rating, the machine must therefore be lengthened to keep down the flux density and so will be expensive.

The number of ampere-turns per pole on both armature and field may be left as desired and the field coils allowed to run hot; then the regulation may be good and the machine not too expensive, but it will be necessary to use materials such as mica and asbestos for the field insulation so that it will not deteriorate due to the high temperature. With such insulation it is usual to design the field coils for a temperature rise by resistance of 100° C. at the maximum field excitation; this corresponds to an increase in resistance of about 40 per cent.

204. Rotor Construction for Turbo Alternators.—Due to the high peripheral velocities required for turbo alternators the centrifugal force acting on a body at the rotor surface of such a machine is very large, for example, a weight of 1 lb. revolving at 1800 r.p.m. with a peripheral velocity of 20,000 ft. per minute is acted on by a centrifugal force of 2000 lb. It is, therefore, necessary to adopt a strong type of construction; one which can be well balanced and which will stay in good balance indefinitely; that is, the rotor windings must be rigidly held so that they cannot move. The type of construction shown in Fig. 191 fulfills the above conditions and has also the additional advantage that since there are no projections on the rotor surface the windage loss and the noise due to stirring up of the air are a minimum.

205. Stresses in Turbo Rotors.—The electrical and mechanical design of a turbo rotor must be carried out together, because the space available for field copper cannot be determined until the section of steel below the rotor coils, required for mechanical strength, has been fixed. The most important of the stresses in the type of rotor shown in Fig. 191 are determined approximately as follows:

Stress at the Bottom of a Rotor Tooth.

Assume that one tooth carries the centrifugal force due to its own weight and to that of the contents of one slot, and also that the total weight of copper, insulation and wedge in a slot is the same as that of an equal volume of steel.

Consider one inch in axial length of the rotor, then in Fig. 192, where all the dimensions are in inches,

$dw = 2\pi r \frac{\alpha}{360} \times dr \times 0.28$ lb. = weight of the cross-hatched piece

$v = \frac{2\pi r}{12} \times \frac{r.p.m.}{60}$ = the peripheral velocity of this piece;

the centrifugal force due to $dw = \frac{dw \times v^2 \times 12}{g \times r}$;

the total centrifugal force carried by a rotor tooth

$$= \int_{r_2}^{r_1} 2\pi r \frac{\alpha}{360} \times dr \times 0.28 \times \left(\frac{2\pi r}{12} \times \frac{r.p.m.}{60}\right)^2 \times \frac{12}{g \times r}$$

$$= \frac{(r_1^3 - r_2^3) r.p.m.^2 \times \alpha}{21.5 \times 10^6} \text{ lb.}$$

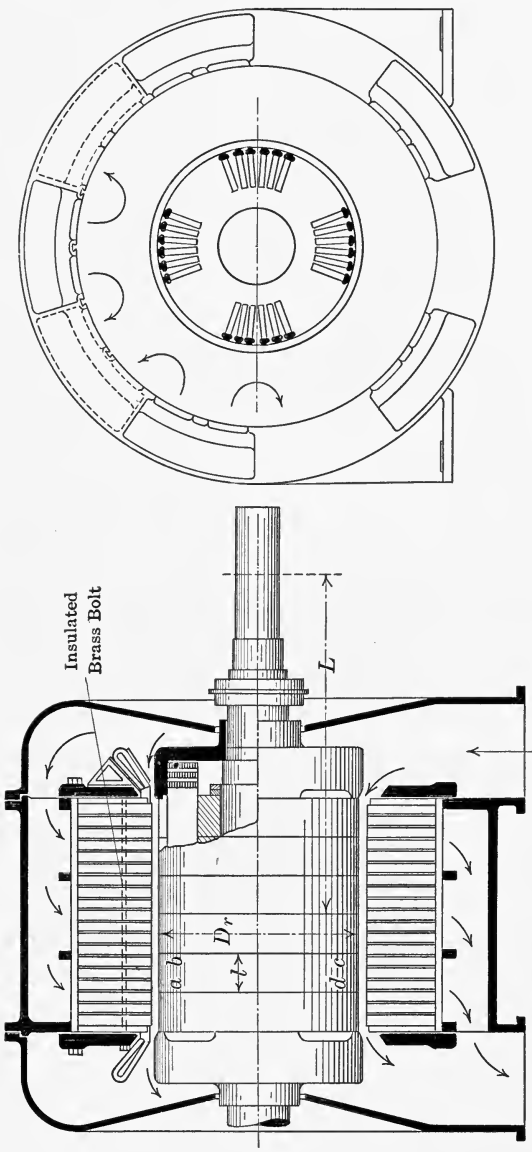


FIG. 191.—Outline of a 5000 k.w., 1800 r.p.m., 60-cycle turbo alternator.

the stress at the bottom of a rotor tooth $= \dfrac{\text{centrifugal force}}{t}$

$$= \frac{(r_1^3 - r_2^3)\, r.p.m.^2}{21.5 \times 10^6 \times t} \times \frac{(t+s) \times 360}{2\pi r_2} \qquad (34)$$

Stress in the Disc.

The centrifugal force of the section of the rotor enclosed in the angle $\beta = \dfrac{(r_1^3 - r_3^3)\, r.p.m.^2\, \beta}{21.5 \times 10^6}$ lb.

the vertical component of this force

$$= \frac{(r_1^3 - r_3^3)\, r.p.m.^2\, \beta}{21.5 \times 10^6} \sin \theta$$

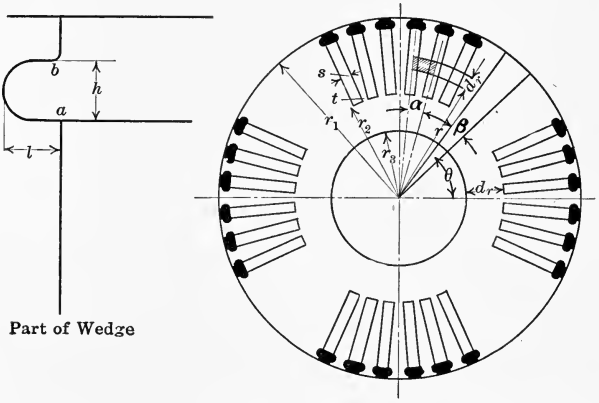


FIG. 192.

the total vertical component due to half the rotor

$$= \frac{(r_1^3 - r_3^3)\, r.p.m.^2 \times 180}{21.5 \times 10^6} \times \text{average value of } \sin \theta$$

$$= \frac{(r_1^3 - r_3^3)\, r.p.m.^2}{1.9 \times 10^5} \text{ lb.}$$

the stress in the section $d_r = \dfrac{(r_1^3 - r_3^3)\, r.p.m.^2}{3.8 \times d_r \times 10^5} \qquad (35)$

Stress in the Wedge.

The total force acting upward on the wedge is the centrifugal force of the contents of the slot

$$= \int_{r_2}^{r_1} s \times dr \times 0.3 \times \left(\frac{2\pi r}{12} \times \frac{r.p.m.}{60} \right)^2 \times \frac{12}{g \times r}$$

$$= \frac{(r_1^2 - r_2^2) \times s \times r.p.m.^2}{2.3 \times 10^5}$$

$$= P$$

the stress at section ab due to shear $= \frac{P}{2 \times h} = S$

the stress at point b due to bending $= \frac{P \times l}{4} \frac{6}{h^2} = B$

the maximum stress in the wedge $= \frac{B}{2} + \sqrt{\frac{B^2}{4} + S^2}$

206. Diameter of the Shaft.—Every shaft deflects due to the weight which it carries so that as it revolves it is bent to and fro once in a revolution. If the speed at which the shaft revolves is such that the frequency of this bending is the same as the natural frequency of vibration of the shaft laterally between its bearings then the equilibrium becomes unstable, the vibration excessive, and the shaft liable to break unless very stiff. The speed at which this takes place is called the critical speed and should not be within 20 per cent. of the actual running speed.

The critical speed for turbo rotors in r.p.m. = a const. $\times \sqrt{\frac{EI}{ML^3}}$

where E = Youngs modulus for the shaft material

I = the moment of inertia of the shaft section about a diameter $= \frac{\pi}{64} d_s^4$

M = the mass of the revolving part $= \frac{\text{weight of rotor}}{g}$

$2L$ = the distance between the bearings, see Fig. 191

[1]the constant = 75 for turbo rotors if inch and lb. units are used

therefore the critical speed $= 100 \times d_s^2 \sqrt{\frac{28 \times 10^6}{\text{rotor weight} \times L^3}}$ (36)

If, instead of vibrating as a whole, the shaft vibrates in two halves with a node in the middle, then the frequency of this harmonic is got by substituting for L in the above equation the value $L/2$ and is equal to

= the fundamental frequency $\times 2^{3/2}$

= 2.8 times the fundamental frequency.

It is found in practice that this harmonic has a value which varies from about 2.4 to 2.6 times the fundamental frequency.

The deflection of the shaft is generally limited to 5 per cent.

[1] Behrend, *Elect. Rev.*, N. Y., 1904, page 375.

of the air-gap clearance so that there shall not be any trouble due to magnetic unbalancing, therefore

$$\text{deflection} = \frac{W \times (2L)^3}{48EI} = 0.05\delta \text{ inches}$$

where W = the weight of the rotor + the unbalanced magnetic pull in lb.; see Art. 346.

The stress in the shaft is determined as follows:

M_b, the bending moment at the center of the shaft $= \frac{WL}{2}$ inch lb.

M_t, the twisting moment $= \frac{\text{watts input}}{746} \times \frac{33000}{2\pi \text{r.p.m.}} \times 12$

$$= \frac{\text{watts input}}{\text{r.p.m.}} \times 85 \text{ inch lb.}$$

M_e, the equivalent bending moment $= \frac{M_b + \sqrt{M^2{}_b + M^2{}_t}}{2}$

$$= \text{stress} \times \frac{\pi}{32} d_s{}^3 \qquad (37)$$

207. Heating of Turbo Rotors.—The assumption made in the following discussion is that all the heat generated in the part *abcd* of the rotor, Fig. 191, is dissipated from the surface $\pi D_r l$, so that each part of the rotor gets rid of its own heat and there is no conduction axially along the winding.

The difference in temperature between the rotor copper and the air which enters the machine

= the difference in temperature from copper to iron

+ the difference in temperature between the iron and the air at the rotor surface

+ the difference in temperature between the air at the rotor surface and that entering the machine, which value may be taken as 15° C.

It was shown in Art. 94, page 109, that in any slot the difference in temperature between the copper and the iron

$$= \frac{\text{amp. cond. per slot}}{\text{cir. mils per amp.}} \times \frac{\text{thickness of insulation}}{2d + s} \times \frac{1}{0.003}$$

$$= \frac{\text{amp. cond. per pole} \times 33}{\text{slots per pole} \times \text{cir mils per amp.}} \times \frac{1}{(2d)}$$

taking the thickness of slot insulation and the slot clearance to be 0.1 in. and neglecting s for the case of strip copper laid flat in the slot, because the heat will not travel down through the layers of insulation.

The difference in temperature between the rotor surface and the air surrounding it is found as follows:
The resistance of a conductor of M cir. mils section, l in. long

$$= \frac{l}{M} \text{ ohm.}$$

The loss in this conductor $= \frac{lI_c^2}{M}$ watts.

The total loss in section $abcd = \frac{lI_c^2}{M}$ cond. per slot $\times$ slots per pole $\times$ poles. $= \frac{\text{amp. cond. per pole}}{\text{cir. mils per amp.}} \times p \times l$

The watts per square inch of radiating surface

$$= \frac{\text{amp. cond. per pole}}{\text{cir. mils per amp.}} \times \frac{p \times l}{\pi D_r l}$$

$$= \frac{\text{amp. cond. per pole}}{\text{cir. mils per amp.}} \times \frac{1}{\tau}.$$

The temperature rise of this surface above that of the air which surrounds it is 10° C. per watt per square inch when the peripheral velocity is 20,000 ft. per minute. This is a lower value than that which would have been obtained by the use of the curves in Fig. 177, page 239, but these curves were based on the results obtained from tests on definite pole machines which stir up the air much better than does the cylindrical type of rotor, and further, the radiating surface was assumed to be the external surface of the coils, whereas it should include the surface of the poles and field ring to be on the same basis as the above figure for cylindrical rotors.

The temperature rise of the rotor surface above that of the surrounding air is therefore

$$= \frac{\text{amp. cond. per pole}}{\text{cir. mils per amp.}} \times \frac{10}{\text{pole-pitch}}.$$

The temperature rise of the rotor copper above that of the air entering the power-house in °C

$$= \frac{\text{amp. cond. per pole}}{\text{cir. mils per amp.}} \left(\frac{10}{\text{pole-pitch}} + \frac{33}{\text{slots per pole } (2d)} \right) + 15$$

and for a temperature rise of 100° C.
cir. mils per amp. =

$$\frac{\text{amp. cond. per pole}}{85} \left(\frac{10}{\text{pole-pitch}} + \frac{33}{\text{slots per pole } (2d)} \right) \quad (38)$$

208. Heating of Turbo Stators.—For its output, the radiating surface of a turbo alternator is generally small, and it is advisable to cool such machines by forced ventilation. At present most turbos are self-contained and have fans attached to the rotor to move the necessary volume of air; these fans are very inefficient, and, in passing through them, the air is heated and raised in temperature from 5 to 10° C. In the machine shown in Fig. 191 the air is supplied by an external fan and is filtered before it enters the generator.

If, in a machine which is cooled by forced ventilation,

t_i = the temperature of the air at the inlet in deg. C.,

t_o = the average temperature of the air at the outlet in deg. C.,

then each pound of air passing through the machine per minute takes with it $0.238(t_o - t_i)$ lb. calories per minute

$$\text{or } 7.5(t_o - t_i) \text{ watts}$$

and each cubic foot of air per minute takes

$$0.536(t_o - t_i) \text{ watts}$$

since the specific heat of air at constant pressure = 0.238 and the volume of 1 lb. of air is approximately 14 cu. ft.

If 100 cu. ft. of air per minute are supplied for each kilowatt loss in the machine then the average rise in temperature of the air will be 19° C.

When air is blown across the surface of an iron core at V ft per minute the watts dissipated per square inch for 1° C rise of the surface $= 0.0245(1 + 0.00127\ V)$. It is not generally advisable to use velocities higher than 6000 ft. per minute, because for higher velocities the air friction loss is large and the air is heated up due to this loss. For this air velocity the watts per square inch for 1° C. rise = 0.21.

The watts per square inch of vent duct surface

= watts per cubic inch $\times X$, Fig. 82, page 104,

where X = half the distance between vent ducts.

For 2-in. blocks of iron, a velocity of 6000 ft. per minute and a difference in temperature between the iron and the air of 10° C.

$$\text{watts per cubic inch} = 0.21 \times 10$$
$$= 2.1$$

and the watts per pound = 7.5 which, as may be seen from Fig. 81, page 102, corresponds to a value of flux density of

75,000 lines per square inch at 60 cycles
and 120,000 lines per square inch at 25 cycles.

As a matter of fact the iron loss in turbo generators is about 0.7 times the value found from the curves in Fig. 81 because the bulk of this loss is in the core behind the teeth and is therefore not affected by filing of the slots; the pole face loss in a turbo generator is also small because the air gap is long and so prevents tufting of the flux.

In a long machine like a turbo generator most of the heat due to stator copper loss has to be conducted through the slot insulation and dissipated from the sides of the vent ducts, and this counteracts the effect of the reduced core loss so far as core heating is concerned. For a new machine the following values of flux density should not be exceeded unless there is considerable information obtained from tests on other machines which would justify the use of higher values.

Frequency	Maximum tooth density, lines per square inch	Maximum core density, lines per square inch
25 cycles	120,000	85,000
60 cycles	100,000	65,000

for ordinary iron 0.014 in. thick.

209. Short-circuits.—When an alternator running at normal speed is short-circuited and the field excitation gradually increased, the current which flows for any field excitation can be found from a short-circuit curve such as that in Fig. 167, page 225, and at the excitation for normal voltage and no-load the armature current on short-circuit will seldom exceed three times full-load current.

The terminal voltage and the power factor are both zero on short-circuit, and the voltage drop, as shown in diagram A, Fig. 193, is made up of the drop E_oE_g due to armature reaction, and of the armature reactance drop IX.

When an alternator, running at normal speed, no-load and excited for normal voltage, is suddenly short-circuited, the armature current increases and tends to demagnetize the poles. Now the flux in the poles cannot change suddenly because the poles are surrounded by the field coils, which are short-circuited through the exciter, so that any decrease in the flux in the poles causes a current to flow round the field coils in such a direction as to

maintain the flux. The armature reaction is therefore not instantaneous in action and at the first instant after the short-circuit the current in the armature is limited only by the armature reactance. The operation of an alternator on a sudden short-circuit is shown in diagram B, Fig. 193.

The maximum value of the current depends on the value of the voltage at the instant of the short-circuit; in Fig. 194, curve 1 gives the value of the generated e.m.f. at any instant, and curve 2 the value of the reactance voltage at any instant during the first few cycles after the short-circuit; the terminal voltage is zero, the armature reaction is dampened out by the field winding, and the reactance voltage is equal and opposite to the generated e.m.f.

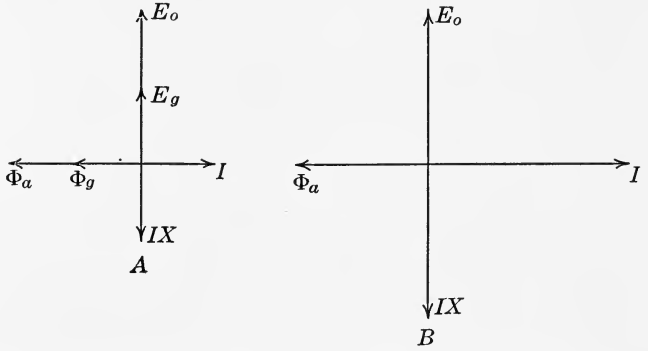


Fig. 193.—Vector diagram for an alternator on short circuit.

The reactance voltage is produced by the change in the short-circuit current and, according to Lenz's law, acts in such a direction as to oppose this change. If then, as in diagram A, Fig. 194, the short-circuit takes place at the instant a, when the generated voltage is a maximum, then between a and b the reactance voltage is negative and must therefore be opposing a growth of current while between b and c the reactance voltage is positive and must be opposing a decay of current. At the instant of short-circuit the current is zero, and curve 3 is the current curve which meets these conditions.

If, as in diagram B, Fig. 194, the short-circuit takes place at the instant f, when the generated voltage is zero, then between f and g the reactance voltage is positive and must therefore be opposing a decay of current while between g and h the reactance

voltage is negative and must be opposing a growth of current. At the instant of short-circuit the current is zero and curve 3 is that current curve which meets these conditions.

Neglecting the leakage reactance of the rotor, the current in the case represented by diagram A, Fig. 194, has an effective

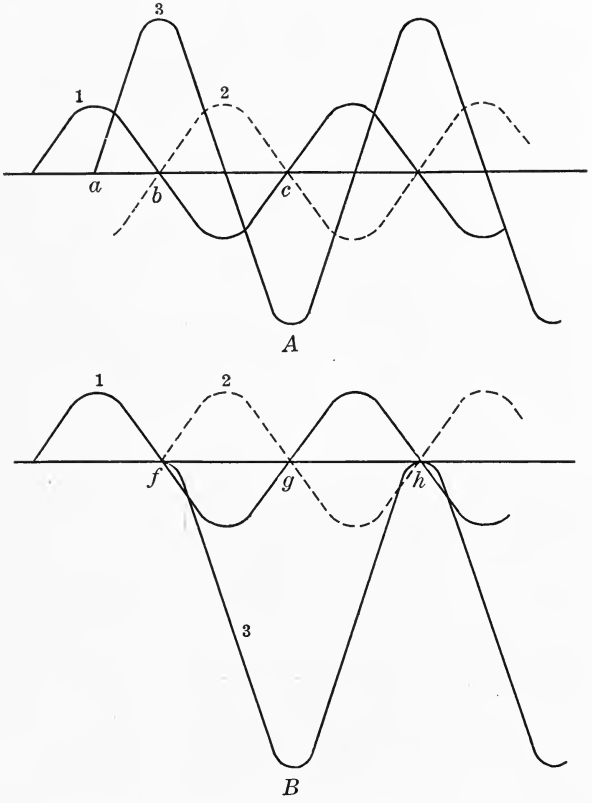


FIG. 194.—Effect on the value of the current of the point of the e.m.f. wave at which the short circuit occurs.

value $= \frac{\text{generated voltage per phase}}{\text{reactance per phase}}$, while in the case represented by diagram B the maximum current is twice as large.

210. Probable Value of the Current on an Instantaneous Short-circuit.—It was shown in the last article that the effective current under the most favorable conditions

$= \frac{\text{generated voltage per phase}}{\text{reactance per phase}}$, and therefore

$$\frac{\text{current on instantaneous short-circuit}}{\text{full-load current}} = \frac{\text{generated voltage}}{\text{reactance voltage}}.$$

The generated voltage per phase

$= 2.22 \times k \times Z\phi_a f 10^{-8}$ formula 25, page 190

$= 2.12(bcp) \times (B_g \psi \tau L_c) f 10^{-8}$ taking $k = 0.96$;

the reactance voltage per phase

$= 2\pi f b^2 c^2 p[\phi_e L_e + (\phi_s + \phi_t) L_c] 10^{-8} \times I$ for a chain winding; formula 28, page 223.

$= 2\pi f b^2 c^2 p \left[\frac{\phi_e L_e}{2} + (\phi_s + \phi_t) L_c\right] 10^{-8} \times I$ for a double layer winding; formula 29, page 223

where $\frac{\phi_e L_e}{n}$ varies with the pole-pitch as shown in Fig. 195

$\frac{\phi_s}{n} = \frac{3.2}{cn}\left[\frac{d_1}{3s} + \frac{d_2}{s} + \frac{2d_3}{s+w} + \frac{d_4}{w}\right]$ and since $c \times n \times s$, the total slot width per pole, is proportional to the pole-pitch, therefore $\frac{\phi_s}{n}$ is approximately inversely proportional to the pole-pitch.

$\phi_t = 0.42\ \phi_{tp} = 1.34 \frac{Ct}{2\delta}$ for machines with definite pole rotors, neglecting the value of ϕ_{ta}

$= 3.2 \frac{Ct}{2\delta}$ for machines with cylindrical rotors, the type generally used for turbo generators. In order that the regulation of the machine may have a reasonable value, δ, the air-gap, is fixed by the value of the armature ampere-turns per pole and is approximately proportional to the pole-pitch, therefore ϕ_t is approximately inversely proportional to the pole-pitch.

The ratio $\frac{\text{reactance voltage}}{\text{generated voltage}} = \frac{3q}{B_g \psi}\left[\frac{\phi_e L_e}{n L_c} + \frac{\phi_s + \phi_t}{n}\right]$ for a chain winding $= \frac{3q}{B_g \psi}\left[\frac{\phi_e L_e}{2n L_c} + \frac{\phi_s + \phi_t}{n}\right]$ for a double layer winding,

where $\frac{\phi_s + \phi_t}{n} = \frac{\text{a constant}}{\text{pole-pitch}}$ for a given number of phases. (39)

The values of ϕ_s and ϕ_t are plotted in Fig. 195 for average machines which have not less than two slots per phase per pole

nor a larger slot pitch than 2.75 in., and for machines with open slots and laminated rotors.

Example.—Determine approximately the per cent. reactance drop at full-load in the following machine:

Normal output in k.v.a	6250
Normal voltage at terminals	2400
Number of phases	3
Current per phase	1500
R.p.m	1800
Frequency	60 cycle
Internal diameter of armature	45 in
Pole-pitch	35.5 in.
Frame length	51 in.
Average gap density	30,000 lines per square inch
Per cent. enclosure	1.0
Ampere conductors per inch	635
Winding	Double layer, Y-connected

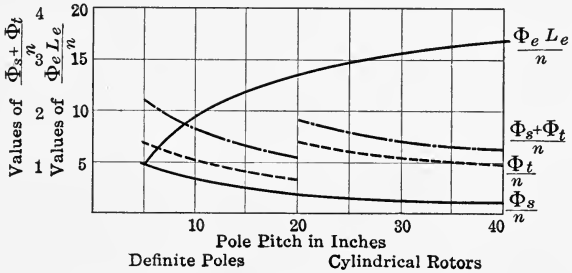


FIG. 195.—Values of the leakage fluxes.

$$\frac{\text{Reactance voltage}}{\text{Generated voltage}} = \frac{3\times 635}{30,000}\left(\frac{17}{2\times 51}+1.3\right) = \frac{1}{10.7}.$$

When a solid pole face is used, as in most turbo alternators with cylindrical rotors, the value of ϕ_t will be lower than that found from Fig. 195, because the flux will be prevented from entering the pole face by eddy currents which it produces therein; this tends to make the short-circuit current larger than the value found from formula 39.

Down to this point the effect of the leakage reactance of the rotor winding has been neglected. As shown in Fig. 159, page 213, the total flux per pole $=\phi_m=\phi_a+\phi_e$ and this remains constant during the first few cycles after an instantaneous short-circuit. In order that ϕ_m remain constant a current must flow in the rotor winding in such a direction as to oppose

the demagnetizing effect of the armature reaction. This current is large, so that the m.m.f. between the poles and, therefore, the leakage flux ϕ_e are much larger immediately after a sudden short-circuit than at no-load.

If ϕ_{eo} = the pole leakage flux at no-load

and ϕ_{es} = the pole leakage flux immediately after a short-circuit, then the generated voltage in the machine, which is produced by the flux ϕ_a, is less immediately after a short-circuit than it was immediately before in the ratio $\frac{\phi_m - \phi_{es}}{\phi_m - \phi_{eo}}$ and this tends to make the short-circuit current smaller than the value found from formula 39.

To reduce the ratio $\frac{\text{current on instantaneous short circuit}}{\text{full-load current}}$

it is necessary as shown in formula 39:

To increase the value of q, the ampere conductors per inch; this will at the same time cheapen the machine but will cause its regulation to be poorer.

Use a chain rather than a double-layer winding so as to increase the end connection reactance.

Use deep, narrow and closed slots so as to increase the slot reactance.

Use as small a pole-pitch as possible until the point is reached at which the term $\frac{\phi_e L_e}{nL_c}$ becomes of more importance than the term $\frac{\phi_s + \phi_t}{n}$.

Use a laminated rotor if possible.

Use a cylindrical rotor rather than one of the definite pole type, because of the larger value of ϕ_t, as may be seen from Fig. 195.

All the above changes are made to increase the reactance of the machine and, therefore, tend to make the regulation poor.

The value of the instantaneous short-circuit current can be reduced by making the pole leakage factor large and this will not affect the regulation seriously if the pole pieces are unsaturated at normal voltage.

211. Supports for Stator End Connections.—The large current due to an instantaneous short-circuit takes several seconds to get down to the value which it would have on a gradual short-circuit, and during this time the force of attraction between adjacent conductors of the same phase is very large and tends

to bunch the end connections of each phase together. The force between the groups of end connections of adjacent phases is also very large and, when the currents in these phases are in opposite directions, this force tends to separate the phase groups of end connections. To prevent any movement of the coils due to this effect it is necessary to brace them thoroughly in some such way as that shown in Fig. 191.

212. The Gap Density.—For the type of rotor shown in Fig. 191 the distribution of flux in the air gap is given by the heavy line curve in Fig. 196; if two more slots per pole are added, as

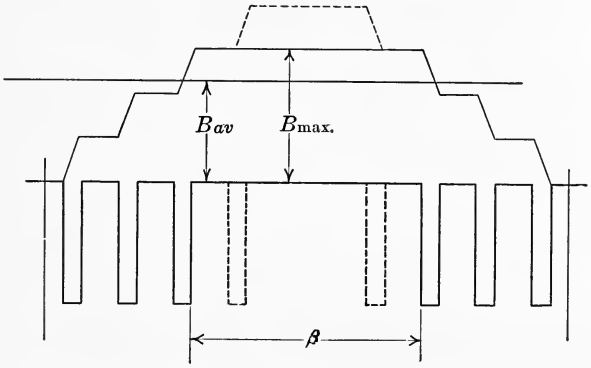


FIG. 196.—Flux distribution in the air gap of a turbo alternator.

shown dotted in Fig. 196, the flux will be increased by the amount enclosed by the dotted curve and, for a considerable increase in rotor copper and rotor loss, only a small increase in the flux per pole will be obtained. The angle β is therefore seldom made less than 30 electrical degrees and for this value the maximum gap density = the average gap density $\times 1.5$ approximately.

The maximum gap density depends on the permissible value of the maximum tooth density since

$$B_{g\ max} = B_{t\ max} \times \frac{L_n}{L_c} \times \frac{t}{\lambda};$$

the blocks of iron in the core are about 2 in. thick, the vent ducts 0.625 in. wide and the stacking factor = 0.9, therefore

$$\frac{L_n}{L_e} = 0.68;$$

an average value for $\frac{\lambda}{t}=1.5$ for turbos, since the slot pitch is generally large

and $B_{t\,max}=100{,}000$ lines per square inch for 60 cycles
$=120{,}000$ lines per square inch for 25 cycles

therefore

$B_{g\,max}=45{,}000$ lines per square inch for 60 cycles
$=54{,}000$ lines per square inch for 25 cycles

213. The Demagnetizing Ampere-turns per Pole at Zero Power Factor.—The distribution of the m.m.f. of armature reaction at two different instants is shown in Fig. 160, page 214, for a machine with six slots per pole and b conductors per slot. In the case of a definite pole machine the portion of this diagram which is cross hatched is effective in demagnetizing the poles, whereas in the type of machine with a cylindrical rotor the whole armature m.m.f. is effective and for this case

$$\begin{aligned} AT_{av}\times 6\lambda &= \text{area of diagram } A \\ &= 2bI_m\lambda + 1.5bI_m\times 2\lambda + bI_m\times 2\lambda \\ &= 7bI_m\lambda \end{aligned}$$

$$\begin{aligned} &= \text{area of diagram } B \\ &= 1.73bI_m\times 3\lambda + 0.866bI_m\times 2\lambda \\ &= 6.92bI_m\lambda \end{aligned}$$

$=6.96bI_m\lambda$ the average value from diagrams A and B

therefore $AT_{av}=1.16bI_m$

$=1.64bI_c$ where I_c is the effective current per conductor

$$=1.64bI_c\left(\frac{\text{slots per pole}}{6}\right)$$

$=0.275\times\text{cond. per pole}\times I_c$

The maximum m.m.f. of the field windings is that at the center of the poles and is equal to the ampere-turns per pole.

The average m.m.f. when β, Fig. 196, is 30 electrical degrees

$$=\frac{\text{maximum m.m.f.}}{1.5}\text{ approximately}$$

$$=\frac{\text{the ampere-turns per pole}}{1.5}$$

therefore the ampere turns per pole required on the field to overcome the demagnetizing effect of the armature

$$= 1.5 \times 0.275 \times \text{cond. per pole} \times I_c$$
$$= 0.41 \times \text{cond. per pole} \times I_c \qquad (40)$$

214. Relation between the Ampere-turns per pole on Field and Armature.—For definite pole machines the value of the ampere-turns for the gap on no-load and normal voltage

$= AT_g$

= 1.5 times the armature ampere-turns per pole for a first approximation, see Art. 183, page 240.

It may be seen by a comparison of formulæ 27 and 40, pages 215 and 290, that, for a machine with a cylindrical rotor, a larger number of field ampere-turns are required to overcome the demagnetizing effect of the armature than for a definite pole machine; and further, it will be seen from the example in Art. 215 that the air gap of a turbo alternator is very long and the number of ampere-turns used up in sending the flux through the poles is, therefore, very small compared with that required for the gap, so that the saturation curve does not bend over and the advantage of a saturated pole, pointed out in Art. 175, page 228, cannot readily be obtained. For these two reasons it is necessary, in order to get reasonably good regulation from a turbo alternator, to make

$AT_g = 1.75$ (armature ampere-turns per pole at full-load) for a first approximation,

a value which is about 25 per cent. larger than for a definite pole machine.

215. Procedure in Turbo Design.—The method whereby the preliminary design of a turbo alternator is worked out can best be seen from the following example.

Work out the preliminary design for a 5000 kw. 2400 volt, three-phase, 1500 ampere, 60 cycle, 1800 r.p.m. machine, to operate at 80 per cent. power factor.

Maximum peripheral velocity, 20,000 ft. per minute assumed

Rotor diameter,	$D_r = 42.5$ in.
Rotor pole pitch,	$\tau = 33.5$ in.
Amp. cond. per inch,	$q = 600$, see Art. 195, page 258
Armature amp.-turn per pole, AT_g	$= \frac{q\tau}{2} = 10{,}000$ $= 1.75 \times 10{,}000$ for a first approximation $= 17{,}500$

Probable value of $C\delta$ $= \dfrac{3.2 \times AT_g}{B_{g\,max}}$
$= 1.25.$ in.

C may be taken $= 1.0$ for such a large air gap

Internal diameter of stator, $D_a = 45$ in.

Total conductors (probable), $Z_c = \dfrac{q \times \pi D_a}{I_c} = 57$

Pole pitch, $\tau = 35.5.$ in.

Slots per pole $= 15$

Total slots $= 60$

Conductors per slot $= 1$

Total conductors (actual), $Z_c = 60$

Flux per pole, $\phi_a = 54.5 \times 10^6$

Slot pitch, $\lambda = 2.36$ in.

Slot width, $s = 0.8$ in.

Tooth width, $t = 1.56$ in.

Tooth area required per pole $= \dfrac{\phi_a}{B_{t\,av}} = \dfrac{\phi_a}{\frac{B_{t\,max}}{1.5}} = 820$ sq. in.

Net length of iron in the core, $L_n = 35$ in.

Gross length of iron in the core, $L_g = 39$ in.

Center vent ducts $= 19$–0.625 in.

Frame length, $Lc = 51$ in.

Average gap density, $B_{g\,av} = \dfrac{\phi_a}{\tau L_c} = 30{,}000$ lines per square inch.

The machine is now drawn approximately to scale and the distance between bearings determined; this distance $= 140$ in.

Rotor Design.

Probable rotor weight $= \left(\frac{\pi}{4} \times 42.5^2 \times 51 \times 0.28\right) 1.5 = 30{,}000$ lb.; the multiplier 1.5 is used to take account of end connections, coil retainers, etc.

Shaft diameter $= 16$ in.
the stress in the shaft $= 2600$ lb. per sq. in. and
the shaft deflection $= 0\ 018$ in.
neglecting the unbalanced magnetic pull.

Critical speeds $= 100 \times 16^2 \sqrt{\dfrac{28 \times 10^6}{30{,}000 \times 70^3}}$ formula 36, page 278

$= 1340$ r.p.m.

and $= 2.4 \times 1340 = 3240$ r.p.m.

Depth below slots $= d_r$ $= \dfrac{(21.25^3 - 8^3)1800^2}{3.8 \times 14{,}000 \times 10^5}$ formula 35, page 277

$= 5.5$ in.

Rotor slot depth $= \dfrac{(42.5 - 16 - 11)}{2}$

$=7.75$ in.

Probable depth of wedge $=1.25$; should be checked after the slot width has been determined

Available slot depth $=6.5$ in.

Armature amp.-turns per pole $=11{,}250$

Max. field amp.-turns per pole $=3.25\times 11{,}250$ assumed

$=37{,}000$

Probable mean turn $=156$ in. from scale drawing

Section of rotor conductor $=\dfrac{37{,}000\times 156}{30}$ formula 7, page 65 for 120 volt excitation

$=193{,}000$; this value should be increased 10 per cent. because of the high temperature at which the field will be run, therefore,

$=210{,}000$ cir mils.

Cir mils per ampere $=\dfrac{2\times 37{,}000}{85}\left(\dfrac{10}{33.5}+\dfrac{33}{6\times 13}\right)$ formula 38, page 280

$=640$ with 6 slots per pole. A smaller conductor could be used if the rotor were made with 8 slots per pole and the two rotors should be worked out together to determine which will be the cheaper.

Maximum exciting current $=\dfrac{\text{total cir mil section of conductor}}{\text{cir mil per ampere}}$

$=\dfrac{210{,}000}{640}$

$=328$ amp.

Ampere conductors per slot $=\dfrac{37{,}000\times 2}{6}$

$=12{,}300$

Conductors per slot $=\dfrac{12{,}300}{328 \text{ amp.}}$

$=38$

The winding to be of strip copper laid flat in the slot; the available depth for copper and insulation $=6.5$, of which 0.2 is used for slot insulation; the available depth for 38 conductors and the insulation between them $=6.3$ in.

Thickness of conductor $=0.15$, and of insulation 0.015

Section of conductor $=210{,}000$ cir mils.

$=0.165$ sq. in.

Width of conductor $=1.1$ in.

Width of slot $=1.3$ in., allowing 0.1 in. per side for insulation and clearance

Width of tooth at the root $=t$ where

Tooth stress 14,000 $=\dfrac{(21.25^3-13.5^3)1800^2\times 360}{21.5\times 10^6\times 2\pi\times 13.5}\times\dfrac{t+s}{t}$ formula 34, page 277

From which t $=0.65$ in.

Stator Core Design.

Conductors per slot	=1
Ampere cond. per slot	=1500
Ampere cond. per inch	=630
Cir mils per ampere	=800; assumed for a first approximation
Cir mils per conductor	=800×1500
	=1,200,000
Section of conductor	=0.95 sq. in.
	=0.55 in.×1.75 in.
	=20 strips each=0.11 in.×0.45 in.; 5 wide and 4 deep in the slot.

Slot depth is found as follows

0.45	depth of each strip
0.024	insulation on each strip
1.9	depth of conductors and conductor insulation
0.16	depth of slot insulation
0.25	depth of wedge
2.3	depth of slot.

Temperature difference copper to iron

$$=\frac{\text{amp. cond. per slot}}{\text{cir. mils per amp.}}\ \frac{\text{insul. thickness}}{2d+s}\ \frac{1}{0.003}$$ formula 18, page 111

$$=\frac{1500}{800}\times\frac{\frac{1}{2}(0.8-0.55)}{5.4}\times 333$$

$$=15^\circ \text{ C.}$$

This figure is probably pessimistic because it neglects the heat that is conducted axially along the copper and dissipated at the end connections, it also neglects the effect of the vent ducts; it does indicate, however, that the section of copper chosen is not too large because, according to the assumptions made in this chapter,

the temperature rise of the air = 19° C. for 100 cu. ft. per minute per kilowatt loss,

the temperature difference from stator iron to air in ducts = 10° C. with a core density of 65,000 lines per square inch,

the temperature difference from copper to iron = 15° C.

If lower temperatures are desired it is necessary to use a larger supply of air, lower core densities, or lower copper densities.

$$\text{The depth of iron behind the slots}=\frac{\text{flux per pole}}{2\times 65000\times\text{net iron}}$$

$$=\frac{54000000}{2\times 65000\times 35}$$

$$=12 \text{ in.}$$

Volume of Air required. It is difficult to predetermine the losses in a turbo with any degree of accuracy; the I^2R loss in

the field and armature can be determined accurately, but the iron loss, the windage loss due to air friction in the ducts, and the load loss which is large in turbo generators, cannot be determined accurately without considerable data on machines previously built and tested. For preliminary design-work Fig. 208, page 318, may be used, which shows that the efficiency of a 5000-kw. turbo at 1800 r.p.m. is approximately 95 per cent., neglecting the bearing loss, which is charged to the turbine; therefore the loss in the machine = 250 kw. and the volume of air required = 25,000 cu. ft. per minute.

The area of the vent duct section

= number of ducts × duct width × core depth
= 19 × 0.5 × 12 square inches
= 0.8 sq. ft.

the vent segments are 0.625 in. thick but the available space for air is only 0.5 in.

As shown in Fig. 191, there are ten air paths through the machine, so that the air velocity

$$= \frac{25000}{10 \times 0.8}$$
= 3200 ft. per minute;

it will be necessary to use a higher air velocity than 3200 ft. per minute in order to cool the core; this may be obtained by cutting down the number of paths through the machine without changing the total volume of air but may be better obtained by increasing the volume of air passing through the machine.

216. Limitations in Design due to Low Voltage.—The larger the flux per pole in a machine the smaller the number of conductors required for a given voltage since

$$E = \text{a constant} \times Z \times \phi_a \times f$$

and for large low-voltage machines it is sometimes difficult to find a suitable winding without considerable change in the machine; for example, in the turbo designed in the last article:

For 2400 volts, three phase, the winding has 60 conductors, 60 slots and 1 conductor per slot Y-connected.

For 600 volts, three phase, the same punching may be used with 1 conductor per slot connected YYYY.

For 500 volts, three phase, a total of 12.5 conductors are required in series and a new punching is needed with 48 slots, 1 conductor per slot connected YYYY.

217. Single-phase Turbo Generators.—In the design of this type of machine a new difficulty presents itself. It was shown in Art 177, page 231, that the armature reaction in a single-phase alternator is pulsating and causes a double frequency pulsation of flux in the poles, and, therefore, a large eddy current and hysteresis loss therein.

These pulsations are dampened out somewhat by the eddy currents, but an eddy-current path in iron is a high-resistance path and the eddy-current loss is, therefore, large. In order to dampen out the flux pulsations with a minimum loss it has been found necessary to surround the rotor with a squirrel-cage winding of copper in which eddy currents will be induced tending to wipe out the pulsating effect of armature reaction, and since the resistance of this squirrel cage can be made low its loss can be small. A suitable squirrel cage may be made by using copper for the slot wedges shown in Fig. 192, dove-tailing similar wedges into the pole face, and connecting them all together at the ends by copper rings.

The following figures[1] show how necessary these dampers are: A 1000-k.v.a., 2-pole, 25-cycle turbo alternator was tested three-phase; one phase was then opened and the machine run with two phases in series, which gave a single-phase winding with 2/3 of the total conductors, see Fig. 124, page 177. The same flux per pole and the same current per conductor were used in each case with the following result:

HEAT RUNS ON FULL-LOAD

Three phase no dampers	Single phase no dampers	Single phase with dampers	
31	122	37	Temperature rise in deg. C.

In the three-phase machine the armature reaction is constant in value and revolves at the same speed and in the same direction as the poles so that dampers are not required.

The pulsating field of armature reaction, as pointed out in Art. 177, page 231, is equivalent to two revolving fields, one of which revolves in the same direction and at the same speed as

[1] Waters, *Trans.* of Amer. Inst. of Elect. Eng., Vol. 29, page 1069.

the poles, while the other, which causes the trouble, revolves at the same speed but in the opposite direction. The e.m.f. induced in an eddy-current path depends on the rate at which the lines of force of this latter field are cut and has a large value in a high-speed turbo alternator but causes little trouble in moderate-speed machines.

CHAPTER XXVI

SPECIAL PROBLEMS ON ALTERNATORS

218. Flywheel Design for Engine-driven Alternators.—When two alternators are operating properly in parallel the currents flow as shown in diagram A, Fig. 197, and the operation is represented by diagram B. The e.m.fs. P and Q are equal and opposite with respect to the closed circuit consisting of the two armatures and the lines connecting them, and there is, therefore, no current circulating between the machines.

Should one of the machines, say Q, slow down for an instant the currents will flow as shown in diagram C, and the operation is represented by diagram D when machine Q lags behind machine P by θ electrical degrees. The two e.m.fs. are no longer opposed to one another and there is a resultant e.m.f. E_r which sends a circulating current round the closed circuit. While this circulating current has no separate existence, because it combines with the current which each machine supplies to the load to give the resultant current in the machine, yet it is convenient to consider its effect separately.

$$\text{This circulating current} = \frac{E_r}{\overline{X}_p + \overline{X}_q} \text{ and lags } E_r \text{ by } 90°,$$

where $\overline{X}_p$ = the synchronous reactance of machine P

$\overline{X}_q$ = the synchronous reactance of machine Q;

the resistances of the armatures and the resistance and reactance of the line are all small compared with the above reactances and so can be neglected. It may be seen from diagram C that this circulating current is in the same direction as the e.m.f. in P, it therefore acts as an additional load on that machine and causes it to slow down; being opposed to the e.m.f. in Q it, therefore, lightens the load on that machine and causes it to speed up; therefore, the two machines tend to come together, until they are in the position shown in diagram B, where the machines are in step and the circulating current is zero. Due, however, to

the inertia of the machines, they will swing beyond the position of no circulating current which current will then be reversed and tend to pull the machines together again. The frequency of this swinging will be that of the natural vibration of the machines; the swinging will gradually die down due to the dampening effect of eddy currents in the pole faces, field coils and dampers.

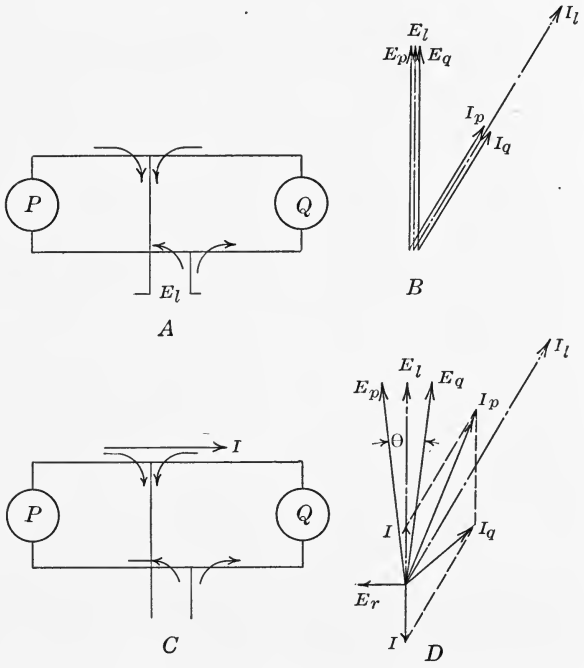


FIG. 197.—Diagrammatic representation of two alternators in parallel.

If one of the machines is direct-connected to an engine whose torque is pulsating, forced oscillations will be impressed on the machine; if their period of vibration is within 20 per cent. of the natural period of vibration, cumulative oscillation will take place and the machines be thrown out of step unless they are powerfully dampened. It is, therefore, of extreme importance to study the natural period of vibration of alternators.

219. Two Like Machines Equally Excited.—When swinging takes place between two such machines they move in opposite directions with the same frequency, so that if Fig. 198 shows the vector diagram of the two machines referred to the closed circuit, then

θ = the angle of displacement between the two machines;

$\alpha = \frac{\theta}{2}$ = the angle of displacement of one machine from the position of zero circulating current or mean position.

$E_r = 2E \sin\frac{\theta}{2}$

I, the circulating current $= \frac{E_r}{2X}$ where X is the synchronous reactance of each machine

$$= \frac{2E \sin\frac{\theta}{2}}{2X}$$

$$= I_{sc} \sin\frac{\theta}{2}$$

where I_{sc} is the short-circuit current at the excitation required at no-load for voltage E and may be found from a short-circuit curve such as that in Fig. 200. I_{sc} is generally about 2.5 times full-load current.

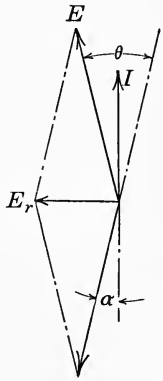


FIG. 198.—Vector diagram for two like machines in parallel.

The synchronising power, or power transferred from one machine to the other, in watts

$= nEI \cos\frac{\theta}{2}$

$= nE\left(I_{sc} \sin\frac{\theta}{2}\right) \cos\frac{\theta}{2}$

$= nEI_{sc} \frac{\sin\theta}{2}$

$= nEI_{sc} \frac{\theta}{2}$ for small oscillations, where θ is the angular displacement between the machines in electrical radians;

$= nEI_{sc}\alpha$ where α is the angular displacement of one machine from its mean position in electrical radians;

$= nEI_{sc}\ \alpha\ \frac{p}{2}$ where α is the same angular displacement in mechanical radians.

The torque corresponding to the above power transfer

$$= n\left(EI_{sc}\alpha\ \frac{p}{2}\right) \times \frac{33000}{746 \times 2 \times \pi \times \text{r.p.m.}} \text{ lb. at 1 ft. radius}$$

$$= n\ EI_{sc}\ \frac{p}{\text{r.p.m.}} \times 3.5\ \alpha$$

$$= K\alpha$$

that is to say, the torque is directly proportional to the displacement and therefore the equation for the small displacements is

$$K\alpha = \frac{Wr^2}{g}\ \frac{d^2\alpha}{dt^2}$$

from which the time of oscillation in seconds

$$= 2\pi\sqrt{\frac{Wr^2}{g \times K}}$$

$$= 2\pi\sqrt{\frac{Wr^2 \times \text{r.p.m.}}{g \times nEI_{sc} \times p \times 3.5}}$$

$$= \sqrt{\frac{Wr^2 \times \text{r.p.m.}}{\text{watts} \times k_1 \times p \times 2.9}} \qquad (41)$$

where Wr^2 = the moment of inertia of one machine in lb. ft.^{2},

r.p.m. = the speed of the machine in revolutions per minute,

watts = the normal output of the machine at unity power factor,

k_1 = the ratio $\frac{\text{short-circuit current}}{\text{full-load current}}$ at the excitation corresponding to voltage E at no-load,

p = the number of poles.

220. One Small Machine in Parallel with Several Large Units.—In this case, if the small machine is driven by an engine, it will swing about its mean position but will not be able to make the large units with which it is in parallel swing in the opposite

direction, so that if Fig. 199 shows the vector diagram of the two machines referred to the closed circuit then

α = the angle of displacement between the small machine and the others with which it is in parallel, and is also the angle of displacement of the small machine from its mean position

$$E_r = 2E \sin \frac{\alpha}{2}$$

I, the circulating current $= \frac{E_r}{X}$ where X is the synchronous reactance of the small machine; the reactance of all the other machines in parallel is very small compared with this value and may be neglected,

$$I = \frac{2E \sin \frac{\alpha}{2}}{X}$$

$$= 2I_{sc} \sin \frac{\alpha}{2}$$

the synchronising power in watts

$$= nEI \cos \frac{\alpha}{2}$$

$$= nE \times 2I_{sc} \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}$$

$$= nEI_{sc} \times 2 \times \frac{\sin \alpha}{2}$$

$$= nEI_{sc}\alpha$$

this is the same value as that obtained for two like machines so that the time of oscillation will be the same as for two like machines.

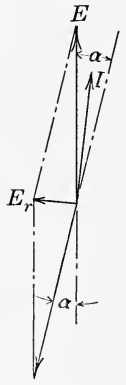


FIG. 199.—Vector diagram for a small machine in parallel with a large station.

Example.[1]—Cross compound engines running at 83 revolutions per minute, with a flywheel effect of 8.5×10^6 lb. ft.^{2}, driving three-phase alternators of 2100 k.v.a. output at 50 cycles, were found to hunt with the periodicity of the revolution. The short-circuit current of the machine for different excitations varied between 2.3 and 3.3 times full-load current.

In such a case there is an impulse impressed on the system every stroke or four impulses per revolution; if any one of these impulses differs in magnitude from the others, due to unequal steam distribution, there will also be a forced oscillation with the periodicity of the revolution.

[1] Rosenberg, *Journal of the Inst. of Elect. Eng.*, Vol. 42, page 549.

For the machine in question the time of one cycle of natural

$$\text{frequency} = \sqrt{\frac{8.5\times 10^6 \times 83}{2100\times 1000\times \begin{bmatrix}2.3\\3.3\end{bmatrix}\times 72 \text{ poles}\times 2.9}}$$

$$= 0.7 \text{ to } 0.84 \text{ seconds.}$$

The number of forced oscillations per minute due to unequal steam distribution is 83 and the corresponding period of vibration is, therefore, $= \frac{60}{83} = 0.72$ seconds, which corresponds very closely with that of the natural frequency. It was found possible to maintain parallel operation long enough to allow tests to be made by carefully equalizing the steam distribution so as to eliminate this low frequency forced oscillation.

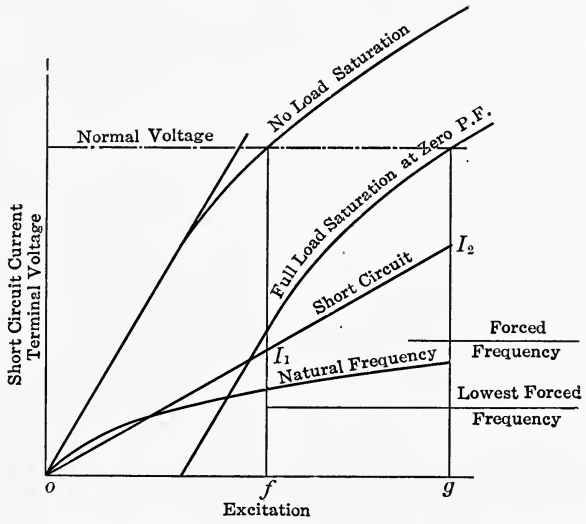


FIG. 200.—Variation of the natural frequency of oscillation with excitation.

It is advisable to design the flywheel so that the natural frequency of vibration of the machine is lower than that of the lowest forced oscillation; this will sometimes require the use of an enormous wheel; as, for example, in the case of alternators operating in parallel and driven by large slow-speed gas engines. Gas engines are generally of the four-cycle type; that is, there is an explosion once in two revolutions. If the engine is of the four-cycle, double-acting, cross-tandem type there are four explosions to the revolution and the forced frequencies in such a case are:

One impulse every two revolutions due to an unequal distribution of gas making one explosion always more powerful than the others; this is not a desirable condition of operation but one that must be provided for.

One impulse per revolution due to the want of perfect balance of the reciprocating parts.

One impulse per quarter revolution due to the four explosions in each revolution.

In order that the natural frequency of oscillation of the alternator be below the frequency of the lowest impulse a very heavy and expensive flywheel is required, so that the wheel is often made with a moment of inertia of such a value that, over the whole range of operation, the natural frequency of the alternator is more than 20 per cent. higher than that of the lowest impulse and more than 20 per cent. lower than that of the impulse of next higher frequency. For example, in Fig. 200 the excitation during operation may vary from *of* to *og* and the value of the short-circuit current from I_1 to I_2. The natural frequency of oscillation is directly proportional to the square root of the short-circuit current, and for the value of W^2r chosen is not within 20 per cent. of the frequency of either of the two lowest impulses for any excitation between *of* and *og*.

221. Use of Dampers.—For gas-engine driven alternators

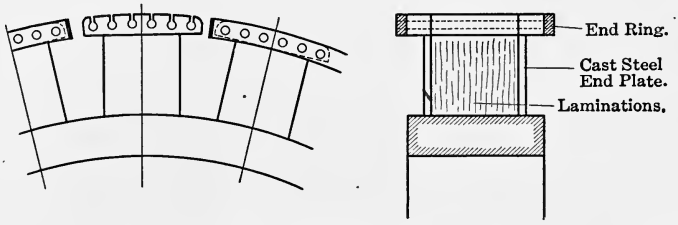


FIG. 201.—Alternator dampers.

powerful dampers are supplied because the applied torque varies so much during each revolution. The type of damper shown in Fig. 201 is that generally used, it consists of a complete squirrel cage around the machine and acts as follows:

The damper rods are embedded in the pole face and so do not cut the main field, therefore any damping effect is due to cutting of the armature field. If the armature field revolves at synchronous speed with a uniform angular volocity then, due to

the impulses of the engine, the poles oscillate about the position of uniform angular velocity and cut this field. The curve in Fig. 202 shows the distribution of the armature field; the poles move relative to it in the direction of the arrow and e.m.fs. are induced in the rotor bars in the direction shown by the dots and crosses. The frequency of these e.m.fs. is that of the oscillation of the machine and is of the order of two cycles per second, so that the reactance of the damper bars can be neglected compared with their resistance if the slots in which they lie are open at the top as shown in Fig. 201. The currents in the bars are, therefore, in phase with the e.m.fs. and so are also represented by the same crosses and dots; it may be seen that the direction

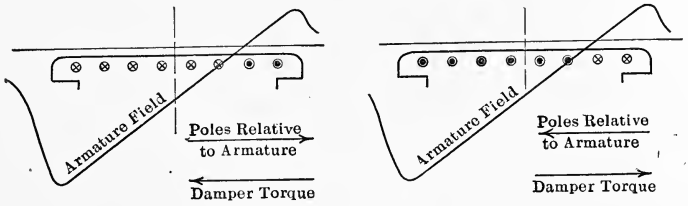


FIG. 202.—Operation of dampers.

of these currents is such that the force exerted on them by the armature field tends to prevent the relative motion of the armature and the damper rods.

The e.m.f. in a damper rod at any instant $= B_{ga} \times L_c \times V_c \times 10^{-8}$ volts.

Where B_{ga} = the gap density at that part of the field which is being cut at the particular instant

L_c = the frame length

V_c = the velocity of the damper rod relative to the armature field in inches per second.

the average value of $V_c = \dfrac{\text{displacement from mean position}}{\text{time of 1/4 cycle}}$

$$= \frac{\beta \times \tau}{180} \times 4 f_n$$

where β = the maximum displacement of the poles, in electrical degrees, from the position of uniform angular velocity

and f_n = the frequency of oscillation;

therefore the effective voltage in a damper rod is approximately

$$= 1.1 \times B_{ga} \times L_c \times \frac{\beta \times \tau}{180} \times 4 f_n \times 10^{-8} \text{ volts.}$$

The resistance of a damper rod of copper $= \frac{L_c}{M}$ ohms

the effective current in a damper rod

$$= B_{ga} \times \frac{\beta \times \tau}{180} \times 4.4 f_n \times M \times 10^{-8}$$

the current density in circular mils per ampere

$$= \frac{10^8 \times 180}{B_{ga} \times \beta \times \tau \times 4.4 f_n}.$$

The dampening effect depends on the value of the total damper current, which, for a given machine, depends on the number of damper rods and on their section. If the section of these rods be increased they will carry a larger current, will have a greater dampening effect and the angle of swing will be reduced so that the larger this section the lower the current density because of the reduction in the value of β.

It is of interest to know the order of magnitude of this current density; for example, assume that

$B_{ga} = 25{,}000$ lines per square inch

$\beta = \pm 5$ electrical degrees which will give a circulating current of about 25 per cent. of full-load current

$\tau = 10$ in., for which the peripheral velocity is 6000 ft. per minute at 60 cycles

$f_n =$ two cycles per second.

then the current density $= \frac{10^8 \times 180}{25{,}000 \times 5 \times 10 \times 4.4 \times 2}$

$= 1640$ circular mils per ampere.

This value is pessimistic in that it neglects the resistance of the end connectors.

It was pointed out above that the dampening effect depends on the total damper section. For gas-engine alternators it is usual to put into the dampers about 25 per cent. of the section of copper that is put into the stator and then, if the damping is not sufficient, or if the dampers get too hot, to look for the cause of the trouble in the governor, flywheel, or load on the system; a pulsating load is equivalent as far as hunting is concerned to a pulsating torque in the engine.

One cause of damper heating must be carefully guarded against. If the dampers are spaced as in Fig. 203, then, when in position *A*, the flux threading between two adjacent dampers is large, while in position *B* this flux is small, so that every time a stator tooth is passed the flux threading between two

damper rods goes through one cycle and the frequency of the current which the induced e.m.f. sends round the closed circuit = the number of stator slots × the revolutions per second. This current is not a damping current and is liable to cause excessive heating and, to prevent the flux pulsation which produces it, the distance between damper rods should be a multiple of the stator slot pitch, as shown at *C*.

For steam-engine driven alternators the strong damping effect of the squirrel cage is seldom required and a cheaper form of

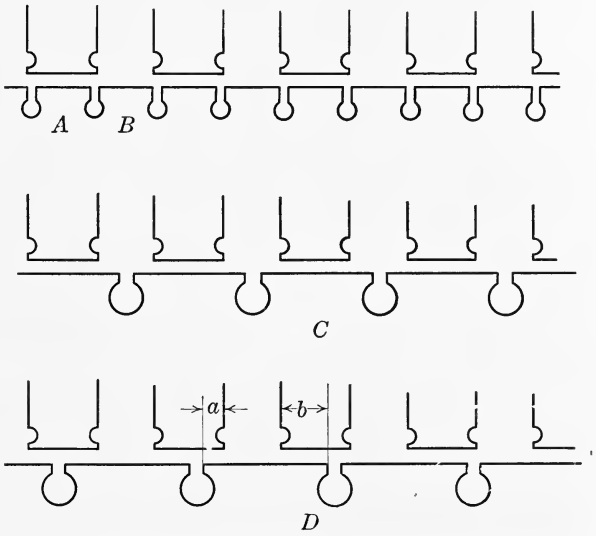


FIG. 203.—Spacing of dampers.

damper is made by surrounding the poles with brass collars as shown in Fig. 140, page 193, where the edges of these collars act exactly like rods threaded across the pole face except that the effect is not so powerful, because they are shielded by the pole tips so that the flux cut is comparatively small.

222. Synchronous Motors for Power-factor Correction.—Consider a synchronous motor running with constant load and constant applied voltage. If the field excitation of the motor be increased its back e.m.f. tends to increase, but this cannot increase much because the applied voltage is constant so that a demagnetizing current must flow in the motor armature to counteract the effect of the increased field excitation; this current must be wattless because the load is constant and to be demagnetizing

it must lag the back generated e.m.f. of the motor and, therefore, lead the applied e.m.f. of the generator.

If on the other hand, the field excitation of the motor be decreased its back e.m.f. tends to diminish and a wattless and magnetizing current must flow in the motor armature; to be magnetizing this current must lead the back generated e.m.f. of the motor and, therefore, lag the applied e.m.f. of the generator.

The size of motor required for a given power-factor correction may be found from a diagram such as Fig. 204 where E is the voltage of the power station I, the total station current per phase, lags E by θ degrees, and the power factor is 70 per cent.

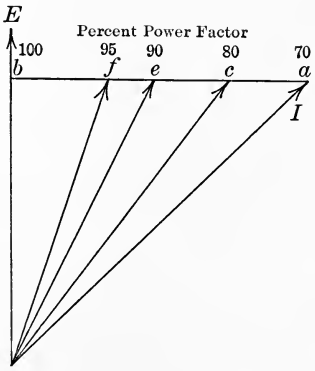


Fig. 204.—Size of motor for power factor correction.

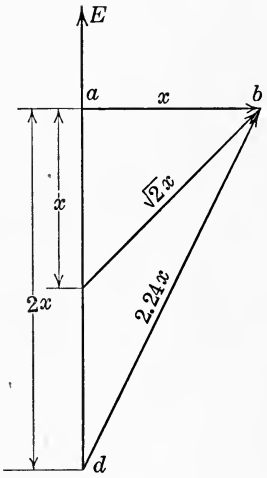


Fig. 205.—Size of motor for power factor correction.

To raise the power factor of this station to 100 per cent. the synchronous motor must draw a leading current $= ab$ and the motor input must be $= \frac{n \times E \times ab}{1000}$ k.v.a. if the motor is running light and the efficiency is 100 per cent.

It may be seen from Fig. 204 that to raise the power factor of the system from

70 to 80 per cent. requires a wattless current ac per phase,
70 to 90 per cent. requires a wattless current ae per phase,
70 to 95 per cent. requires a wattless current af per phase,
70 to 100 per cent. requires a wattless current ab per phase.

The improvement in power factor from 95 to 100 per cent. is, therefore, obtained at a considerable cost.

When synchronous motors are used for power-factor correction it is advisable to arrange that some of the load on the system is carried by these machines; for example, if *ab*, Fig. 205, is the wattless current per phase required for power factor correction, then in order to carry a mechanical load of the same value in k.v.a. the rating of the motor would not be doubled but increased by only 41 per cent.; or for the case shown by triangle *abd*, with an increase in current of 12 per cent. over the value required for the mechanical load a power-factor correction effect of 50 per cent. may be obtained.

223. Design of Synchronous Motors.—Diagram A, Fig. 206, shows some of the saturation curves of an alternator taken at constant current and varying power factor. If this machine were used as a synchronous motor then

for the maximum power-factor correction effect the power factor of the motor would be zero and the excitation $= of$

for zero power-factor correction effect the power factor of the motor would be 100 per cent. and the excitation $= og$

for 80 per cent. load and 60 per cent. power-factor correction effect the excitation would be $= oh$.

Synchronous motors are generally high-speed machines and have, therefore, few poles, so that, as pointed out in Art. 195, page 258, they will be troubled with field heating if designed in the same way as synchronous generators, because for power-factor correction work they are operated with large excitation. In a synchronous motor, however, close regulation is not required so that for these machines the value of q, the ampere conductors per inch, may be run about 20 per cent. higher than the value given in Fig. 184 and the ratio $\frac{\text{maximum field excitation}}{\text{armature } AT. \text{ per pole}}$ may be made less than 3, which value was suggested as a first approximation for synchronous generators in Art. 176, page 229. Diagram B, Fig. 206, shows saturation curves for a synchronous motor of the same rating as that of the generator whose curves are given in diagram A.

Since the flywheel effect of a synchronous motor is generally small, no extra flywheel being supplied, its natural frequency of vibration will generally be comparatively high and the machine therefore liable to be set in violent oscillation by some of the

forced frequencies on the system, for that reason synchronous motors are generally supplied with dampers.

224. Self-starting Synchronous Motors.—These are polyphase machines which are used for motor generator sets and for driving

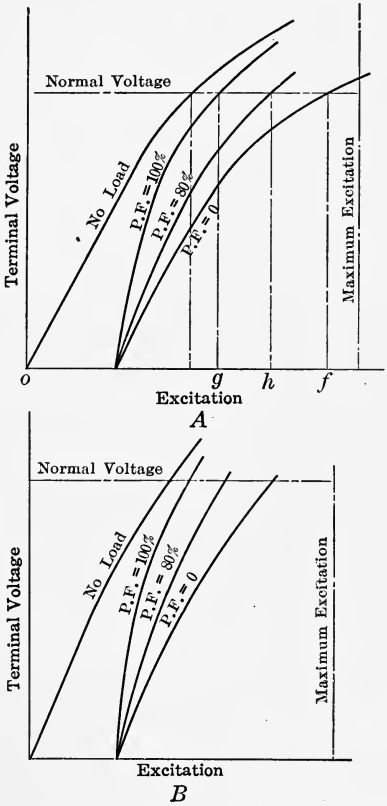


FIG. 206.—Saturation curves of an alternator.

apparatus which requires a small starting torque, when a special starting motor is not desired. They are built exactly like gas-engine alternators and consist of a standard synchronous motor supplied with a squirrel-cage winding on the poles. The method whereby this squirrel cage is calculated so as to meet the required starting conditions will be understood after a study of the induction motor; the following points of importance, however, must be noted here:

Polyphase currents in the armature winding produce a revolv-

ing field which tends to pull the squirrel cage round with it. In order that e.m.fs. may be induced in the squirrel cage by the revolving field the flux must enter the poles, therefore the poles should be laminated and, during the starting period, the field coils should be open circuited.

The pole enclosure must be such that the air-gap under the pole has a constant reluctance for all positions of the pole relative to the armature. If the pole were made as shown in Fig. 207 then in position A the air-gap reluctance would be a minimum and in position B would be a maximum; in such a case the machine would lock in position A and would require a large force to move it out of this locking position.

In Art. 283 on the induction motor, it is shown that if the rotor slot pitch is a multiple of that of the stator, locking will take

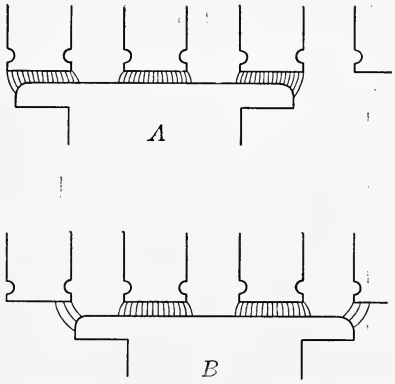


FIG. 207.—Effect of the pole arc on the air gap reluctance.

place due to the leakage fields and that in order to prevent this locking the rotor slot pitch must differ from that of the stator. It was pointed out in Art. 221 that to prevent useless circulating currents the rotor slot pitch should be a multiple of that of the stator, so that a compromise must be made and the rods spaced as shown in diagram D, Fig. 203, where t_r, the rotor tooth, is made equal to the stator slot pitch, for which value it will be found that the air-gap reluctance under a rotor tooth, which is proportional to $a+b$, is constant in all positions of the rotor.

Since the frequency of the flux which, due to the revolving field, passes through the poles, is very high at starting, being equal to the frequency of the applied e.m.f., there will be high voltages

generated in the field coils because the number of turns per coil is large, so that for self-starting synchronous motors the excitation voltage should be as low as possible so as to keep down the number of turns per pole and the field coils must be better insulated than for ordinary synchronous machines; in the case of machines with a large total flux it is sometimes advisable to supply a break-up switch which will open up the field circuit in several places at starting so that the starting e.m.fs. in the different poles will not add up. When the motor is nearly up to speed the field circuit is closed and a small excitation applied which tends to bring the motor into synchronism and ensures that it comes into step with the proper polarity.

CHAPTER XXVII

SPECIFICATIONS

225. The following is a typical specification for an alternator.

SPECIFICATION FOR AN ALTERNATING CURRENT GENERATOR

Rating.

Rated capacity in kilovolt-amperes	400
Power factor	100 per cent.
Normal terminal voltage	2,400
Phases	3
Amperes per terminal	96
Frequency in cycles per second	60
Speed in revolutions per minute	600

Construction.—The generator is for direct connection to a horizontal type of water-wheel and shall be of the internal revolving field type, supplied with base, two pedestal bearings bolted to the base, and a horizontal shaft extended for a flange coupling. The machine must be so constructed that the stator can be shifted sideways to give access to both armature and field coils.

Stator.—The stator coils must be insulated complete before being put into the slots, shall be thoroughly impregnated with compound, and must be readily removable for repairs. Shields must be supplied to protect the coils where they project beyond the core.

Bidders must state the type of slot to be used whether open, partially closed, or completely closed.

Rotor.—The rotor must be strong mechanically and able to run at 75 per cent. overspeed with safety. Any fans or projecting parts on the rotor must be screened in such a way that a person working around the machine is not liable to be hurt.

Workmanship and Finish.—The workmanship shall be first class, all parts shall be made to standard gauge and interchangeable, and all surfaces not machined are to be dressed, filled, and rubbed down to present a smooth finished appearance.

Exciter.—The alternator shall be separately excited and the exciter, which must be direct-connected, is described in a separate specification.

Rheostat.—A suitable field rheostat with face plate and sprocket wheel for distance control is to be supplied.

Foundation bolts will not be furnished.

Coupling.—This is to be supplied by the builder of the water-wheel who shall send one-half to the alternator builder to be pressed on the alternator shaft.

General.—Bidders shall furnish plans or cuts with descriptive matter from which a clear idea of the construction may be obtained; they shall also furnish the following information:

Stator net weight,

Rotor net weight,

Net weight of base and pedestals,

Shipping weight,

Efficiency at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, full- and $1\frac{1}{4}$ non-inductive load,

Regulation at normal output, at 100 per cent. and also at 85 per cent. power factor,

Exciting current, at normal voltage and speed, for normal k.v.a. output, at 100 per cent. and at 85 per cent. power factor. The exciter voltage is 120.

Efficiency.—The losses in the machine shall be taken as:—

Windage, friction, and core loss, which shall be determined by driving the machine by an independent D.-C. motor, the output of which may be suitably determined, the alternator being run at normal speed and excited for normal voltage at no-load.

Field loss, which shall be taken as the exciting current corresponding to the different loads multiplied by the corresponding voltage at the field terminals, the field coils being at their full-load temperature.

The armature I^2R loss; the armature resistance being measured immediately after the full-load heat run.

The load loss, which is determined from a short-circuit core loss test, the machine being short-circuited through an ammeter, run at normal speed, and the field excitation adjusted to send the currents corresponding to the different loads through the armature. One-third of the difference between the loss so found and the I^2R loss for the same current, shall be taken as the load loss.

Regulation.—This shall be taken as the per cent. increase in terminal voltage when the load at which the guarantee is made is reduced to zero, the speed and excitation being kept constant. It shall be found by assuming the synchronous impedence constant for a given excitation, this impedence to be determined

from saturation curves at no-load and also at full-load and zero power factor.

Temperature.—The machine shall carry its normal load in k.v.a., at normal voltage and speed and at 100 per cent. power factor, continuously, with a temperature rise that shall not exceed 40° C. by thermometer on any part of the machine, and, immediately after the full-load heat run, shall carry 25 per cent. overload for 2 hours, at the same voltage, speed and power factor, with a temperature rise that shall not exceed 55° C. by thermometer on any part of the machine.

The temperature rise of the bearings shall not exceed 40° C. as measured by a thermometer in the oil well, either at normal load or at the overload.

No compromise heat run, other than one at normal voltage, zero power factor, and with the armature current for which the guarantee is made, will be accepted. If the manufacturer cannot load the machine, a heat run will be made within 3 months after its erection to find out if it meets the heating guarantee; the test to be made by the alternator builder who shall supply the necessary men and instruments.

Overload Capacity.—The machine must be able to carry at least 50 per cent. overload at 80 per cent. power factor with the normal exciter voltage of 120.

The machine shall be capable of standing an instantaneous short-circuit when operating at normal voltage, normal speed, and no-load, and must be able to carry this short-circuit for 10 seconds without injury.

Insulation.—The machine shall stand the puncture test recommended in the standardization rules of the American Institute of Electrical Engineers (latest edition) and the insulation resistance of the armature and field windings shall each be greater than one megohm.

Testing Facilities.—The builder shall provide the necessary facilities and labor for testing the machine in accordance with this specification.

226. Notes on Alternator Specifications.—The builder of the alternator generally inserts the following clause in the specification.

"We guarantee successful parallel operation between our generators, without racing, hunting, or pumping, whether driven by water-wheels or steam engines, provided that the variation of the angular velocity of the prime movers does not produce

between two generators operating in parallel a displacement of more than five (5) electrical degrees ($2\frac{1}{2}°$ on either side of the mean position), one electrical degree being equal to one mechanical degree divided by half the number of poles."

This clause is inserted to ensure that the engine builder shall supply such a flywheel and governor that the variation of the angular velocity of the engine during one revolution shall be within reasonable limits. As shown in Art 218, however, hunting may take place if the natural period of oscillation of the machine is approximately the same as that of any of the forced oscillations on the system. In practically every case the burden of proof when hunting takes place is put on to the alternator builder.

In the case of alternators for direct connection to gas engines it is often advisable for the alternator builder to check up the size of the proposed flywheel and make sure that it is not such as shall cause resonance.

Wave Form.—It is often specified that the e.m.f. when plotted on a polar diagram, shall not deviate radially more than 5 per cent. from a circle; it must be remembered, however, that the serious trouble, due to a bad wave form, is generally due to the higher harmonics and these harmonics are usually well within the 5 per cent. specified.

Temperature Rise.—In turbo generators the temperature rise by thermometer is misleading, particularly in the case of the rotor, and specifications for these machines should call for the temperature rise to be determined by the increase in electrical resistance; a convenient figure to remember is that 10 per cent. increase in the resistance of copper is equivalent approximately to 25 per cent. increase in temperature.

In high voltage machines the stator temperature should be determined from resistance measurements, and in the case of large and important machines it is sometimes specified that the internal temperature of the machine at the hottest part shall be measured by a thermo-couple built into a coil or by the increase in resistance of a number of turns of fine wire wound round a coil before it is insulated; this coil is placed as near the neutral of the machine as possible.

227. Effect of Voltage on the Efficiency.—Consider the case of two machines built on the same frame and for the same output and speed but for different voltages.

The windage and friction loss is independent of the voltage since it depends on the speed, which is constant.

The excitation loss and the iron loss are independent of the voltage, the winding being made so that the flux per pole is the same for all voltages.

In order to have the same flux per pole the total number of conductors must be directly proportional to the voltage, and if the total amount of copper in the machine is kept constant the size of the conductor will be inversely proportional to the voltage and therefore directly proportional to the current, so that the current density in the conductors, and therefore the copper loss, is independent of the voltage. If, however, as is generally the case, the total amount of armature copper decreases as the voltage increases, on account of the space taken up by insulation, then in order to keep the copper loss constant the current rating must decrease more rapidly than the voltage increases and the output become less the higher the voltage.

If then the total section of copper is constant at all voltages, the losses, output and efficiency will be independent of the voltage; but if the total section of copper decreases as the voltage increases then, while the losses remain constant, the output and the efficiency decrease with increase of voltage.

228. Effect of Speed on the Efficiency.—Consider two machines built for the same kw. output, one of which runs at twice the speed of the other, and assume that

$$\frac{\text{kw.}}{\text{r.p.m.} \times D_a^2 L_c} = \text{a constant}$$

also that

$$\frac{\text{pole-pitch}}{\text{frame length}} = \text{a constant}$$

both of which are approximately true for machines which have more than 10 poles, then the relative dimensions of the two machines are given in the following table:

	Machine A	Machine B
Output	kw.	kw.
Speed	r.p.m.	2(r.p.m.)
Poles	p	1/2 (p)
Internal diameter of armature	D_a	$\frac{D_a}{1.6}$
Frame length	L_c	1.25(L_c)
Pole-pitch	τ	1.25 (τ)
Core depth	d_a	1.25 d_a

Core Loss.—The core weight is approx. = a const. $\times D_a \times L_c \times d_a$ and is the same for each machine, so that for the same flux density in the core the core loss is independent of the speed.

$$\textbf{Copper loss} = \frac{nZI_c^2\ L_b}{\text{cir. mil per cond.}} \quad \text{where } L_b = (L_c + 1.5\tau) \text{ approx.}$$

$$= \frac{\text{amp. cond. per inch}}{\text{cir. mils per ampere}} \times \pi\ D_a\ (L_c + 1.5\tau)$$

The ratio $\frac{\text{amp. cond. per. inch}}{\text{cir. mils per ampere}}$ is limited by heating as shown in Fig. 181, page 253, and since the peripheral velocity is only 25 per cent. higher in machine B than in machine A, it may be assumed that this ratio has approximately the same value in the two machines and the copper loss is therefore

a const. $\times D_a(L_c + 1.5\tau)$ in machine A

and a const. $\times \frac{D_a}{1.6}$ $(1.25L_c + 1.9\tau)$ in machine B

= a const. $\times D_a(0.80L_c + 1.2\tau)$

so that the higher the speed for a given output the lower is the copper loss.

Windage Loss is approximately proportional to the surface of the rotor multiplied by the (peripheral velocity)^3. The peripheral velocity of machine B is 25 per cent. greater than that of machine A, and the windage loss is greater for the high-speed than for the low-speed machine.

Bearing Friction Loss. = a constant $\times d \times l \times (V_b)^{\frac{3}{2}}$, formula 13, page 97. Since the torque on the shaft of machine B is approximately half that on machine A, and the projected area of the bearing $= d \times l$ is directly proportional to this torque, therefore, the bearing friction loss

= a const. $\times d \times l \times (V_b)^{\frac{3}{2}}$ in machine A

= a const. $\times \frac{d \times l}{2} \times (\sqrt{2}V_b)^{\frac{3}{2}}$ in machine B

= a const. $\times d \times l \times (V_b)^{\frac{3}{2}} \times 0.85$ in machine B

so that the higher the speed, the lower the bearing friction loss.

Excitation Loss.—The radiating surface of the field coil is approximately = 2(pole waist + frame length) $\times$ radial length of field coil. Machine B has 25 per cent. greater pole waist, 25 per cent. longer frame length, and 10 per cent. longer poles radially than has machine A, but it has also half the number of poles and, therefore, 0.7 times the radiating surface.

The permissible watts per square inch in machine B is approxi-

mately 25 per cent. greater than in machine A, therefore, if the total permissible watts excitation loss in machine A $= W$
the permissible excitation loss in machine B $= W \times 0.7 \times 1.25$
$= 0.88\ W$
therefore the higher the speed, the lower the field excitation loss.

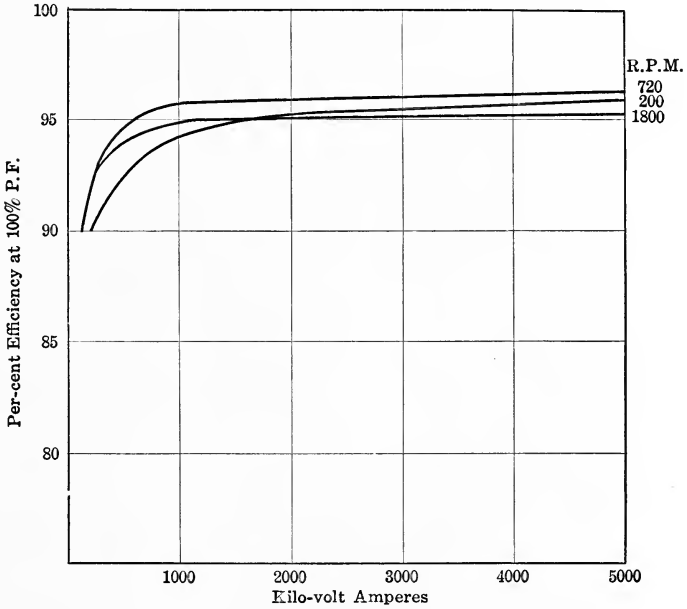


FIG. 208.—Efficiency curves for 2400 volt alternators.

It may therefore be stated as a general rule, that the higher the speed of a machine for a given rating in kilowatts the lower are the losses and the higher the efficiency.

Fig. 208 shows curves of efficiency for polyphase alternators wound for 2400 volts.

CHAPTER XXVIII

ELEMENTARY THEORY OF OPERATION

229. The Revolving Field.—*P*, Fig. 209, shows the essential parts of a two-pole two-phase induction motor. The stator or stationary part carries two windings *M* and *N* spaced 90 electrical degrees apart. These windings are connected to a two-phase alternator and the currents which flow at any instant in the coils *M* and *N* are given by the curves in diagram *Q*; at instant *A* for example, the current in phase $1 = +I_m$ while that in phase 2 = zero.

The windings of each phase are marked *S* and *F* at the terminals and these letters stand for start and finish respectively; a + current is one that goes in at *S*, and a − current one that goes in at *F*.

The resultant magnetic field produced by the windings *M* and *N* at instants *A*, *B*, *C* and *D* is shown in diagram *R* from which it may be seen that, although the windings are stationary, a revolving field is produced which is of constant strength and which moves through the distance of two pole-pitches while the current in one phase passes through one cycle.

To reverse the direction of rotation of this field it is necessary to reverse the connections of one phase.

P, Fig. 210, shows the winding for a two-pole three-phase motor; *M*, *N* and *Q*, the windings of the three phases, are spaced 120 electrical degrees apart. These windings are connected to a three-phase alternator and the currents which flow at any instant in the coils *M*, *N* and *Q* are given by the curves in diagram *R*.

The resultant magnetic field that is produced by the windings at instants *A*, *B*, *C* and *D* is shown in diagram *S* from which it may be seen that, just as in the case of the two-phase machine, a revolving field is produced which is of constant strength and which moves through the distance of two pole-pitches while the current in one phase passes through one cycle.

To reverse the direction of rotation of this field it is necessary

to interchange the connections of two of the phases; for example, to connect phase 2 of the motor to phase 3 of the alternator and phase 3 of the motor to phase 2 of the alternator.

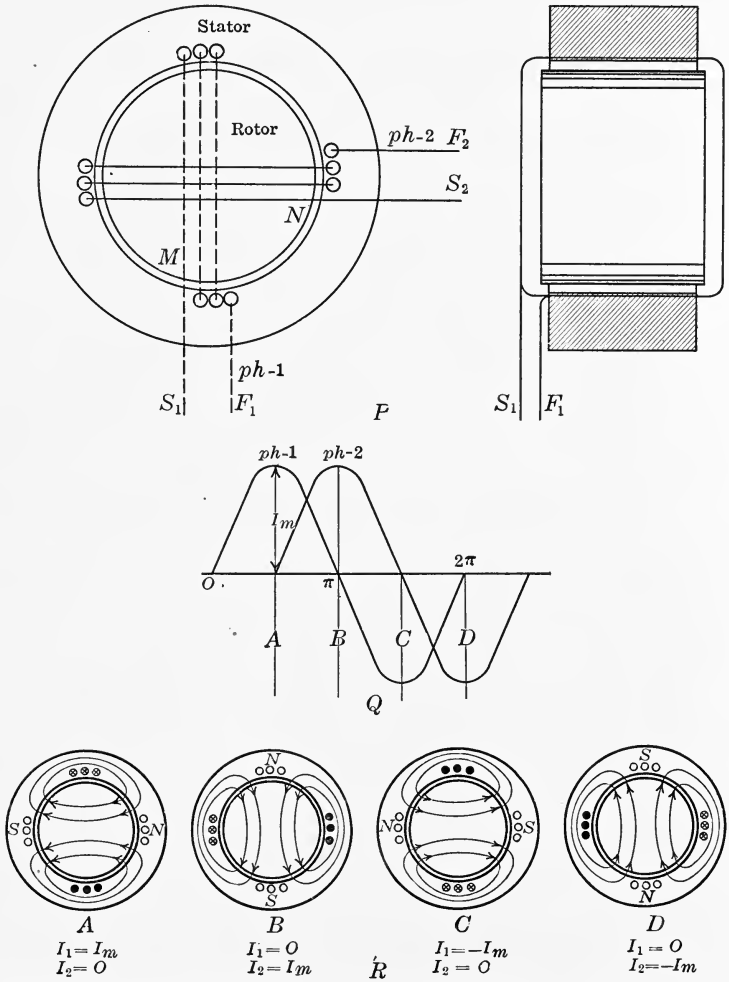


FIG. 209.—The revolving field of a two-pole two-phase induction motor.

230. Multipolar Motors.—Fig. 211 shows the winding for a four-pole, two-phase motor and also the resultant magnetic field at the instants *A* and *B*, Fig. 209. The field moves through the distance of 1/2(pole-pitch) while the current in one phase passes through 1/4(cycle).

Fig. 212 shows the winding for a four-pole, three-phase motor and also the resultant magnetic field at the instants A and B, Fig. 210. In this case the field moves through the distance of

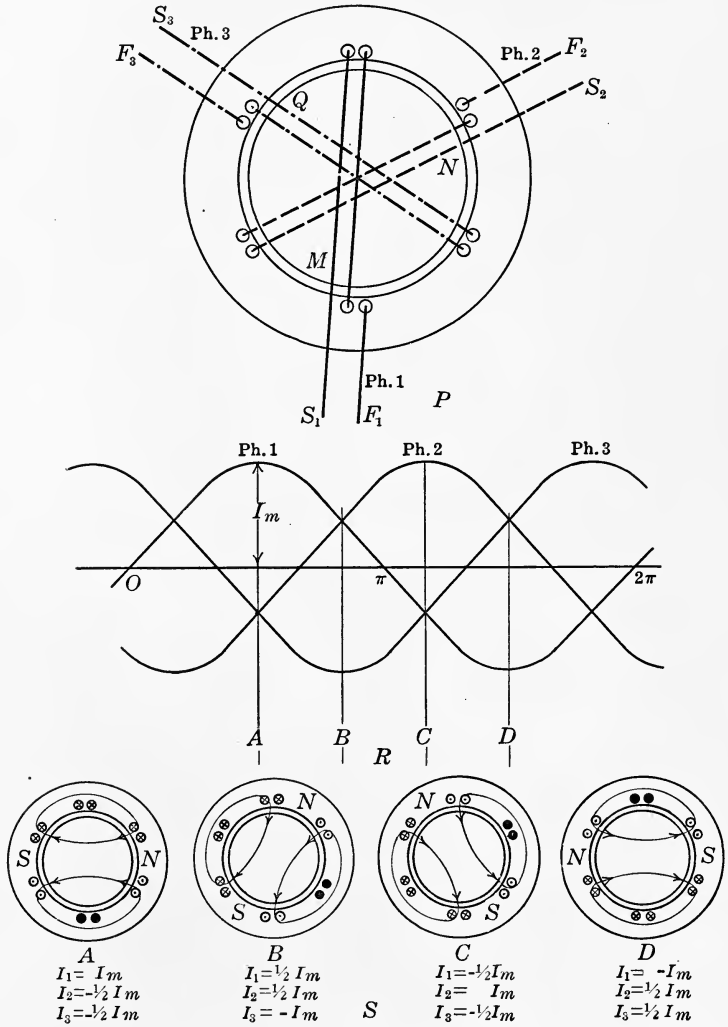


FIG. 210.—The revolving field of a two-pole three-phase induction motor.

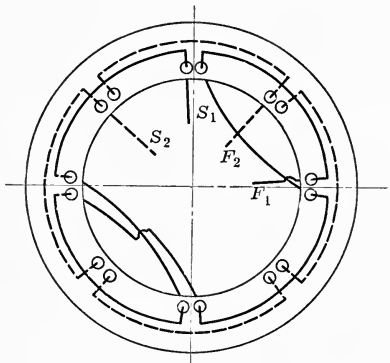
1/3(pole-pitch) while the current in one phase passes through 1/6(cycle).

In general the field moves through the distance of two pole-

pitches or through $\frac{2}{p}$ of a revolution while the current in one phase passes through one cycle, therefore, the speed of the revolving field $= \frac{2}{p} \times f$ revolutions per second

$$= \frac{120 \times f}{p} \text{ revolutions per minute} \qquad (42)$$

this is called the synchronous speed.



Diagrammatic Representation of a 4 Pole 2 Phase Induction Motor

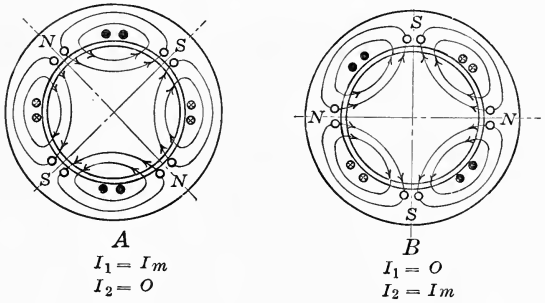


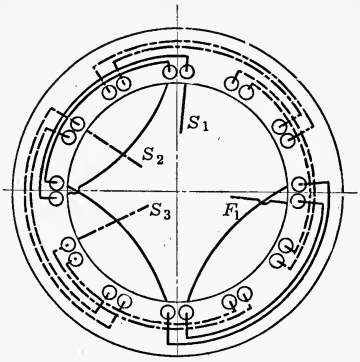
FIG. 211.—The revolving field of a four-pole two-phase induction motor.

231. Induction Motor Windings.—The conditions to be fulfilled by these windings are that the phases should be wound alike, should have the same number of turns, and should be spaced 90 electrical degrees apart in the case of a two-phase winding and 120 electrical degrees apart in the case of a three-phase winding.

The above are the conditions that have to be fulfilled by alternator windings so that the diagrams developed in Chapter XVII apply to induction motors as well as to alternators.

Fig. 213 shows an actual stator with its windings arranged in phase belts. The ends of the coils are bent back so that the rotating part of the machine, called the rotor, can readily be put in position.

The type of rotor which is in most general use is shown in Fig. 214 and is called the squirrel-cage type. It consists of an iron core slotted to carry the copper rotor bars; these bars are



Diagrammatic Representation of a 4 Pole 3 Phase Induction Motor

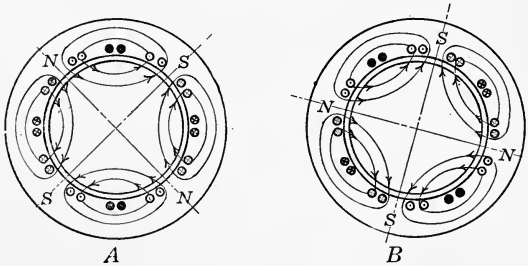


FIG. 212.—The revolving field of a four-pole three-phase induction motor.

joined together at the ends by brass end connectors so as to form a closed winding.

232. Rotor Voltage and Current at Standstill.—The actions and reactions of the stator and rotor will be taken up later; it is necessary, however, to point out here that, since the applied voltage per phase is constant, the back voltage per phase must be approximately constant at all loads. This back voltage is produced by the revolving field, therefore, the actions and reactions of the

stator and rotor must be such as to keep the revolving field approximately constant at all loads.

Let the revolving field be represented by a revolving north and south pole as shown in diagram A, Fig. 215, and then for convenience let this figure be split at *xy* and opened out on to a plane; the result will be diagram B, of which a plan is shown in diagram D.

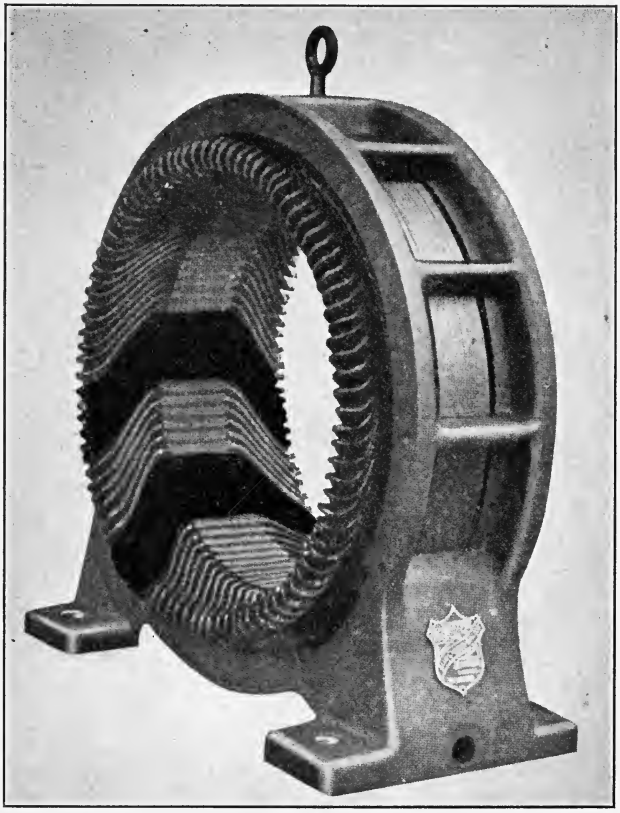


Fig. 213.—Stator of an induction motor.

The distribution of flux on the rotor surface due to the revolving field is given by the curve in diagram B at a certain instant. The field is moving in the direction of the arrow; it therefore cuts the rotor bars and generates in them, e.m.fs. which are shown in magnitude at the same instant in diagram D. Since the rotor

circuit is closed these e.m.fs. will cause currents to flow in the rotor bars.

The frequency of the e.m.fs. in the rotor bars at standstill $= \frac{p \times \text{syn. r.p.m.}}{120}$ cycles per second = the frequency of the stator applied e.m.f., so that the frequency of the rotor currents is high, and the reactance of the rotor, which is proportional to this frequency, is large compared with its resistance; the rotor current

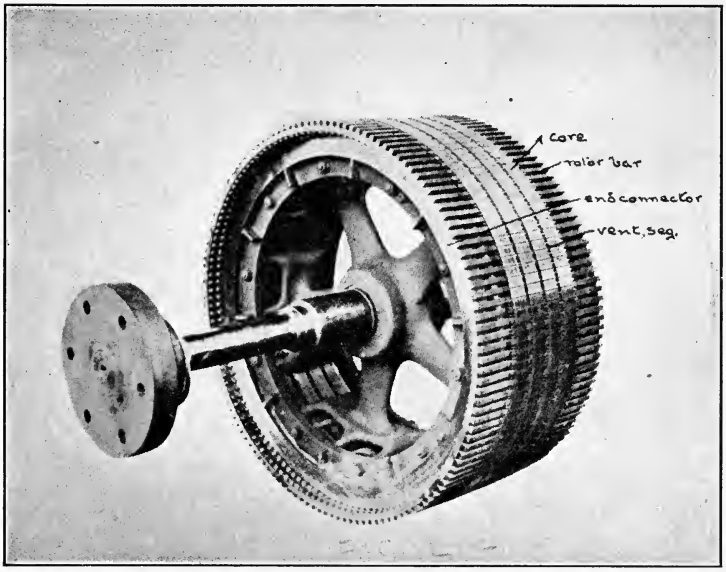


FIG. 214.—Squirrel cage rotor.

therefore lags considerably behind the rotor voltage as shown by the curve in diagram D and also by crosses and dots in diagram C. The value of this current at standstill

$$= \frac{\text{rotor voltage at standstill}}{\text{rotor impedance at standstill}}$$

and this is usually about 5.5 times the full-load rotor current.

233. Starting Torque.—As shown in diagrams C and D the rotor bars are carrying current and are in a magnetic field so that a force is exerted tending to move them; this force when multiplied by the radius of the rotor gives the torque. The relative torque at different points on the rotor surface is given by the curve in diagram D, which is got by multiplying the value of flux density

at different points on the rotor surface by the value of the current in the rotor bar at these points. It may be seen from this curve, and also from diagram C, that at some points on the rotor surface the torque is in one direction and at other points is in the opposite direction so that the resultant torque is quite small.

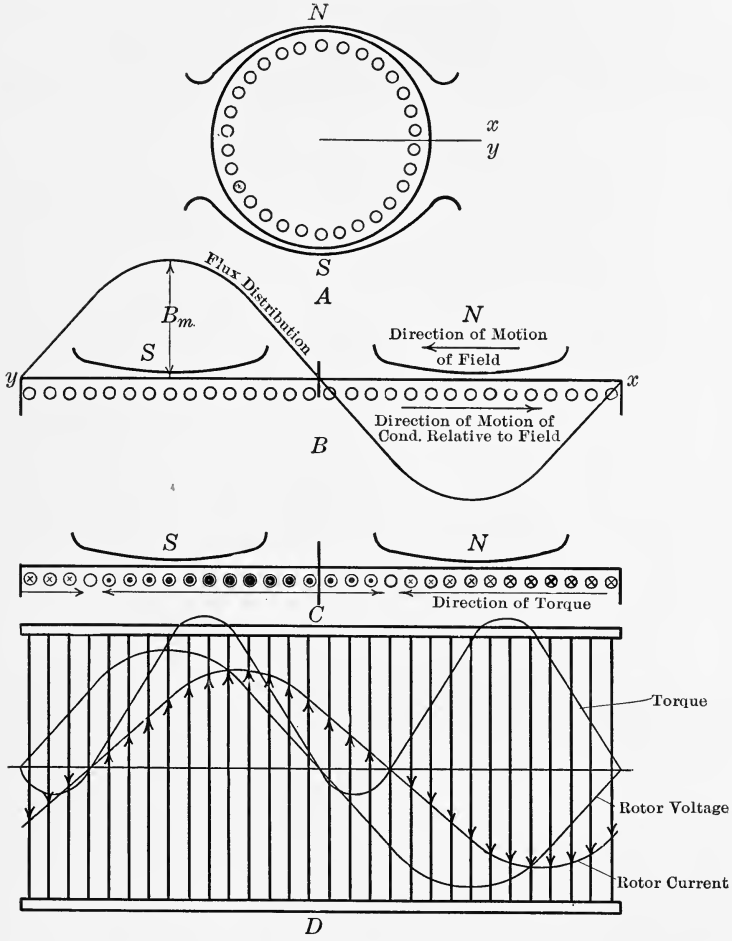


FIG. 215.—Production of torque in an induction motor.

As a rule the starting torque of a squirrel-cage motor is about 1.5 times full-load torque when the rotor current is about 5.5 times the full-load rotor current. Diagram C shows that the resultant torque is in such a direction as to tend to make the rotor follow up the revolving field.

The starting torque can be increased for a given current if that current be brought more in phase with the voltage, this can readily be seen from the curves in Fig. 215. The angle of lag of the rotor current can be decreased by increasing the rotor resistance, and sufficient resistance is generally put in the rotor circuit to bring the current down to its full-load value; the angle of lag is then so small that full-load torque is developed.

When a motor is running under load a large rotor resistance is undesirable because it causes large loss, low efficiency, and

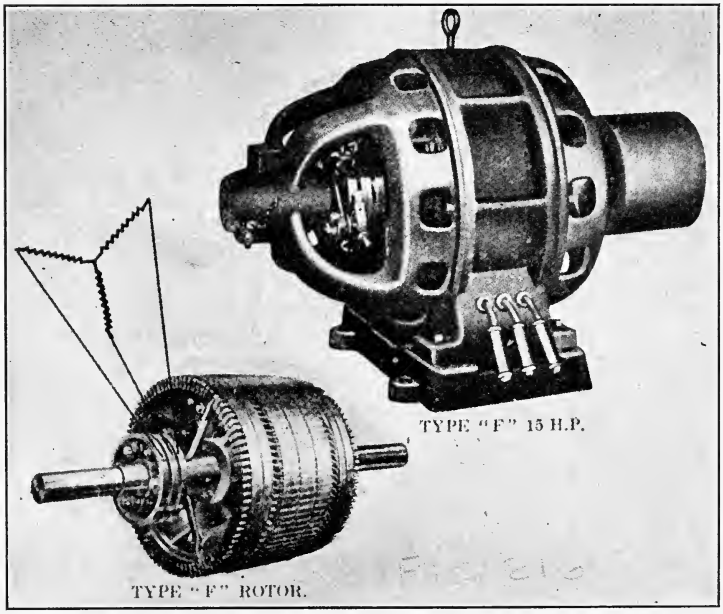


FIG. 216.—Wound rotor type of induction motor.

excessive heating. To get over this difficulty the wound rotor motor was developed. This type of motor has the same stator as that used for the squirrel-cage machine but its rotor is as shown in Fig. 216; the rotor bars are connected together to form a winding, but this winding is not closed on itself as in the squirrel-cage machine, it is left open at three points which are connected to three slip rings, and the winding is closed outside of the machine through resistances which can be adjusted. The winding is finally short circuited at the slip rings when the motor is up

to speed. In this way it is possible to have the advantage of high rotor resistance for starting and low rotor resistance when the motor is running at full speed.

234. Running Conditions.—It was pointed out in the last article that the resultant torque is in such a direction as to make the rotor follow up the revolving field. When the motor is not carrying any load the rotor will revolve at practically synchronous speed, that is, at the speed of the revolving field. If the motor is then loaded it will slow down and slip through the revolving field, the rotor bars will cut lines of force, the e.m.fs. generated in these bars will cause currents to flow in them and a torque will be produced. The rotor will slow up until the point is reached at which the torque developed by the rotor is equal to the torque exerted by the load.

The ratio $\frac{\text{syn. r.p.m.} - \text{r.p.m. of rotor}}{\text{syn. r.p.m.}}$ is called the per cent. slip and is represented by the symbol s, its value at full-load is generally about 4 per cent.

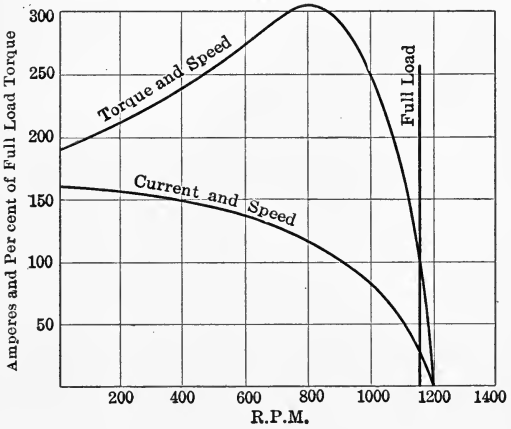


Fig. 217.—Characteristics of a 25-h.p., 440-volt, 3-phase, 60-cycle, 1200 syn. r.p.m. induction motor.

As the load is increased the rotor drops in speed and the slip, rotor current, frequency and lag of rotor current all increase. The torque developed by the rotor tends to increase due to the increase in rotor current and to decrease due to the increase in current lag. Up to a certain point, called the break-down point or point of maximum torque, the effect of the current is greater

than that of the current lag, beyond that point the effect of the current lag is the greater, so that after the break-down point is passed the torque actually decreases even although the current is increasing.

The relation between speed, torque, and current is shown in Fig. 217 for a 25-h. p., 440-volt, three-phase, 60-cycle, 1200-syn. r.p.m. induction motor.

235. Vector Diagram at No-load.—At no-load a motor runs at synchronous speed so that the slip, rotor voltage and rotor current are all zero.

Consider the winding S_1F_1 of one phase of the stator as shown in Fig. 209 or 210. The voltage E_1 applied to this phase causes a current I_0, called the magnetizing current, to flow in the winding. This current, along with the magnetizing currents in the other phases, produces a revolving field ϕ_f of constant strength.

While the revolving field ϕ_f is of constant strength the flux ϕ_g which threads the winding S_1F_1 is an alternating flux and is a maximum and $= \phi_f$ when the magnetizing current in the winding S_1F_1 is a maximum; this can be ascertained by examination of diagram R, Fig. 209 and of diagram S, Fig. 210, so that the flux which threads the winding S_1F_1 is in phase with the magnetizing current I_0 in that winding.

The alternating flux ϕ_g generates an alternating e.m.f. E_{1b} called the back e.m.f. in the winding S_1F_1, and this voltage lags the flux ϕ_g by 90°. To overcome this back e.m.f. the applied e.m.f. must have a component which is equal and opposite to E_{1b} at every instant; the other component of the applied e.m.f. must be large enough to send a current I_0 through the impedance of the winding.

In Fig. 218

I_0 is the magnetizing current in one phase,

ϕ_g is the flux per pole threading that phase,

E_{1b} is the back e.m.f. in that phase of the stator and lags ϕ_g which produces it by 90°,

E_{11} is the component of the applied e.m.f. which is equal and opposite to E_{1b},

I_0Z_1 is the component of the applied e.m.f. which is required to send the current I_0 through the stator impedance Z_1,

E_1 is the applied voltage per phase and is the resultant of E_{11} and I_0Z_1.

236. Vector Diagram at Full-load.—At full-load the speed of

the rotor = r.p.m.$_2$ = $(1-s)$r.p.m.$_1$, and the speed of the revolving field relative to the rotor surface = s(r.p.m.$_1$), so that the frequency of the e.m.f. which is generated in the rotor winding by the revolving field $= \frac{s(\text{r.p.m.}_1)\times p}{120} = sf_1$.

The flux ϕ_g threads the rotor coils and generates in each phase of the rotor winding a voltage E_2 which lags ϕ_g by 90°. This voltage causes a current $I_2 = \frac{E_2}{Z_2}$ to flow in the closed rotor circuit; I_2 lags the voltage E_2 by an angle whose tangent $= \frac{sX_2}{R_2}$, where Z_2 is the rotor impedance at full-load,

X_2 is the rotor reactance per phase at standstill,

R_2 is the rotor resistance per phase,

s, the per cent. slip, is approximately = 4 per cent.,

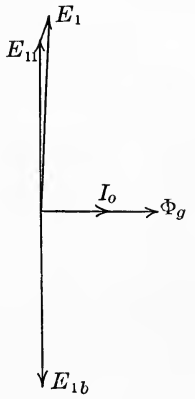


FIG. 218.—No-load vector diagram.

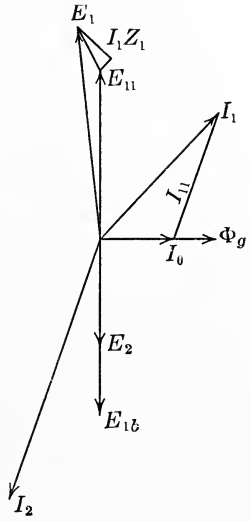


FIG. 219.—Vector diagram at full load.

with this value of s the angle of lag of the rotor current at full load seldom exceeds 20°.

In Fig. 219

I_0 is the magnetizing current, which has the same value as in Fig. 218,

ϕ_g is the flux per pole threading one phase of both rotor and stator windings,

E_{1b} is the back e.m.f. of the stator,
E_2 is the e.m.f. generated in one phase of the rotor by flux ϕ_g,
I_2 is the current in that phase,
E_{11} is the component of the applied e.m.f. which is equal and opposite to E_{1b}.,

Now E_1, the applied voltage, is constant and is the resultant of E_{11} and I_1Z_1; this latter quantity is comparatively small even at full-load, so that it may be assumed that E_{11} has the same value at full-load as at no-load and therefore E_{1b}, which is equal and opposite to E_{11} and ϕ_g which produces E_{1b}, are approximately constant at all loads, so that the resultant magnetizing effect of the stator and rotor currents must be equal to the magnetizing effect of the current I_o. The stator current may therefore be divided into two components, one of which I_{11} has a m.m.f. equal and opposite to that of the rotor current I_2; the other component of stator current must be I_o, because this is the necessary condition for a constant value of ϕ_g and of E_{1b}; therefore, in Fig. 219, I_{11} is a component of primary current whose m.m.f. is equal and opposite to that of I_2.
I_1 is the primary current and is the resultant of I_o and I_{11}.
E_1 is the applied voltage and is the resultant of E_{11} and I_1Z_1.

CHAPTER XXIX

GRAPHICAL TREATMENT OF THE INDUCTION MOTOR

237. Current Relations in Rotor and Stator.—It is shown in Art. 236 that the m.m.f. of the rotor opposes that of the stator, and that the resultant m.m.f., namely, that of the magnetizing current, is just sufficient to produce the constant revolving field ϕ_f.

Fig. 220 shows the position of the windings of one phase of both rotor and stator, and also the direction of the currents in these

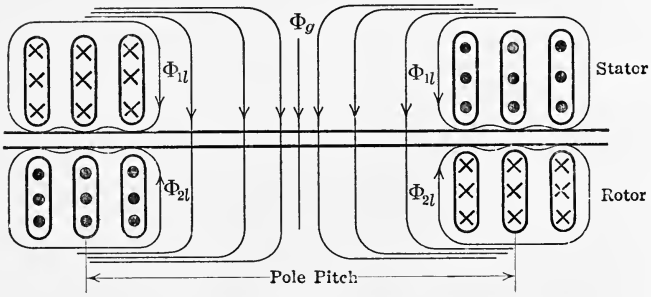


FIG. 220.—The magnetic fields in an induction motor.

windings, at the instant that the rotor and stator m.m.fs. are directly opposing one another. As the rotor revolves relative to the stator there will be positions on either side of that shown in Fig. 220, in which the rotor and stator m.m.fs. do not quite oppose one another; two such positions are shown in Fig. 221; this overlapping of the phases is called the *belt effect* and is discussed more fully in Chap. 33.

Due to the revolving field ϕ_f an alternating flux ϕ_g threads one phase of both rotor and stator windings. It is shown in Fig. 220 that, in addition to the flux ϕ_g which crosses the air-gap, there is a leakage flux ϕ_{1l} which links the stator coils but does not cross the air-gap; ϕ_{1l} is proportional to the current I_1 which produces it. There is also a leakage flux ϕ_{2l} which links the rotor

coils but does not cross the air-gap; ϕ_{2l} is proportional to the current I_2 which produces it.

238. The Stator and Rotor Revolving Fields.—

If f_1 = the frequency of the stator current,

s = the per cent. slip,

r.p.m.$_1$ = the synchronous speed,

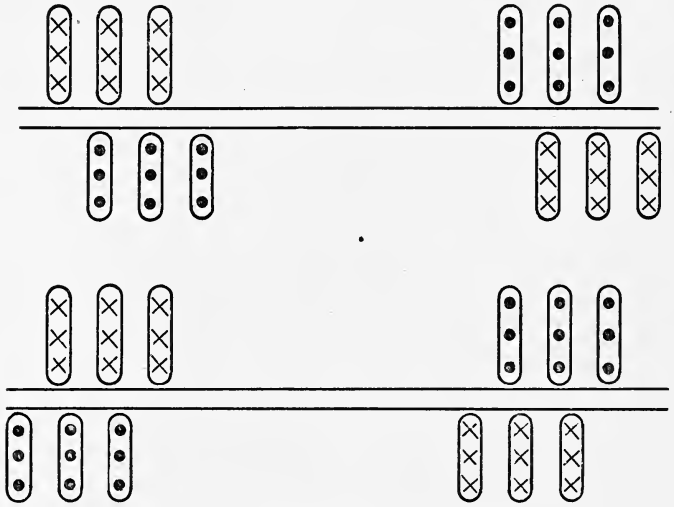


FIG. 221.—Overlapping of the phases.

then f_2, the frequency of the rotor current = sf_1, see Art. 236, and $(1-s)$r.p.m.$_1$ = the rotor speed.

The stator current acting alone produces a field which revolves at synchronous speed, namely r.p.m.$_1$; the rotor current acting alone produces a field which revolves at a speed of s(r.p.m.$_1$) relative to the rotor surface or at s(r.p.m.$_1$) + $(1-s)$r.p.m.$_1$ = r.p.m.$_1$ relative to the stator surface; that is, the two fields revolve at the same speed in space and so can be represented on the same diagram.

239. The Voltage and Current Diagram.—Fig. 224 shows the voltage and current relations in an induction motor.

E_1 = the voltage per phase applied to the stator windings,

I_1 = the stator current per phase, and lags E_1 by θ degrees; the locus of I_1 is a circle.

240. The Flux Diagram.—Consider the stator current acting alone, then in Fig. 222:

ϕ_1 = the flux per pole threading one phase of the stator and, as pointed out in Art. 235, page 329, is in phase with I_1,

ϕ_{1l} = the stator leakage flux per phase per pole, see Fig. 220,

ϕ_{1g} = that part of ϕ_1 which crosses the gap and threads one phase of the rotor $= v_1\phi_1$.

Consider the rotor current I_2 acting alone, then:

ϕ_2 = the flux per pole threading one phase of the rotor,

ϕ_{2l} = the rotor leakage flux per phase per pole,

ϕ_{2g} = that part of ϕ_2 which crosses the gap and threads one phase of the stator $= v_2\phi_2$.

Under load conditions, when both I_1 and I_2 are acting,

ϕ_{1s} = the actual flux per pole threading one phase of the stator,

ϕ_{2r} = the actual flux per pole threading one phase of the rotor,

ϕ_g = the actual flux per pole in the gap between rotor and stator,

E_{1b} = the stator e.m.f. per phase generated by ϕ_{1s} and consists of two components;

E_{1g} generated by the flux ϕ_g

$E_{1l} = I_1X_1$, generated by the flux ϕ_{1l}

E_{2g} = the rotor e.m.f. per phase generated by ϕ_g. This e.m.f. sends a current I_2 through the rotor winding which produces the leakage flux ϕ_{2l} and generates the e.m.f. $E_{2l} = I_2X_2$ in that winding.

E_{2g}, therefore, consists of two components, namely,

E_{2r} to overcome the resistance per phase and a voltage equal and opposite to E_{2l} to overcome the reactance per phase.

Since E_{2r} is in phase with I_2 and therefore with ϕ_2, the angle aoh = angle oab = 90°.

The applied voltage must be equal and opposite to E_{1b} if the stator resistance per phase and therefore the voltage I_1R_1 is sufficiently small to be neglected. Since E_1 is constant, therefore E_{1b}, and the flux ϕ_{1s} which produces E_{1b}, must be constant in magnitude.

241. Geometrical Proof of the Circle Law.—In Fig. 222 draw dk perpendicular to fd so as to cut the line of produced at k.

In triangles aob and fdk,

$$\text{angle } oab = \text{angle } fdk = 90°,$$
$$\text{angle } oba = \text{angle } dfk,$$

therefore,
$$\frac{fk}{ob} = \frac{fd}{ab} = \frac{fd}{ac - cb} = \frac{fd}{oh - cb}$$

and

$$fk = \frac{ob \times fd}{oh - cb}$$

$$= \frac{(v_1 \times of)(v_2 \times oh)}{oh(1 - v_1 v_2)}$$

$$= \frac{of(v_1 v_2)}{1 - v_1 v_2}$$

= a constant, since v_1, v_2, and of are all constant.

Since fk is constant and angle fdk is an angle of 90°, the locus of d, or of ϕ_1, is a circle whose center is on the line ok and whose diameter $= fk$.

If the magnetic circuit be not saturated, and this is the case at the actual flux densities due to ϕ_{1s} and ϕ_{2r}, the actual fluxes in the machine, then Fig. 222 may be transformed into Fig. 223,

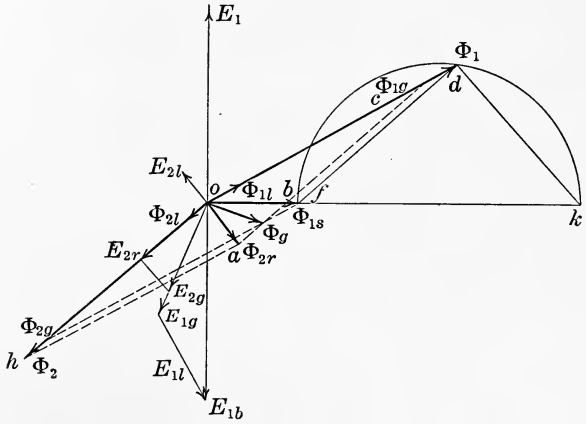


FIG. 222.—Voltage and flux diagram.

which is a m.m.f. and voltage diagram, by making $n_1 b_1 c_1 I_1$ proportional to ϕ_1 and $n_2 b_2 c_2 I_2$ proportional to ϕ_2.

Fig. 223 may be transformed into a voltage and current diagram as shown in Fig. 224, and in this form, with a slight modification, it is generally used.

242. Special Cases.—1. The motor is running without load and, therefore, at synchronous speed, so that the rotor e.m.f. and the rotor current are zero. Under these conditions Fig. 224 becomes Fig. 225.

2. The motor is at standstill and the rotor resistance is assumed to be zero. Under these conditions Figs. 222 and 224 give Fig. 227. Since R_2 is zero therefore E_{2r} and ϕ_{2r} are both zero, and E_{2g} must be equal and opposite to E_{2l} and therefore equal to I_2X_2.

Now $E_{1b}=E_{1g}+E_{1l}$

$=E_{1g}+I_1X_1$

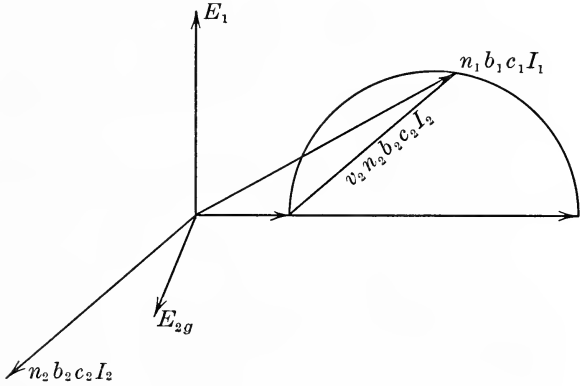


FIG. 223.—Voltage and m.m.f. diagram.

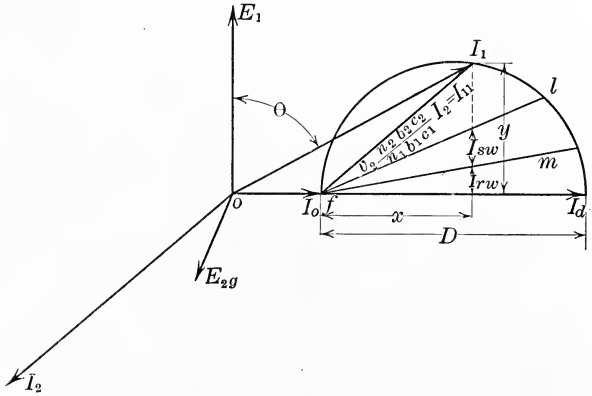


FIG. 224.—Voltage and current diagram

and $\dfrac{E_{1g}}{E_{2g}}=\dfrac{b_1c_1}{b_2c_2}$ since they are produced by the same flux ϕ_g

therefore $E_1=E_{2g}\left(\dfrac{b_1c_1}{b_2c_2}\right)+I_1X_1$

$=I_2X_2\times\left(\dfrac{b_1c_1}{b_2c_2}\right)+I_1X_1$

$$= I_1 X_2 \times \frac{n_1}{n_2}\left(\frac{b_1 c_1}{b_2 c_2}\right)^2 + I_1 X_1$$

$$= I_1\left[X_1 + \frac{n_1}{n_2}\left(\frac{b_1 c_1}{b_2 c_2}\right)^2 X_2\right] \qquad (43)$$

the value of I_1 in this case is I_d the maximum current, Fig. 224.

243. Representation of the Losses on the Circle Diagram.—The losses in an induction motor are:

Constant losses { Mechanical losses—windage and friction loss. Iron or core loss—hysteresis and eddy-current loss.

Variable losses { Stator copper loss. Rotor copper loss. Load losses.

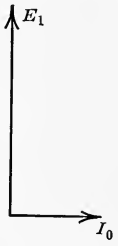


Fig. 225.

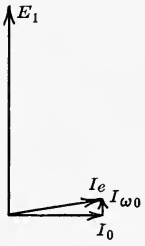


Fig. 226.

No-load conditions.

Constant Losses.—As a motor is loaded, the rotor drops in speed and the slip and therefore the frequency of the flux in the rotor core increases. Due to the drop in speed the windage and friction loss decrease and due to the increase in frequency of the rotor flux the iron loss in the rotor core increases, so that it is reasonable to assume that the sum of these two losses is constant at all speeds.

To overcome the constant loss a stator current I_{wo} is required, in phase with the applied voltage and of such a value that $n_1 E_1 I_{wo}$ = the constant loss in watts. To take account of this in the circle diagram the no-load conditions have to be changed from those of Fig. 225 to those of Fig. 226.

Variable Losses.—These are $n_1 I^2{}_1 R_1$ and $n_2 I^2{}_2 R_2$.

In Fig. 224 the stator current $= I_1$ and the corresponding rotor current $= I_2 = I_{11} \times \frac{1}{v_2} \times \frac{n_1 b_1 c_1}{n_2 b_2 c_2}$

The Rotor Copper Loss.—This loss $= n_2 I_2{}^2 R_2$ watts

$$= n_2 I^2{}_{11}\left(\frac{1}{v_2}\times\frac{n_1 b_1 c_1}{n_2 b_2 c_2}\right)^2 R_2$$

$$= I^2{}_{11}\times \text{a constant.}$$

Now $I^2{}_{11} = x^2 + y^2$, Fig. 224

$$= x^2 + x(D - x)$$

$$= xD;$$ where D is a constant

therefore the rotor loss $= x$(a constant).

To allow for the rotor loss on the circle diagram take any stator current I_1, find the corresponding rotor current I_2 and the rotor copper loss $n_2 I^2{}_2 R_2$ in watts; set up from the base line

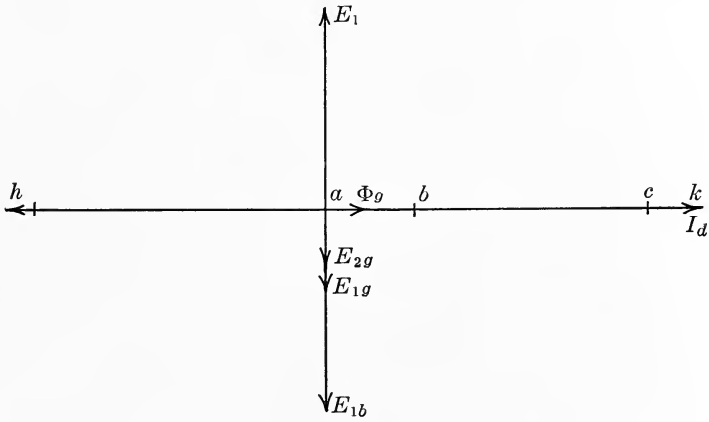


FIG. 227.—Conditions at standstill with zero rotor and stator resistance.

oI_d a current I_{rw} as shown in Fig. 224, in phase with E_1, such that $n_1 E_1 I_{rw}$ = the rotor copper loss in watts. Draw a straight line through I_o and I_{rw}. The vertical distance from the base to this line is proportional to x and is a measure of the rotor loss.

The Stator Copper Loss.—This loss $= n_1 I^2{}_1 R_1$ watts. $I^2{}_1$ is assumed to be $= I^2{}_o + I^2{}_{11}$; the error due to this assumption is comparatively small. The part of the copper loss $n_1 I^2{}_o R_1$ is added to the constant losses, the other part $n_1 I^2{}_{11} R_1$ is proportional to x just as in the case of the rotor loss.

To allow for the stator loss on the circle diagram take any stator current I_1, find the corresponding stator copper loss $n_1 I^2{}_1 R_1$ in watts; set up from the line fm, Fig. 224, a current I_{sw} in phase with E_1, such that $n_1 E_1 I_{sw}$ = the stator copper loss in watts. Draw a straight line through I_o and I_{sw}.

Load Losses.—These consist of various eddy-current losses which cannot be accurately predetermined and which are made as small as possible by careful design; they are usually allowed for by increasing the variable losses by a certain percentage found from tests on similar machines.

244. Relation Between Rotor Loss and Slip.—Let the curve in Fig. 228 represent the distribution of the actual rotor revolving field ϕ_r corresponding to the alternating flux ϕ_{2r}, Fig. 222.

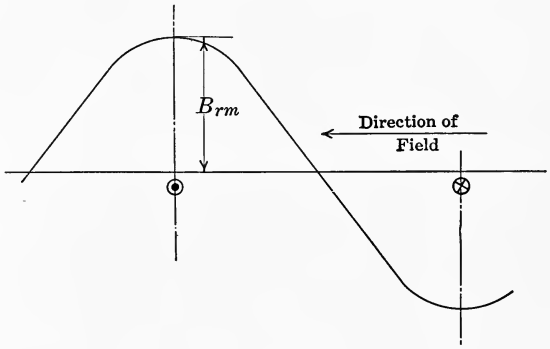


FIG. 228.—Torque on a rotor conductor.

The e.m.f. generated in a rotor conductor by this flux

$$=2.22\ \phi_r f_2\ 10^{-8} \text{ volts}$$

$$=2.22\ \left(B_{r\,m}\frac{2}{\pi}\tau L_c\right)\ (sf_1)10^{-8} \text{ volts}$$

and is in phase with the flux density.

It is shown in Fig. 222 that I_2 is in phase with the voltage E_{2r} so that the loss per conductor

$$=E_{2r}\times I_2 \text{ watts}$$

$$=2.22\ B_{r\,m}\frac{2}{\pi}\tau L_c s f_1 I_2 10^{-8} \text{ watts.}$$

Since I_2 is in phase with E_{2r} it is also in phase with the flux density so that the average force on a rotor conductor

$$=B_{r\,eff}\frac{I_2}{10}\times L_c \text{ dynes, where } B_r \text{ and } L_c \text{ are in centimeter units}$$

and the work done by this conductor in ergs is therefore

$$=B_{r\,eff}\times\frac{I_2}{10}\times L_c\times \text{cond. velocity in centimeters per second}$$

$$= \frac{B_{r\,m}}{\sqrt{2}} \frac{I_2}{10} \times L_c \times \pi \text{ (rotor diameter} \times \text{revolutions per second)}$$

$$= \frac{B_{r\,m}}{\sqrt{2}} \frac{I_2}{10} \times L_c \times p\tau \times \frac{2(1-s)\, f_1}{p}$$

or in watts

$$= \frac{B_{r\,m}}{\sqrt{2}} \frac{I_2}{10} \times L_c \times \tau \times (1-s) f_1 \times 2 \times 10^{-7}$$

therefore, $\dfrac{\text{rotor loss}}{\text{rotor output}} = \dfrac{s}{1-s}$

and $\dfrac{\text{rotor input}}{\text{rotor output}} = \dfrac{1}{1-s}$

$$= \frac{\text{synchronous speed}}{\text{actual speed}}.$$

245. The Final Form of the Circle Diagram.—Fig. 229 shows the form in which the circle diagram is generally used.

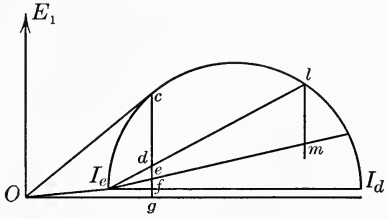


FIG. 229.—The circle diagram.

For a stator current $I_1 = oc$ = full-load current

gc = the power component of the primary current

fg = that part of gc required to overcome the constant losses

ef = that part of gc required to overcome the stator copper loss

de = that part of gc required to overcome the rotor copper loss

cd = that part of gc required to overcome the mechanical load

$\dfrac{cJ}{oc}$ = the power factor

$\dfrac{cd}{cJ}$ = the efficiency

$\dfrac{dc}{ce} = \dfrac{\text{rotor output}}{\text{rotor input}} = \dfrac{\text{actual speed}}{\text{synchronous speed}}$

$\dfrac{de}{ce} = \dfrac{\text{rotor loss}}{\text{rotor input}} = s$ = per cent. slip

the h. p. output $= \dfrac{n_1 E_1 (dc)}{746}$.

the corresponding torque, $T = \frac{n_1 E_1 (dc)}{746} \times \frac{33000}{2\pi r.p.m._2}$ lb. at 1 ft. radius.

At synchronous speed this same torque would produce

$$\frac{T \times 2\pi r.p.m._1}{33000} \text{ horse-power}$$

$$= \frac{n_1 E_1 (ce)}{746} \text{ horse-power; this is called the}$$

Synchronous Horse-power due to full-load torque.

At standstill the synchronous horse-power due to the starting torque $= \frac{n_1 E_1 (lm)}{746} = \frac{\text{the rotor copper loss in watts}}{746}$.

The maximum horse-power output of the motor

$$= \frac{n_1 E_1 (\text{maximum value of } dc)}{746}$$

The maximum torque is that which gives a synchronous horse-power $= \frac{n_1 E_1 \text{ (maximum value of } ce)}{746}$

The use of this circle diagram is shown by an example which is worked out fully in the next chapter.

CHAPTER XXX

CONSTRUCTION OF THE CIRCLE DIAGRAM FROM TESTS

The no-load saturation and the short-circuit tests are those that are usually made on an induction motor in order to determine its characteristics.

246. The No-load Saturation Curve.—The figures necessary for the construction of this curve are got by running the motor at normal frequency and without load. Starting at 50 per cent. above normal voltage, the voltage is gradually reduced until the motor begins to drop in speed and simultaneous readings are taken of voltage, current and total watts input.

The results of such a test on a 50 h. p., 440-volt, three-phase, 60-cycle, eight-pole, Y-connected motor are given below.

NO-LOAD SATURATION

E_t Terminal volts	I_l Line current	P Watts input
600	32.3	2,940
562	29.0	2,650
521	25.9	2,350
470	22.6	2,100
422	19.8	1,860
375	17.0	1,650
318	14.2	1,450
275	12.5	1,300
222	10.5	1,150
175	8.9	1,060
120	7.5	960

In Fig. 230:—

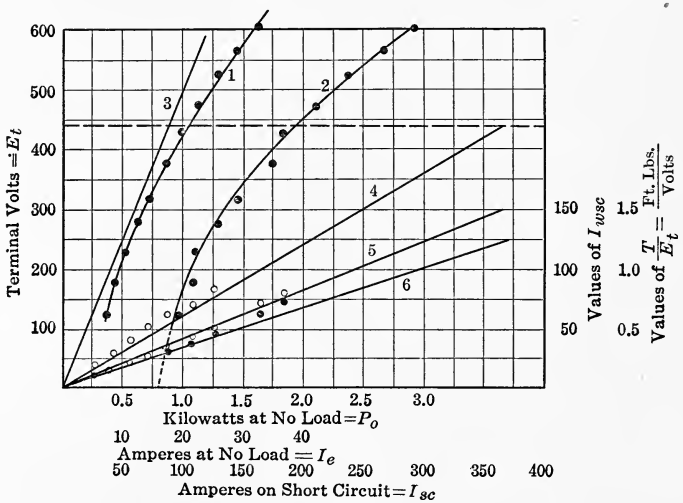
The relation between E_t and I_l is shown in curve 1,

The relation between E_t and P is shown in curve 2,

The straight line, curve 3, shows the relation between E_t and that part of the exciting current which is required to send the magnetic flux across the air gap; this curve is calculated by

the method explained in Chap. XXXII. At normal voltage the exciting current is 21 amperes, the magnetizing current required to send the magnetic flux across the air-gap is 17.6 amperes, and the ratio $\frac{21}{17.6}=1.20=k_o$ which is called the iron factor.

Curve 1 is not tangent to curve 3 because, in addition to the true magnetizing current, the exciting current contains the power



Curve 1.—Terminal volts and exciting current.

Curve 2.—Terminal volts and no-load loss.

Curve 4.—Terminal volts and short-circuit current.

Curve 5.—Short-circuit current and power component of short-circuit current.

Curve 6.—Short-circuit current and $\frac{\text{torque in ft.-lbs.}}{\text{terminal volts.}}$

FIG. 230.—Test curves on a 50-h.p., 440-volt, 3-ph., 60-cycle, 900-syn. r.p.m. induction motor.

component I_{wo} which is required to overcome the losses P. I_{wo} is quite large at low voltages; for example, at 120 volts

I_e $=7.2$ amperes

P $=960$ watts

$$I_{wo}=\frac{960}{1.73\times 120}=4.6 \text{ amperes}$$

I_o $=\sqrt{7.2^2-4.6^2}=5.6$ amperes $=$ the true magnetizing current. The difference between I_o and I_e is comparatively small at normal voltage, for example, at 440 volts

I_e = 21 amperes
P = 1950 watts
I_{wo} = 2.54 amperes
$I_o = \sqrt{21^2 - 2.54^2} = 20.8$ amperes

P contains the windage and friction loss, the core loss, and the small stator copper loss due to the current I_e. If curve 2 be produced so as to cut the axis as shown, then the watts loss at zero voltage = 800 watts, must be the windage and friction loss since the flux, and therefore the core loss, is zero and the stator copper loss can be neglected.

247. The Short-circuit Curve.—This curve shows the relation between stator voltage and current, the rotor being at standstill. The figures necessary for its construction are obtained by blocking the rotor so that it cannot revolve and applying voltage to the stator windings at normal frequency; the rotor is sometimes held by means of a prony brake so that readings of starting torque may also be taken. The applied voltage is gradually raised from zero to a value that will send about twice full-load current through the motor, and simultaneous readings are taken of voltage, current, total watts input, and torque.

The results of such a test on the 50 h. p. motor on which the saturation test was made are given below.

SHORT CIRCUIT

E_t Terminal volts	I_{sc} Line current	P Watts input	I_{wsc}	T Torque in pounds at 1 ft. radius	$\frac{T}{E_t}$
222	185	3,000	78	155	.7
201	165	2,400	69	122	.61
178	143	1,820	59	94	.53
162	127	1,420	51	71	.44
138	109	990	41	51	.37
120	88	680	33	35	.29
101	72	440	25	23	.23
80	58	270	20	14	.18
58	42	140	14	7	.12
38	27	60	9	.....	.....

On short-circuit $I_{sc} = \dfrac{E_t}{X_1 + X_2\left(\dfrac{b_1 c_1}{b_2 c_2}\right)^2 \dfrac{n_1}{n_2}}$ formula 43, page 337, so that theoretically the relation between E_t and I_{sc} in the above table should be represented by a straight line. The actual relation is shown in curve 4, Fig. 230; the curve gradually bends away from the straight line due to the gradual saturation of the iron part of the leakage path.

The watts input on short-circuit $= 1.73 E_t\ I_{wsc}$, it is also $= kI_{sc}^2$, because it is all expended in copper loss, and since E_t is proportional to I_{sc}, see curve 4, therefore I_{wsc} also is proportional to I_{sc} and the relation is plotted, for the tests in the above table, in curve 5.

In Art. 245, it was shown that the starting torque is proportional to $n_2 I_2^2 R_2$, the rotor copper loss, where I_2 is the rotor current on short-circuit, and since I_2 is proportional to I_{sc} which is proportional to E_t therefore $\dfrac{\text{starting torque}}{E_t}$ is proportional to I_{sc} and the relation is plotted, for the tests in the above table, in curve 6.

Curves 4, 5 and 6 are produced as shown, so that the probable values at normal voltage of I_{sc}, I_{wsc}, and $\dfrac{\text{torque}}{\text{volts}}$ may be determined; at 440-volts $I_{sc} = 370$ amperes

$I_{wsc} = 150$ amperes

$$\frac{\text{torque in lb. ft.}}{\text{volts}} = 1.2$$

torque in pounds at 1 ft. radius $= 1.2 \times 440 = 528$,

torque in synchronous horse-power $= \dfrac{528 \times 2\pi \times 900}{33000} = 90$

248.—Construction of the Circle Diagram.—The results obtained from the curves in Fig. 230 are to be used in order to construct the circle diagram, Fig. 231, from which the characteristics of the motor will be determined.

1. Locate the short-circuit point. With o as center and radius $I_{sc} = 370$ amperes describe the arc of a circle; of the total current I_{sc} the power component $I_{wsc} = 150$ amperes. These two values of current give l, the short-circuit point.

2. Locate the no-load point. At 440 volts the exciting current $= I_e = 21$ amperes and the value of P at the same voltage $=$ 1950 watts, so that the power component of the exciting current

$= \frac{1950}{1.73 \times 440} = 2.54$ amperes. These two values of current give the point I_e, the no-load point.

3. Draw the circle. The circle must pass through the two points l and I_e and must have its center on the constant loss line.

4. Find the stator and rotor copper losses. Of the total copper loss represented by lp the part mp represents the stator copper loss and is determined as follows: The resistance of the stator winding from terminal to neutral, measured by direct current = 0.112 ohms, so that the stator copper loss on short-circuit = $370^2 \times 0.112 \times 3 = 46000$ watts, and the corresponding power com-

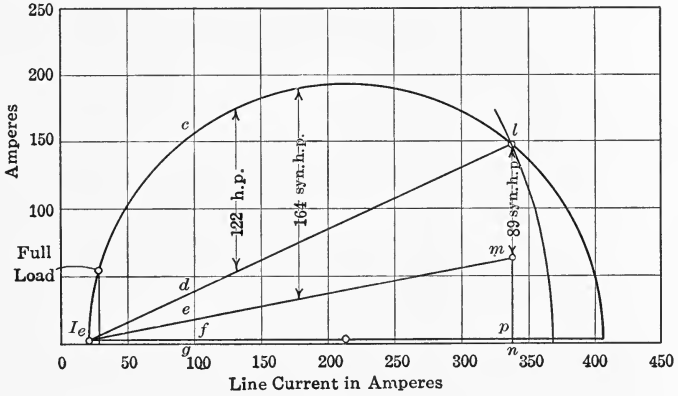


FIG. 231.—Circle diagram for a 50-h.p., 440-volt, 3-ph., 60-cycle, 900-syn. r.p.m. induction motor.

ponent of current $= \frac{46000}{1.73 \times 440} = 60$ amperes $= mp$.

For any stator current oc

the constant loss $= pn \times 1.73 \times 440$ watts
the stator copper loss $= fe \times 1.73 \times 440$ watts
the rotor copper loss $= de \times 1.73 \times 440$ watts
the mechanical output $= cd \times 1.73 \times 440$ watts

249. To Find the Characteristics at Full-load.—The full-load = 50 h. p. and the value of cd corresponding to this output = $\frac{50 \times 746}{1.73 \times 440} = 49$ amperes; the point c on the circle is then found to suit and the following values scaled off

$ce = 51$ amp.
$cg = 55$ amp.
$oc = 62$ amp.

from which the efficiency $= \frac{cd}{cg} = \frac{49}{55} = 89$ per cent.

the power factor $= \frac{cg}{oc} = \frac{55}{62} = 89$ per cent.

the actual speed = syn. speed $\times \frac{cd}{ce} = 900 \times \frac{49}{51} = 865$ r.p.m.

the slip $= \frac{de}{ce} = \frac{2}{51} = 4$ per cent.

The maximum value of $cd = 120$ amperes so that the maximum horse-power $= \frac{120 \times 1.73 \times 440}{746} = 122$.

The maximum value of $ce = 160$ amperes so that the maximum torque in synchronous horse-power $= \frac{160 \times 1.73 \times 440}{746} = 164 =$ about 3.25 times full-load torque.

The value of $ml = 87.5$ amperes so that the starting torque in synchronous horse-power $= \frac{87.5 \times 1.73 \times 440}{746} = 89$ or about 1.8 times full-load torque, which checks closely with the value found by brake readings.

In plotting test results it is advisable to plot line current and terminal volts rather than current per phase and voltage per phase, because then the diagram becomes independent of the connection, since three-phase power $= 1.73\ E_l I_l$ for either Y or $\triangle$ connection, and two-phase power $= 2E_l I_l$ for either star or ring connection.

CHAPTER XXXI

CONSTRUCTION OF INDUCTION MOTORS

250. Fig. 232 shows the type of construction that is generally adopted for motors up to 200 horse-power at 600 r.p.m. The particular machine shown is a squirrel-cage motor.

251. The Stator.—*B*, the stator core, is built up of laminations of sheet steel 0.014 in. thick which are separated from one another by layers of varnish and have slots *E* punched on their inner periphery to carry the stator coils *D*.

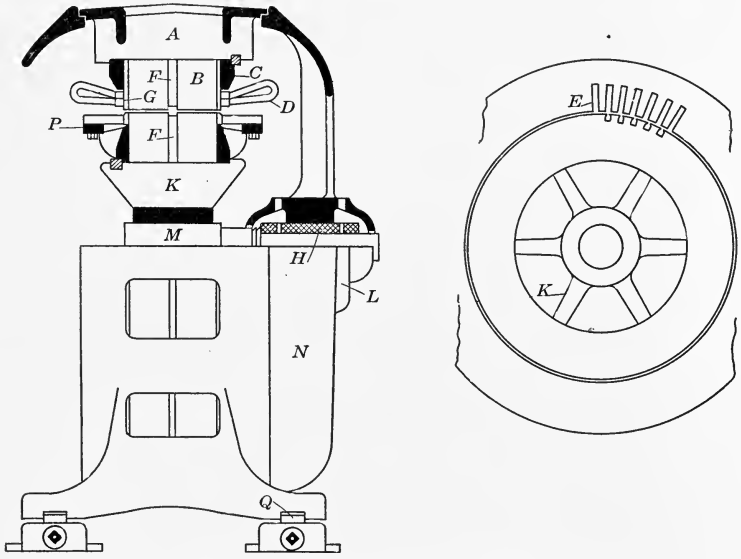


FIG. 232.—Small squirrel-cage induction motor.

Two kinds of slot are in general use and both are shown in Fig. 233. The partially closed slot is used for all rotors and has the advantage that it causes only a small reduction in the air-gap area; the open slot is generally used for stators because the stator coils are subject to comparatively high voltages, and the open slot construction allows the coils to be fully insulated

before they are put into the machine, it also allows the coils to be easily and quickly repaired in case of breakdown.

The core is built up with ventilating segments spaced about

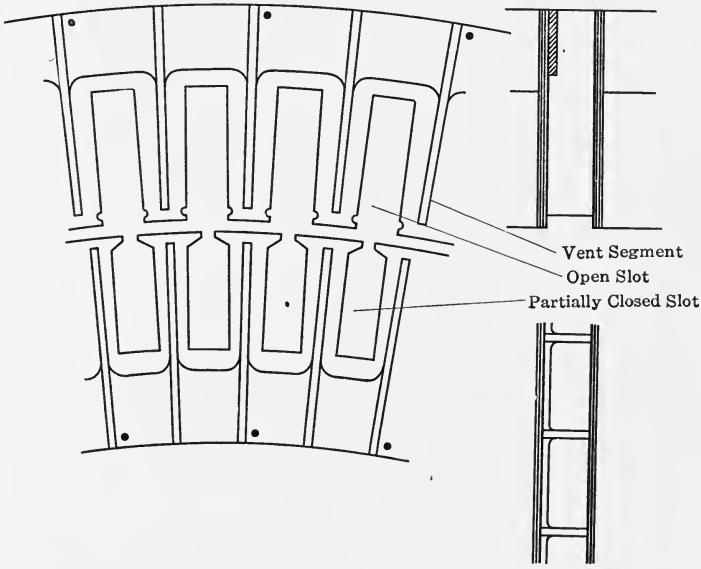


Fig. 233.—Slots and vent segments.

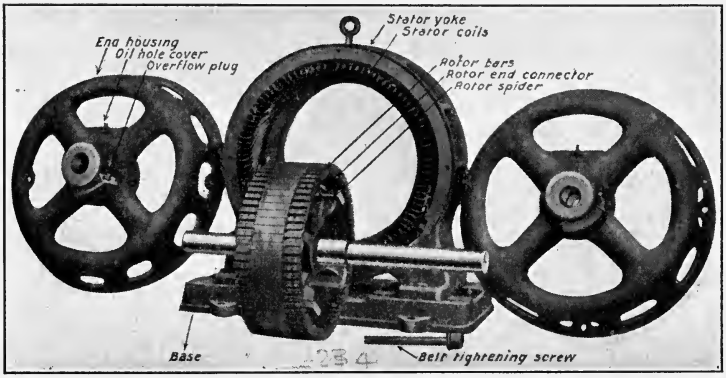


Fig. 234.—Parts of an Allis Chalmers Bullock squirrel-cage induction motor.

3 in. apart; one segment is shown at *F*, Fig. 232, and in greater detail in Fig. 233, and consists of a light brass casting which is riveted to the adjacent lamination of the core; the

lamination for this purpose is usually made of 0.025 in. steel. The laminations are clamped tightly between two cast-iron end heads C, and in order to prevent the core from spreading out on the inner periphery, tooth supports G, of strong brass, are often placed between the end heads and the core.

The stator yoke A carries the stator core and the bearing housings N. When the motor has to be mounted on a wall, or on a ceiling, the housings N must be rotated through 90° or 180° respectively; this is necessary because ring oiling is always used for motor bearings, and the oil well L must always be below the shaft. The bearing housings help to stiffen the whole machine and allow the use of a fairly light yoke. The shape of the housings, as may be seen from Fig. 234, is such that it is a simple matter to close the openings between the arms with perforated sheet metal so as to form a *semi-enclosed motor*, or all the openings in the machine with sheet metal so as to form a *totally enclosed motor*.

252. The Rotor.—The rotor core also is built up of laminations of sheet steel, which are usually 0.025 in. thick; the use of such thick sheets is permissible because the frequency of the magnetic flux in the rotor is low and the rotor core loss is small.

The core with its vent ducts is clamped between two end heads and mounted on a spider K, Fig. 232. In the case of squirrel-cage rotors the depth of the rotor slot is usually so small that a tooth support is not required.

The rotor bars are carried in partially closed slots and are connected together at the ends by rings P called end connectors; these rings are usually supported on fan blades which are carried by the end heads.

The shaft M is extra stiff because the clearance between the stator and rotor is very small, and the bearings H are extra large so as to give a reasonably long life to the wearing surface.

The whole machine is carried on slide rails Q, which are rigidly fixed to the foundation and by sliding the motor along the rails the belt may be tightened or slackened. Slide rails are not used for geared or direct-connected motors.

253. Large Motors.—Fig. 235 shows the type of construction for large motors. Pedestal bearings are used, so that the yoke has to be extra stiff in order to be self-supporting. The stator coils are tied to a wooden coil support ring R which is carried by brackets attached to the end heads; coil supports should be used

even on small motors if the coils are flimsy and liable to move due to vibration.

Both squirrel-cage and wound-rotor constructions are shown in Fig. 235 and it may be seen that the same spider U can be used in either case; the two rotors differ in the number and size of slots and in the shape of the end head. S is a band of steel

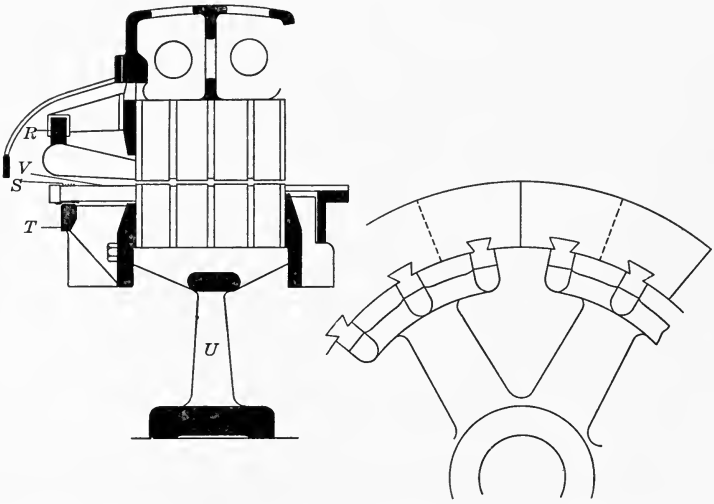


FIG. 235.—Large induction motor; both squirrel-cage and wound-rotor construction shown.

wire used to bind down the rotor coils of the wound-rotor machine on to the coil support T which is carried by the end heads; this coil support acts as a fan and helps to keep the machine cool. When the rotor diameter is greater than 30 in. the rotor core is generally built up of segments which are carried by dovetails on the rotor spider.

CHAPTER XXXII

MAGNETIZING CURRENT AND NO-LOAD LOSSES

254. The E.M.F. Equation.—The revolving field generates e.m.fs. in the stator windings which are equal and opposite to those applied, therefore, as shown in Art. 148, page 189,

$E = 2.22\ kZ\phi_a f 10^{-8}$ volts, for full-pitch windings

where E = the voltage per phase

k = the distribution factor from the table on page 189

Z = the conductors in series per phase

ϕ_a = the flux per pole of the revolving field

f = the frequency of the applied e.m.f.

so that, if the winding of an induction motor and also its operating voltage and frequency are known, the value of the revolving field can be found from the above equation.

255. The Magnetizing Current.—Diagram A, Fig. 236, shows an end view of part of the stator of a three-phase induction motor which has twelve slots per pole. The starts of the windings of the three phases are spaced 120 electrical degrees apart and are marked S_1, S_2 and S_3.

Diagram B shows the value of the current in each phase at any instant.

Diagram C shows the direction of the current in each conductor at the instant F and the corresponding distribution of m.m.f.

Diagram D shows the direction of the current in each conductor at the instant G and the corresponding distribution of m.m.f.

It will be seen that the revolving field is not quite constant in value but varies between the two limits shown in diagrams C and D. The average m.m.f., AT_{av}, is found as follows:

$AT_{av} \times 12\lambda$ = area of curve in diagram C

$= 4.0\ bI_m \times \lambda$

$3.5\ bI_m \times 2\lambda$

$3.0\ bI_m \times 2\lambda$

$2.5\ bI_m \times 2\lambda$

$2.0\ bI_m \times 2\lambda$

$1.0\ bI_m \times 2\lambda$

$28\ bI_m\lambda$

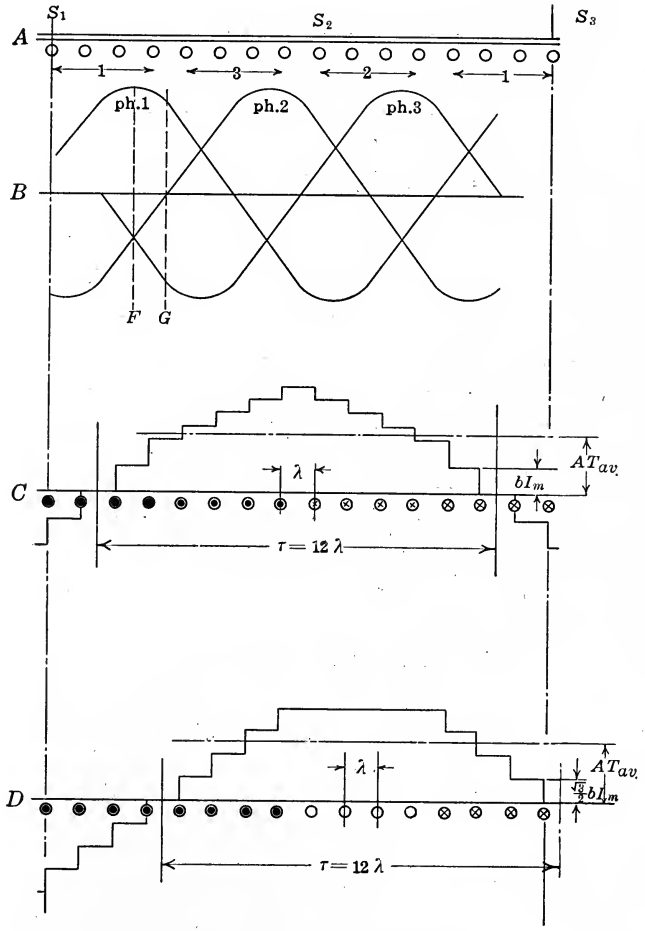


FIG. 236.—The revolving field.

$$\text{also } AT_{av} \times 12\lambda = \text{area of curve in diagram D}$$
$$= 4.0 \times 0.866\, bI_m \times 5\lambda$$
$$3.0 \times 0.866\, bI_m \times 2\lambda$$
$$2.0 \times 0.866\, bI_m \times 2\lambda$$
$$1.0 \times 0.866\, bI_m \times 2\lambda$$
$$27.7\, bI_m\lambda$$

therefore, taking the mean of these two values

$$AT_{av} \times 12\lambda = 27.85\, bI_m\lambda$$
$$\text{and } AT_{av} = 2.32\, bI_m$$
$$= 2.32 \times \left(\frac{\text{slots per pole}}{12}\right) \times b \times \sqrt{2} I_c$$

since there are 12 slots per pole

$$= 0.273 \text{ (cond. per pole) } I_c$$

The value of AT_{av} can be found in a similar way for any number of slots per pole and for both two- and three-phase windings and varies slightly from the above figure for different cases, but in general the value of AT_{av} used for all polyphase windings $= 0.273$ (cond. per pole) I_c.

The relation between the flux per pole and the magnetizing current per phase is found from the formula

$$\phi_a = 3.2 \frac{AT_{av}}{\delta} \frac{\tau L_g}{C}$$
$$= 0.87 \text{ (cond. per pole) } I_c \frac{\tau L_g}{\delta C}$$
$$\text{or } I_o = \frac{1}{0.87 \text{ (cond. per pole)}} \times \frac{\phi_a}{\tau L_g} \times \delta C \qquad (44)$$

where $I_o =$ the magnetizing current per phase (effective value)
$\phi_a =$ the flux per pole of the revolving field
$\tau =$ the pole-pitch in inches
$L_g =$ the axial length of air-gap in inches and is used instead of L_c because, on account of the small air-gap, there is little fringing into the vent ducts
$\delta =$ the air-gap clearance in inches
$C =$ the Carter coefficient, see Fig. 40, page 44.

To allow for the m.m.f. required to send the flux through the iron part of the magnetic circuit, the value of current found from the last formula has to be multiplied by a factor which is found from the results of similar machines that have already been tested; where no such information is available the factor is taken as 1.2.

256. No-load Losses.—These losses are similar to and are figured out in the same way as the no-load losses in a D.-C. machine.

$$\textbf{Bearing Friction Loss} = 0.81\, dl\left(\frac{V_b}{100}\right)^{3/2} \text{watts}$$

where d = the bearing diameter in inches
l = the bearing length in inches
V_b = the rubbing velocity of the bearing in feet per minute.

Brush Friction.—This loss is found only in wound rotor motors; it is very small, and $= 1.25\, A\, \frac{V_r}{100}$ watts

where A is the total brush rubbing surface in square inches,
V_r is the rubbing velocity in feet per minute.

Windage Loss.—This loss cannot readily be separated out from the bearing friction loss so that its value is not known, it is generally so small that it can be neglected.

The Iron Loss.—The frequency of the flux in the stator teeth and core $= f_1$, the frequency of the applied voltage, while that in the rotor teeth and core $= sf_1$, where s is the per cent. slip.

At no-load the slip is practically zero so that the rotor loss due to the main flux is very small and is not calculated.

The iron loss at no-load includes:
The hysteresis and eddy-current loss due to the main flux,
Loss due to filing of the slots,
Loss due to alternating leakage fluxes in the yoke and end heads,
Additional loss due to non-uniform flux distribution in the core.

In addition to the above there is also the pulsation loss which is peculiar to the induction motor. In Fig. 237, A shows part of a motor which has a larger number of slots in the rotor than in the stator. It may be seen that the flux density in the rotor tooth x is a maximum while that in the tooth y is a minimum, so that there is a pulsation of flux in the rotor teeth which goes through one cycle while the rotor passes through the distance of one stator slot, or the frequency of this pulsating flux = the number of stator slots $\times$ the revolutions per second of the rotor

In the same way there is a pulsation of flux in the stator teeth with a frequency = the number of rotor teeth $\times$ the revolutions per second of the rotor.

These frequencies are of the order of 1000 cycles per second, and, therefore, the loss due to these pulsations of flux is large;

the tests plotted in Fig. 238 give some idea of their value. The machine on which the tests were made was a 60-cycle, wound-rotor motor. Curve I shows the loss with the rotor circuit open and therefore the machine at a standstill, the rotor current zero, the stator current being the exciting current, and the frequency of the flux in the rotor core the same as that in the stator core, then the loss *ab* at any voltage = the hysteresis and eddy-current loss due to the main field in both stator and rotor. Curve 2 shows the loss with the rotor circuit closed and the

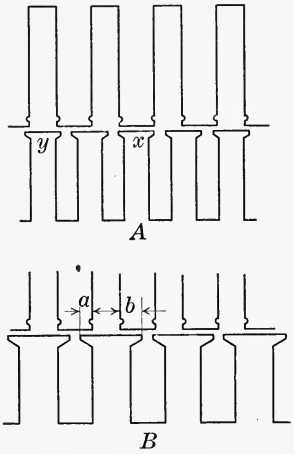


Fig. 237.—Flux pulsation in the rotor teeth.

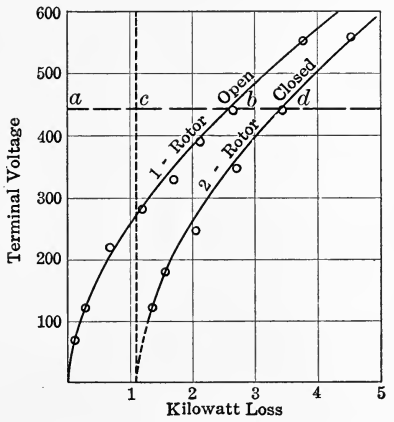


Fig. 238.—Iron loss tests on a 3-phase, 60-cycle, wound-rotor induction motor.

motor running idle at synchronous speed. In this case the rotor core loss due to the main field is zero and the loss *cd* at the same voltage as before is the hysteresis and eddy-current loss in stator only and also the pulsation loss: the pulsation loss is about 50 per cent. of the total no-load loss.

257. Rotor Slot Design.—To make the pulsation loss a minimum the reluctance of the air-gap under each tooth should remain constant. This can be accomplished by the use of totally closed slots for both rotor and stator; even partially closed slots will give approximately this result if the slits at the top of the slots be made narrow.

When open slots are used in the stator the size of rotor tooth which makes the pulsation loss a minimum is found in the follow-

ing way: The rotor slot is made partially closed and the rotor tooth is made equal to the stator slot pitch as shown in *B*, Fig. 237, where it will be seen that the pulsation of flux in a rotor tooth is zero because the area of air gap under a rotor tooth $=a+b=$a constant. The pulsation of flux in the stator teeth is small because the rotor slots are almost closed and the fringing makes them equivalent to totally closed slots so far as pulsation is concerned.

258. Calculation of the Iron Loss.—Due to all the additional losses the calculation of the total iron loss is exceedingly difficult and is usually carried out by the help of the curves in Fig. 81, page 102, which curves are found to apply to induction motors as well as to D.-C. machines. The value so found is approximately correct for machines with open stator slots and partially closed rotor slots. When partially closed slots are used for both stator and rotor the value found from the curves is about 50 per cent. too large. Where the number of slots is larger in the rotor than in the stator, and where the stator slots are open so that the pulsation loss must be large, the value of iron loss found from the curve will probably be too small.

259. Calculation of Magnetizing Current, Core Loss and Bearing Friction.

A 50-h. p., 440-volt, three-phase, 60-cycle, 900-syn. r.p.m. induction motor is built as follows:

	Stator	Rotor
External diameter	25 in.	18.94 in.
Internal diameter	19 in.	15.5 in.
Frame length	6.375 in.	6.375 in.
Vent ducts	1–3/8 in.	1–3/8 in.
Gross iron	6 in.	6 in.
Slots, number	96	79
Slots, size	0.32 in. $\times$ 1.5 in.	0.45 in. $\times$ 0.4 in.
Cond. per slot, number	6	1
Cond. per slot, size	0.14 in. $\times$ 0.2 in.	0.4 in. $\times$ 0.35 in.
Connection	*Y*	Squirrel cage
Air-gap clearance	0 .03 in.	

A stator and rotor slot are shown to scale in Fig. 242.

The voltage per phase $=\frac{440}{1.73}=254$ since the connection is *Y*.

The flux per pole is found from the formula

$$E=2.22\ kZ\phi_a f\ 10^{-8}, \text{ see page 352,}$$

therefore $254=2.22\times 0.96\times\left(6\times\frac{96}{3}\right)\times\phi_a\times 60\times 10^{-8}$

from which $\phi_a=1{,}040{,}000$.

C_1 the stator Carter coefficient is found as follows:

$$\frac{s}{\delta}=10.6$$

$f=0.33$ from Fig. 40, page 44

$$C_1=\frac{0.302+0.32}{0.302+0.33\times0.32}=1.52$$

C_2 the rotor Carter coefficient is found in the same way

$$\frac{s}{\delta}=2.3$$

$$f=0.7$$

$$C_2=\frac{0.68+0.07}{0.68+0.7\times0.07}=1.03$$

The pole-pitch $=\frac{\pi\times19}{8}=7.48$ in.

The magnetizing current $I_o=\frac{1}{0.87\text{ (cond. per pole)}}\times\frac{\phi_a}{\tau L_g}\times\delta\ C$

$$=\frac{1}{0.87\ (6\times12)}\times\frac{1040000}{7.48\times6.0}\times0.03\times1.52\times1.03$$

$=17.6$ amperes for air-gap alone

this has to be increased 20 per cent. to allow for the iron part of the magnetic circuit, therefore, the magnetizing current $=17.6\times1.2=21$ amperes.

The Flux Densities.—Fig. 239 shows the flux distribution in a four-pole induction motor.

FIG. 239.—Flux distribution in a four-pole induction motor.

The average stator tooth density $=\frac{\text{flux per pole}}{\text{tooth area per pole}}$ and the maximum tooth density, namely, that at point A

$=$ average tooth density $\times\frac{\pi}{2}$

$$= \frac{\text{flux per pole}}{\text{tooth area per pole}} \times \frac{\pi}{2}.$$

The tooth area per pole = teeth per pole $\times t \times$ net iron
$= 12 \times 0.302 \times 5.4$
$= 19.6$ sq. in.

The maximum tooth density $= \frac{1040000}{19.6} \times \frac{\pi}{2}$
$= 83{,}000$ lines per square inch.

The core area = the core depth $\times$ net iron
$= 1.5 \times 5.4$
$= 8.1$ sq. in.

The maximum core density = the core density at B, Fig. 239
$= \frac{\text{flux per pole}}{2 \times \text{core area}}$
$= \frac{1040000}{2 \times 8.1}$
$= 64{,}000$ lines per square inch.

The total weight of stator teeth = 76 lb.
The flux density in stator teeth = 83,000 lines per square inch.
The corresponding loss per pound = 9 watts from Fig. 81, page 102.
Therefore, the total stator tooth loss = 680 watts.
This result is slightly pessimistic, because the maximum tooth density is 83,000 lines per square inch only at the tip of the tooth and is less than that further back.

The total weight of stator core = 166 lb.
The maximum core density = 64,000 lines per square inch.
The corresponding loss per pound = 6 watts from Fig. 81.
Therefore, the total loss in the core = 1000 watts.
The total no-load iron loss = tooth loss + loss in core
= 1680 watts.

The size of bearings is 3 in. $\times$ 9 in.

The loss in each bearing $= 0.81\, dl \left(\frac{V_b}{100}\right)^{3/2}$ watts, see page 355, where V_b, the rubbing velocity at 900 r.p.m. = 700 ft. per minute.
Therefore, the loss in two bearings $= 2 \times 0.81 \times 3 \times 9 \times (7)^{\frac{3}{2}}$
= 810 watts.

CHAPTER XXXIII

LEAKAGE REACTANCE

260. Necessity for an Accurate Formula.—To calculate the reactance voltage of the coils undergoing commutation in a D.-C. machine an approximate formula is used. A more accurate formula is of little extra value because commutation depends on so many other things such as brush contacts, shape of pole tips, etc., about the effect of which comparatively little is known.

In the case of an alternator 20 per cent. error in the calculation of the armature reactance has comparatively little effect on the value of the regulation determined by calculation since the reactance drop is only a part of the total voltage drop, the remainder being due principally to armature reaction.

In the case of an induction motor, however, the maximum current I_d, and therefore the starting torque and the overload capacity, depend entirely on the leakage reactance, as shown in formula 43, page 337, so that it is necessary to develop a formula which will give this reactance accurately.

261. The Leakage Fields.—The leakage in an induction motor consists of the end connection and the slot leakage, which are also found in alternators, and the leakage ϕ_z, Fig. 240, which links the conductors and passes zig-zag along the air-gap.

Except for the end-connection leakage these fluxes have no separate existence but form part of the total flux in the machine; since, however, the magnetic circuit of an induction motor is not saturated, their effect may be considered separately.

262. Reactance Formula.—If

ϕ_e = the lines of force that circle 1 in. length of the belt of end connections for each ampere conductor in that belt.

ϕ_s = the lines of force that cross the slots and circle 1 in. length of the phase belt of conductors for each ampere conductor in that belt.

ϕ_z = the lines of force that zig-zag along the air-gap and circle 1 in. length of the phase belt of conductors for each ampere conductor in that belt.

then, just as in the case of the alternator, the stator reactance per phase in ohms

$$=2\pi fb^2c^2p[\phi_e L_e+(\phi_s+\phi_z)L_g]10^{-8} \text{ for a chain winding}$$

$$=2\pi fb^2c^2p[\frac{\phi_e L_e}{2}+(\phi_s+\phi_z)L_g]10^{-8} \text{ for a double-layer winding;}$$

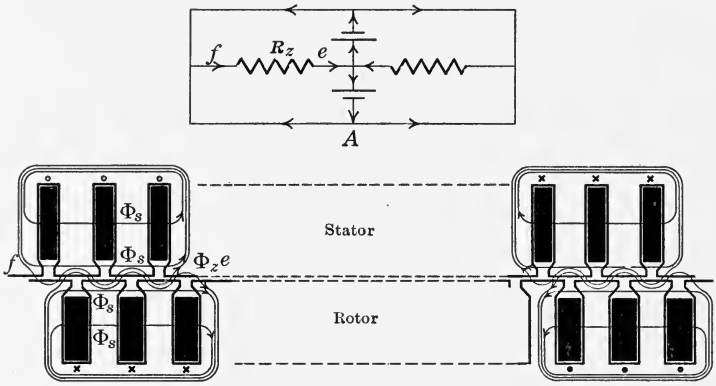


FIG. 240.—The leakage fields.

because of its lower reactance the double-layer winding is largely used for induction-motor work.

263. Zig-zag Reactance.[1]—Fig. 240 shows part of the winding of one phase of the stator and the corresponding part of the

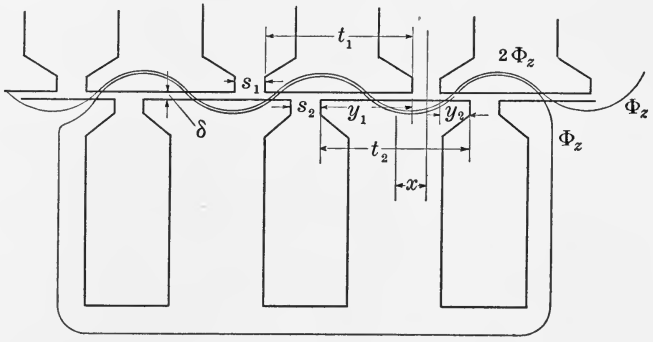


FIG. 241.—Zig-zag leakage.

rotor winding of a three-phase induction motor. It was shown in Art. 237, page 332, that the m.m.f. of the rotor is opposed to and practically equal to that of the stator, so that the direction of the currents in the conductors is as shown by the crosses and

[1] Adams, *Trans.* of A. I. E. E., Vol. 24, page 327.

dots at the bottom of the slots, and a peculiar magnetic circuit is produced of which the electrical equivalent is shown in diagram A, Fig. 240.

The m.m.f. between e and $f = bci$ ampere-turns, therefore the flux along one path $R_z = \dfrac{3.2bci}{R_z}$

where R_z is the reluctance of one of the zig-zag paths.

In order to find R_z it is assumed for simplicity that there are the same number of stator as of rotor slots so that $\lambda_1 = \lambda_2 = \lambda$; also that

$t_1 =$ the width of the stator tooth and its magnetic fringe

$t_2 =$ the width of the rotor tooth and its magnetic fringe

so that $\dfrac{\lambda_1}{t_1} = C_1$, the Carter coefficient for the stator

$\dfrac{\lambda_2}{t_2} = C_2$, the Carter coefficient for the rotor

y_1 Fig. 241 $= \dfrac{t_2}{2} + x - \dfrac{s_1}{2} = m + x$

$y_2 = \dfrac{t_2}{2} - x - \dfrac{s_1}{2} = m - x$

then $\dfrac{1}{R_z} = \dfrac{L_g}{c\left(\dfrac{\delta}{y_1} + \dfrac{\delta}{y_2}\right)}$

$= \dfrac{L_g}{c\left(\dfrac{\delta}{m+x} + \dfrac{\delta}{m-x}\right)}$

$= L_g \dfrac{m^2 - x^2}{2cm\delta}$

this value varies over the range of half a slot pitch between a maximum when x is zero and zero when $x = \dfrac{t_2}{2} - \dfrac{s_1}{2} = m$

and the average value of $\dfrac{1}{R_z} = \dfrac{2}{\lambda}\int_o^m L_g \dfrac{(m^2 - x^2)}{2cm\delta}\, dx$

$= \dfrac{2}{\lambda}\left(\dfrac{2}{3} \times \dfrac{L_g m^2}{2c\delta}\right)$

$= \dfrac{4 L_g}{6c\delta\lambda}\left(\dfrac{t_2}{2} - \dfrac{s_1}{2}\right)^2$

$= \dfrac{L_g}{6c\delta\lambda}(t_2 - \lambda + t_1)^2$

$= \dfrac{L_g \lambda}{6c\delta}\left(\dfrac{t_2}{\lambda} - 1 + \dfrac{t_1}{\lambda}\right)^2$

$$=\frac{L_g\lambda}{6\,c\delta}\left(\frac{1}{C_1}+\frac{1}{C_2}-1\right)^2$$

herefore, the leakage flux along one path R_z

$$=3.2\,bci\frac{L_g\lambda}{6\,c\delta}\left(\frac{1}{C_1}+\frac{1}{C_2}-1\right)^2$$

and ϕ_z the lines per ampere conductor per 1 in. length of stator or rotor

$$=3.2\frac{\lambda}{6\,c\delta}\left(\frac{1}{C_1}+\frac{1}{C_2}-1\right)^2\times 1/2$$

$$=0.26\frac{\lambda}{c\delta}\left(\frac{1}{C_1}+\frac{1}{C_2}-1\right)^2$$

since half the lines link the stator coils and the other half link the rotor coils.

264. Final Formula for Reactance.—For double-layer windings the stator reactance per phase

$$=2\pi f_1 b_1{}^2 c_1{}^2 p\left[\frac{\phi_e L_e}{2}+(\phi_s+\phi_z)L_g\right]10^{-8}$$

the rotor reactance per phase

$$=2\pi f_2 b_2{}^2 c_2{}^2 p\left[\frac{\phi_e L_e}{2}+(\phi_s+\phi_z)L_g\right]10^{-8}$$

the maximum current per phase

$$=I_d=\frac{\text{volts per phase}}{X_1+X_2\left(\frac{b_1c_1}{b_2c_2}\right)^2\frac{n_1}{n_2}}$$

$$=\frac{E}{X_{eq}}$$

where X_{eq} is the equivalent reactance per phase and

$$=2\pi f_1 b_1{}^2 c_1{}^2 p^2 n_1\left[\frac{\phi_e L_e}{2n_1p}+\left(\frac{\phi_s+\phi_z}{n_1p}\right)L_g+\frac{\phi_e L_e}{2n_2p}+\left(\frac{\phi_s+\phi_z}{n_2p}\right)L_g\right]10^{-8}$$

$$=2\pi f_1 b_1{}^2 c_1{}^2 p^2 n_1\left[\frac{\phi_e L_e}{2n_1p}+\left(\frac{\phi_s+\phi_z}{n_1p}+\frac{\phi_s+\phi_z}{n_2p}\right)L_g\right]10^{-8} \qquad (45)$$

where $\dfrac{\phi_s+\phi_z}{n_1p}=\dfrac{1}{n_1c_1p}\left[3.2\left(\dfrac{d_1}{3s}+\dfrac{d_2}{s}+\dfrac{2d_3}{s+w}+\dfrac{d_4}{w}\right)+0.26\,\dfrac{\lambda_1}{\delta}\left(\dfrac{1}{C_1}+\dfrac{1}{C_2}-1\right)^2\right]$

and $\dfrac{\phi_s+\phi_z}{n_2p}=\dfrac{1}{n_2c_2p}\left[3.2\left(\dfrac{d_1}{3s}+\dfrac{d_2}{s}+\dfrac{2d_3}{s+w}+\dfrac{d_4}{w}\right)+0.26\,\dfrac{\lambda_2}{\delta}\left(\dfrac{1}{C_1}+\dfrac{1}{C_2}-1\right)^2\right]$

the end-connection reactance in formula 45 is the equivalent reactance of both rotor and stator. For squirrel-cage motors the end-connection reactance of the rotor may be neglected and the curve for $\frac{\phi_e L_e}{n}$ in Fig. 244 used directly. For wound-rotor motors on the other hand it might seem at first that this value should be doubled, but it must be remembered that the m.m.fs. of rotor and stator are opposing and the leakage flux has to get through the space V, Fig. 235, between the two layers of windings, so that the closer together these windings are the more restricted is the space V and the smaller the leakage flux; for ordinary motors it is satisfactory to use the value for $\frac{\phi_e L_e}{n}$ in Fig. 244 and increase it by 35 per cent.

Example of Calculation.—Find the maximum current I_d for the following machine.

Rating.—50 h. p., 440 volts, three-phase, 60 cycle, 900 syn. r.p.m. The construction of the machine is as follows:

	Stator	Rotor
External diameter	25 in.	18.94 in.
Internal diameter	19 in.	15.5 in.
Frame length	6.375 in.	6.375 in.
Vent ducts	1–3/8 in.	1–3/8 in.
Gross iron	6 in.	6 in.
Slots, number	96	79
Slots, size	0.32 in. × 1.5 in.	0.45 in. × 0.4 in.
Cond. per slot, number	6	1
Cond. per slot, size	0.14 in. × 0.2 in.	0.4 in. × 0.35 in.
Connection	Y	Squirrel cage
Winding	Double layer	
Air-gap clearance	0.03 in.	

A stator and a rotor slot are shown to scale in Fig. 242.

The calculation is carried out in the following way:

pole-pitch $= 7.5$ in.

$$\frac{\phi_e L_e}{2n} = 4.3 \text{ from Fig. 244}$$

$$\lambda_1 = 0.622$$

$$C_1 = 1.52$$

$$C_2 = 1.03$$

$$\frac{\phi_s + \phi_z}{n_1 p} = \frac{1}{96}\left[3.2\left(\frac{1.3}{3\times 0.32} + \frac{0.2}{0.32}\right) + 0.26\frac{0.622}{0.03}\left(\frac{1}{1.52} + \frac{1}{1.03} - 1\right)^2\right]$$

$$= \frac{1}{96}[6.3 + 2.1] \text{ for the stator}$$

$$\lambda_2 = 0.76$$

$$\frac{\phi_s+\phi_z}{n_2 p}=\frac{1}{79}\left[3.2\left(\frac{0.40}{3\times 0.45}+\frac{2\times 0.07}{0.45+0.07}+\frac{0.03}{0.07}\right)+0.26\,\frac{0.76}{0.03}\left(\frac{1}{1.52}+\frac{1}{1.03}-1\right)^2\right]$$

$$=\frac{1}{79}\,[3.2+2.6] \text{ for the rotor}$$

$$X_{eq}=2\times\pi\times 60\times 6^2\times 4^2\times 8^2\times 3\left[\frac{4.3}{8}+\left(\frac{8.4}{96}+\frac{5.8}{79}\right)\times 6\right]10^{-8}$$

$$=0.415(0.54+0.96)$$

$$=0.62 \text{ ohms}$$

The voltage per phase $=\frac{440}{1.73}=254$ since the connection is Y and the maximum current per phase $=\frac{254}{0.62}=410$ amperes.

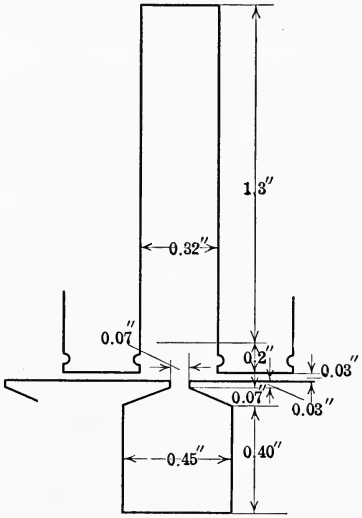


FIG. 242.—Stator and rotor slot for a 50-h.p 900-syn. r.p.m. induction motor.

265. Belt Leakage.[1]—In addition to the end connection, slot, and zig-zag leakage, there is another which enters into the reactance formula for a wound-rotor motor, namely, the belt leakage.

In developing the formula on page 363 it was assumed that the m.m.f. of the rotor was equal and opposite to that of the stator at every instant. This cannot be the case in a wound-rotor machine because, as shown in Fig. 221, the winding is arranged in phase belts and the stator and rotor belts sometimes overlap one another.

When the stator and rotor are in the relative position shown in diagram A, Fig. 243, the currents in the stator phase belts are

[1] Adams, *Trans.* of International Electrical Congress, 1904, Vol. 1, page 706.

exactly opposed by those in the rotor phase belts. The starts of the windings of the three phases are spaced 120 electrical degrees apart, and it may be seen from diagram A, that the currents in the three stator belts S_1, S_2 and S_3, and also in the three rotor belts R_1, R_2 and R_3 are out of phase with one another by 60 degrees; they are represented by vectors in diagram B.

When the stator and rotor are in the relative position shown in diagram C the currents in the belts have the phase relation shown in diagram D.

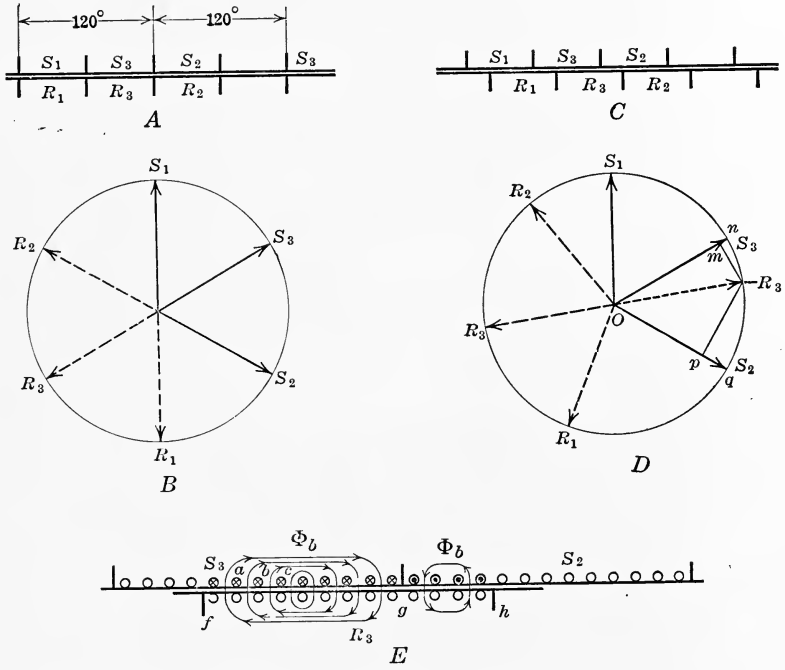


FIG. 243.—Belt leakage in 3-phase machines.

Part of diagram C is shown to a large scale in diagram E where it may be seen that belt R_3 overlaps belt S_3 by a distance fg and belt S_2 by a distance gh.

In the belt fg the current in the stator conductors is S_3, diagram D, and that in the rotor conductors is R_3, of which the component om opposes the stator current; mn, the remaining part of the stator current, is not opposed by an equivalent rotor current and is represented by crosses in diagram E.

In the belt gh the current in the stator conductors is S_2, dia-

gram D, and that in the rotor conductors is R_3, of which the component *op* opposes the stator current; *pq*, the remaining part of the stator current, is not opposed by an equivalent rotor current and is represented by dots in diagram E.

The currents represented by crosses and dots in diagram E set up the flux ϕ_b, which is in phase with the belt of conductors which it links and is therefore the same in effect as a leakage flux; it is called the belt leakage. ϕ_b varies through one cycle while the rotor moves, relative to the stator, through the distance of one-phase belt, and this belt flux is the cause of the variation in short-circuit current with constant applied voltage that is found in wound rotor motors, when the rotor is moved relative to the stator.

The belt flux per ampere conductor in the phase belt and per inch axial length of core depends on the reluctance of the belt-leakage path and is directly proportional to the pole-pitch, inversely proportional to the air-gap clearance and to the Carter fringing constant, and is the greater the smaller the number of phases and therefore the wider the phase belt, so that the average value of the belt leakage that circles 1 in. length of the phase belt per ampere conductor in that belt,

$$= \text{a const.} \times \frac{\tau}{\delta \times C}$$

and the average belt reactance per phase which must be added to formula 45, page 363, in the case of wound-rotor motors

$$= 2\pi f b^2 c^2 p^2 n L_g 10^{-8} \left(\text{a const.} \times \frac{\text{pole-pitch}}{p \times \delta \times C}\right). \qquad (46)$$

In addition to varying with the number of phases, the constant depends on the number of slots per phase per pole because, as shown in diagram E, the belt flux which links conductors *a* and *b* is smaller than if these conductors were concentrated in slot *c*. The value of the constant, which is found theoretically, is given in the following table:

Stator slots per pole	Two-phase motors	Three-phase motors
6	0.0052×3	0.00107×3
12	×1.5	×1.5
18	×1.25	×1.25
24	×1.15	×1.15
30	×1.10	×1.1
Infinity.	×1.0	×1.0

For motors which have two-phase stators and three-phase rotors a mean value should be used.

266. Approximate Values for the Leakage Reactance.—For a machine with a double-layer winding the equivalent reactance per phase

$$=2\pi f b_1^2 c_1^2 p n_1 \left[\frac{\phi_e L_e}{2n_1}+\left(\frac{\phi_s+\phi_z}{n_1}+\frac{\phi_s+\phi_z}{n_2}\right)L_g\right]10^{-8}$$ from formula 45, page 363, where

$\frac{\phi_e L_e}{2n_1}$ varies with the pole-pitch as shown in Fig. 244.

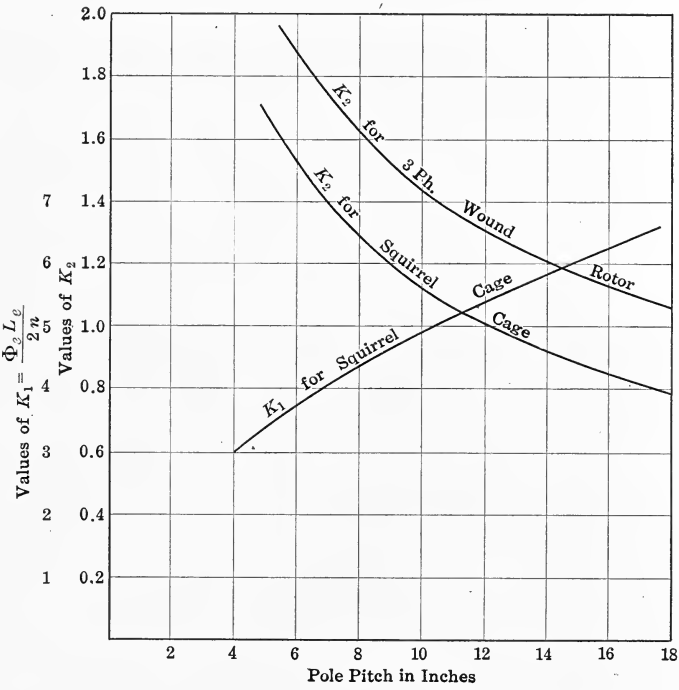


FIG. 244.—Leakage constants.

$\frac{\phi_s}{n}=\frac{3.2}{cn}\left(\frac{d_1}{3s}+\frac{d_2}{s}+\frac{2d_3}{s+w}+\frac{d_4}{w}\right)$, and since $c\times n\times s$, the total slot width per pole, is proportional to the pole-pitch, therefore $\frac{\phi_s}{n}$ is approximately inversely proportional to the pole-pitch. $\frac{\phi_z}{n}=\frac{1}{nc}\times 0.26\frac{\lambda}{\delta}\left(\frac{1}{C_1}+\frac{1}{C_2}-1\right)^2$; the ratio $\frac{\lambda}{\delta}$ is approximately

constant, the maximum value being limited by humming as shown in Art. 282, page 387, and that being the case the Carter fringing constants C_1 and C_2 are also approximately constant. The number of slots per pole $=n\times c$ is approximately proportional to the pole-pitch, therefore $\frac{\phi_z}{n}$ is approximately inversely proportional to pole-pitch.

The reactance per phase then

$$=2\pi f b^2 c^2 pn(K_1+K_2L_g)10^{-8} \qquad (47)$$

where K_1 and K_2 are plotted in Fig. 244 against pole-pitch, from the results of tests on a large number of machines with open

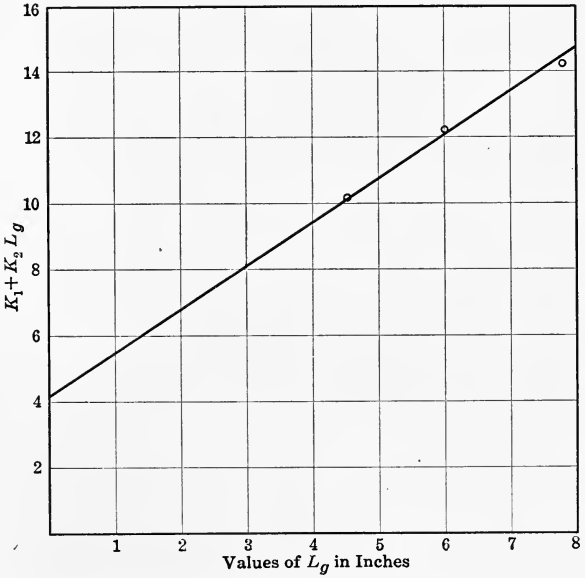


FIG. 245.—Variation of the leakage flux with frame length.

stator slots, partially closed rotor slots, and double-layer windings. The reason for the large value for wound-rotor motors compared with that for squirrel-cage machines is that in the former there is the additional end-connection leakage of the rotor, the belt leakage, and the larger rotor-slot leakage due to the deep slots required to accommodate the rotor conductors and insulation.

The method whereby these constants were determined may be understood from the following example.

Three machines were built as follows:

	A	B	C
Terminal voltage	440	440	440
Phases	3	3	3
Cycles	60	60	60
Poles	8	8	8
Stator internal diameter	19 in.	19 in.	19 in
Pole-pitch	7.5 in.	7.5 in.	7.5 in.
Gross iron	4.5 in.	6 in.	7.75 in.
Slots per pole	12	12	12
Conductors per slot	8	6	8
Connection	Y	Y	Δ
Maximum current I_d from test	270	400	575

The results are worked up as follows:

Voltage per phase	254	254	440
Current per phase	270	400	334
Reactance per phase	0.94	0.63	1.32
$K_1+K_2L_g$	10.2	12.2	14.2

These results are plotted against the values of L_g in Fig. 245 from which it may be seen that K_1, the part which is independent of the frame length, $=4.2$ while $K_2=1.30$; these values check closely with the curves in Fig. 244.

CHAPTER XXXIV

THE COPPER LOSSES

267.—Copper Losses in the Conductors.

If L_{b_1} = the length of a stator conductor in inches

I_{c_1} = the effective current per conductor

M_1 = the section of each conductor in cir. mils

then the resistance of one conductor $= \frac{L_{b_1}}{M_1}$ ohms;

the loss in one conductor $= \frac{L_{b_1} I^2_{c_1}}{M_1}$ watts

and the total stator copper loss $= \frac{\text{total cond.} \times L_{b_1} \times I^2_{c_1}}{M_1}$

$$= \frac{N_1 b_1 L_{b_1} I^2_{c_1}}{M_1} \text{ watts} \qquad (48)$$

Similarly the total rotor copper loss $= \frac{N_2 b_2 L_{b_2} I^2_{c_2}}{M_2}$ watts (49)

268. The Rotor End-connector Loss.—Fig. 246 shows the distribution of current in part of the rotor of a squirrel-cage induction motor.

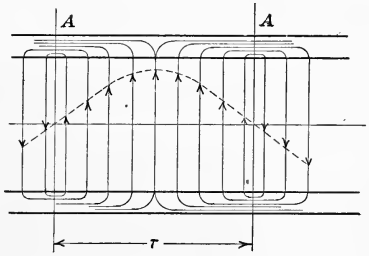


FIG. 246.—Current distribution in the rotor end connectors.

The effective current in each rotor bar $= I_{c_2}$

The average current in each rotor bar $= \frac{I_{c_2}}{1.11}$

The current in the ring at A = the maximum current in the ring

= average current per cond. $\times 1/2$ (cond. per pole)

$$= \frac{I_{c_2}}{1.11} \times \frac{N_2 b_2}{2p}.$$

The effective current in each ring $= \frac{I_{c2}}{1.11} \frac{N_2 b_2}{2p} \times \frac{1}{\sqrt{2}}$.

The resistance of each ring $= \frac{\pi D_r \times k_r}{A_r \times 1270000}$

where D_r = the mean diameter of the ring in inches

A_r = the area of the ring in square inches

$A_r \times 1{,}270{,}000$ = the area of the ring in cir. mils

$k_r = \frac{\text{the specific resistance of the ring material}}{\text{the specific resistance of copper}}$.

$$\text{The loss in two rings} = 2 \times \left(\frac{I_{c2} N_2 b_2}{1.11 \times 2p} \times \frac{1}{\sqrt{2}} \right)^2 \times \frac{\pi D_r k_r}{A_r \times 1270000}$$

$$= 0.5 \left(\frac{N_2 b_2 I_{c2}}{1000p} \right)^2 \times \frac{D_r k_r}{A_r} \text{ watts} \qquad (50)$$

In addition to the above copper losses there are eddy-current losses in the conductors due to the leakage flux which crosses the slot horizontally, as described in Art. 189, page 248. To prevent these losses from having a large value it is necessary to laminate the conductors horizontally.

When the rotor is running at full speed the eddy-current loss in the rotor bars is small, because the rotor frequency is low; even at standstill, when the rotor frequency is the same as that of the stator, the eddy-current loss in the rotor bars is still low because the conductors are not very deep.

Such eddy-current loss at standstill causes an increase in the starting torque without a sacrifice of the running efficiency, since the frequency is low and the eddy current loss negligible at full-load speed. A number of patents have been taken out on different methods of exaggerating this eddy-current loss in the rotor, but motors built under these patents have not come into general use.

Example of Copper Loss Calculation

A 50-h. p., 440-volt, 3-phase, 60-cycle, 900-syn. r.p.m. induction motor is built as follows:

	Stator	Rotor
External diameter	25 in.	18.94 in.
Internal diameter	19 in.	15.5 in.
Frame length	6.375 in.	6.375 in.
Slots, number	96	79

Slots, size	0.32 in. × 1.5 in.	0.45 in. × 0.4 in.
Cond. per slot, number	6	1
size	0.14 in. × 0.2 in.	0.4 in. × 0.35 in.
Connection	Y	Squirrel cage

Section of each end ring, A_r.. 0.75 sq. in.
Mean diameter of end rings..17.5 in.
Resistance of end-ring material is 5 times that of copper.

The circle diagram for the machine is shown in Fig. 247; it is required to draw in the copper loss lines.

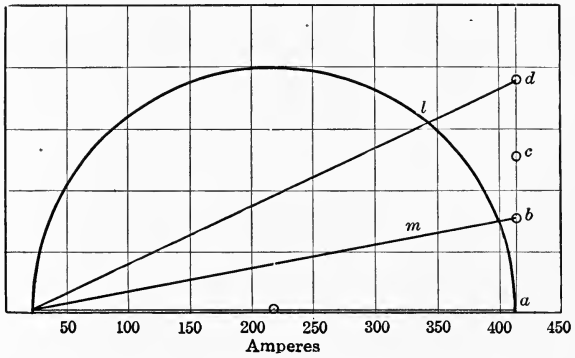


FIG. 247.—Calculated circle diagram for a 50-h.p., 440-volt, 3-phase, 60-cycle, 900-syn. r.p.m. induction motor.

L_{b1} the length of stator conductor = 20.5 in., from Fig. 84

L_{b2} the length of rotor conductor = frame length + 4 in.
= 10.5 in.

The maximum stator current per cond. = the maximum current in the line, since the connection is Y
= 415 amp.

The maximum rotor current per conductor $= (415 - 21) \times \frac{96 \times 6}{79}$
$= 394 \times \frac{96 \times 6}{79}$
= 2880 amp.

The maximum stator cond. loss $= \frac{96 \times 6 \times 20.5 \times 415^2}{0.14 \times 0.2 \times 1270000}$
From formula 48.
= 57 kw.

The maximum rotor cond. loss $= \frac{79 \times 1 \times 10.5 \times 2880^2}{0.4 \times 0.35 \times 1270000}$
From formula 49.
= 39 kw.

The maximum rotor ring loss $= 0.5 \left(\frac{79 \times 2880}{1000 \times 8}\right)^2 \times \frac{17.5 \times 5}{0.75}$
From formula 50.
= 47 kw.

Therefore in Fig. 247

$1.73 \times 440 \times ab = 57{,}000$

and $ab = 75$ amp.

$1.73 \times 440 \times bc = 39{,}000$

and $bc = 51$ amp.

$1.73 \times 440 \times cd = 47{,}000$

and $cd = 62$ amp.

From these figures the loss lines can readily be drawn in. The final circle compares closely with that found from test and plotted in Fig. 231.

CHAPTER XXXV

HEATING OF INDUCTION MOTORS

269. Heating and Cooling Curves.—The losses in an electrical machine are transformed into heat; part of this heat is dissipated by the machine and the remainder, being absorbed, causes the temperature of the machine to increase. The temperature becomes stationary when the heat absorption becomes zero, that is, when the point is reached where the rate at which heat is generated in the machine is equal to the rate at which it is dissipated by the machine.

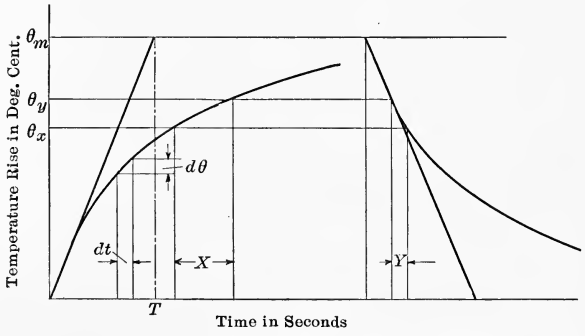


FIG. 248.—Heating and cooling curves.

The rate at which heat is dissipated by any machine depends on θ, the difference between the temperature of the machine and that of the surrounding air. During the first interval after a machine has been started up, θ is small, very little of the generated heat is dissipated, therefore a large part is absorbed and the temperature rises rapidly. As the temperature increases, that part of the heat which is dissipated increases, therefore the part which is absorbed decreases and the temperature rises more slowly. The relation between temperature rise and time is shown in Fig. 248. The equation to this curve is derived as follows:

In a given machine let $d\theta$ be the increase in temperature in the time dt.

The heat generated during this time $= Q \times dt$ lb. calories

where $Q = 0.53$(kw. loss)

The heat absorbed by the machine $= W \times s \times d\theta$ lb. calories

where W is the weight of the active part of the machine in pounds and s, its specific heat $= 0.1$ approximately

The heat dissipated $= A(a + bV)\theta \times dt$ lb. calories

where A is the radiating surface of the machine

V is the peripheral velocity

a and b are constants.

The heat generated = the heat dissipated + the heat absorbed

or

$$Q dt = A(a + bV)\theta dt + W s d\theta$$

and the temperature rise is a maximum and $= \theta_m$ when the heat absorbed is zero or when

$$Q dt = A(a + bV)\theta_m dt$$

therefore

$$A(a + bV)\theta_m dt = A(a + bV)\theta dt + W s d\theta$$

and

$$\int_o^t dt = \int_o^\theta \frac{W s d\theta}{A(a + bV)(\theta_m - \theta)}$$

from which

$$t = -\frac{Ws}{A(a + bV)}(\log_e(\theta_m - \theta) - \log_e \theta_m,)$$

$$= \frac{Ws}{A(a + bV)} \log_e \frac{\theta_m}{\theta_m - \theta}$$

and

$$\frac{\theta_m}{\theta_m - \theta} = e^{\frac{t(A(a+bV))}{Ws}}$$

therefore

$$\theta = \theta_m \left(1 - e^{\frac{-t(A(a+bV))}{Ws}}\right)$$

$$= \theta_m \left(1 - e^{-\frac{t}{T}}\right) \tag{51}$$

where

$$T = \frac{Ws}{A(a + bV)}$$

$$= \frac{Ws\theta_m}{A(a + bV)\theta_m}$$

$$= \frac{Ws\theta_m}{Q}$$

or

$$QT = Ws\theta_m \tag{52}$$

Therefore T is the time that would be taken to raise the temperature of the machine θ_m deg. if all the heat were absorbed.

The cooling curve is the reciprocal of the heating curve and its equation is derived as follows:
if the temperature falls $d\theta$ deg. in dt seconds then
the heat dissipated $= -W \times s \times d\theta = A(a+bV)\theta \times dt$

therefore $$dt = -\frac{Ws}{A(a+bV)} \frac{d\theta}{\theta}$$

or $$\int_o^t dt = -\int_{\theta_m}^{\theta} T \frac{d\theta}{\theta}$$

and $$t = -T \log \frac{\theta}{\theta_m}$$

from which $$\theta = \theta_m e^{-\frac{t}{T}} \qquad (53)$$

If the motor is standing still while cooling the temperature drops much more slowly, as shown in Fig. 249.

Consider the following example: An induction motor is run at full load and the final temperature rise $=45°$ C.
The current density in the stator cond. $=480$ cir. mils per ampere
The iron loss at no-load and normal voltage $=1000$ watts
The weight of iron in the stator $=115$ lb.
The iron loss per pound $=8.7$ watts
It is required to find the time constant T.

The resistance of a copper wire L in. long and M cir. mils section

$$= \frac{L}{M} \text{ ohms.}$$

The loss in this wire due to a current I

$$= \frac{LI^2}{M} \text{ watts}$$

$$= \frac{LI^2}{M} \times 5.3 \times 10^{-4} \text{ lb. calories per second.}$$

The weight of this wire $= L \times M \times 2.5 \times 10^{-7}$ lb.
The specific heat of copper $=0.09$.

Since $$QT = Ws\theta_m$$

therefore $$\frac{LI^2}{M} \times 5.3 \times 10^{-4} \times T = L \times M \times 2.5 \times 10^{-7} \times 0.09 \times \theta_m$$

and $$\frac{\theta_m}{T} = \frac{2.3 \times 10^4}{(\text{cir. mils per amp.})^2} \text{ deg. Cent. rise per second.}$$

In the given problem $M=480$ and $\theta_m=45°$ C.
therefore T for the coils $=450$ seconds.

Consider now the iron loss:

The loss per pound of iron $=P$ watts

$=P\times 5.3\times 10^{-4}$ lb. calories per second

The specific heat of iron $=0.1$ approximately

Since $QT=Ws\theta_m$

therefore $P\times 5.3\times 10^{-4}\times T=1\times 0.1\times\theta_m$

and $$\frac{\theta_m}{T}=\frac{P}{190}$$ deg. Cent. rise per second.

In the given problem $P=8.7$ watts per pound and $\theta_m=45°$ C.
therefore T for the iron of the stator $=1000$ seconds.

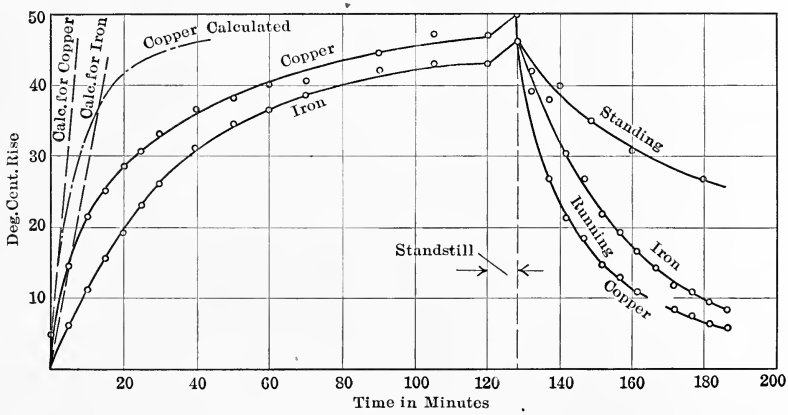


FIG. 249.—Heating and cooling curves.

For the value of T found for the copper, and for $\theta_m=47°$ C., the relation between temperature and time is plotted in Fig. 249, and it will be seen that the calculated curve differs considerably from that found from test.

The actual temperature rise in a given time is less than that calculated because:—

a. It has been assumed that all the iron loss is in the stator whereas a portion of it which cannot readily be separated out is due to rotor pulsation loss.

b. Part of the heat developed in the active part of the machine is conducted to and absorbed by the frame.

c. The temperature of the copper rises more rapidly than that of the iron and there is a transfer of heat which tends to bring them to the same temperature.

d. The thermal capacity of the insulation has been neglected.

270. Time to Reach the Final Temperature.—Due to the chances of error pointed out above it is difficult to predetermine the rate of increase of temperature.

It may be seen from formula 51 that the larger the value of T, which is called the time constant, the smaller is the rate of increase of temperature. T is the time that would be taken to raise the temperature of the machine to its final value if all the heat were absorbed, so that the lower the copper and iron densities the larger the value of T and the smaller the rate of increase of temperature.

Slow-speed machines have poor ventilation and therefore low copper densities so that for such machines the rate of increase of temperature is comparatively small.

Low-frequency machines, in which the flux density in the core is limited by permeability rather than by the iron loss, have the loss per pound of iron low and the rate of increase of temperature comparatively small.

The current density in the field coils of D.-C. machines is usually of the order of 1200 cir. mils per ampere so that such coils heat up slowly.

271. Intermittent Ratings.—Suppose that a motor is operating on a continuous cycle, X seconds loaded and Y seconds without load, the final temperature will vary, during each cycle, between θ_x and θ_y, Fig. 248, where these temperatures are such that the temperature increase in time X is equal to the temperature decrease in time Y. Under such conditions of service therefore θ_y, the highest temperature, is lower than θ_m, the maximum temperature which would be obtained on continuous operation under load. For such service, therefore, a motor may have higher copper and iron densities than it would have if designed for the same load but for continuous operation.

272 Heating of Squirrel-cage Motors at Starting.—The loss in a squirrel-cage rotor at starting is very large and equals the full-load output of the machine $\times$ the per cent. of full-load torque required to start the load. Thus, if a 20 h. p. squirrel-cage motor has to develop full-load torque at starting the rotor loss under these conditions must be 20 h. p. This loss, in the form of heat,

has to be absorbed by the rotor copper, and the temperature rises rapidly unless there is sufficient body of copper to absorb this heat during the starting period.

The stator current also is large at starting; an average squirrel-cage motor with normal voltage applied to the terminals develops about 1.5 times full-load torque and takes about 5.5 times full-load current; when started on reduced voltage it develops full-load torque with about 4.5 times full-load current, since the starting torque is proportional to the rotor loss and therefore to the square of the current.

As shown in Art. 269, the temperature rise when the heat is all absorbed may be found from the following formulæ.

$$\text{Degrees Cent. rise per sec.} = \frac{2.3 \times 10^4}{(\text{cir. mils per amp.})^2} \text{ for copper wire}$$

$$= \frac{\text{watts per lb.}}{190} \text{ for iron bodies}$$

$$= \frac{\text{watts per lb.}}{170} \text{ for copper bodies.}$$

An average value for the cir. mils per ampere at full-load is 500, for both rotor and stator. The starting current in the motor for full-load torque is about 4.5 times full-load current, the corresponding value of cir. mils per ampere is 110, and the temperature rise 1.9° C. per second.

The proper weight of end connector to be used in any particular case depends on the starting torque required and the time needed to bring the motor up to full speed. For motors from 5 h. p. to 100 h. p. the loss per pound of end connector is generally taken about 1 kw. and, corresponding to this loss, the temperature rise of the end connectors

$$= \frac{1000}{170}$$

$$= 6° \text{ C. per second approximately.}$$

273. Stator Heating.—The temperature rise of the stator of an induction motor is fixed in the same way as that of the armature of a D.-C. machine.

For induction motors built with iron of the same grade as used in D.-C. machines and of a thickness of 0.014 in., so that the iron loss curves are as shown in Fig. 81, the following flux densities

may be used for a machine whose temperature rise at normal load must not exceed 40° C.

Frequency	Maximum tooth density in lines per sq. in.	Maximum core density in lines per sq. in.
60 cycles	85,000	65,000
25 cycles	100,000	85,000

These figures represent standard practice for machines with open stator slots and partially closed rotor slots when the rotor slot is designed as pointed out in Art. 257, page 356, for minimum pulsation loss. When both stator and rotor slots are partially closed these densities may safely be increased 15 per cent.

The end connection heating is limited by keeping the value of the ratio $\frac{\text{amp. cond. per inch}}{\text{cir. mils per amp.}}$ below that given in Fig. 250, which curve applies to wound-rotor motors, and to squirrel-cage motors with less than 4 per cent. slip, of the type shown in Figs. 232 and 235.

274. Rotor Heating.—At full-load and normal speed the frequency of the flux in the rotor is very low so that comparatively high flux densities may be used. The rotor tooth density is not carried above 120,000 lines per square inch if possible, because, for greater values, the m.m.f. required to send the flux through these teeth becomes large and causes the power factor to be low; for the same reason the rotor core density is seldom carried above the point of saturation, namely about 85,000 lines per square inch. So far as heating is concerned the rotor core loss is so small that it may be neglected.

The rotor copper loss is limited by making the ratio $\frac{\text{amp. cond. per inch}}{\text{cir. mils per amp.}}$ the same for the rotor as for the stator, and then, if the motor is of the squirrel-cage type, any additional rotor resistance that is required to give the necessary starting torque is put in the end connectors, which are easily cooled.

Since the rotor ventilation is better than that of the stator, because it is revolving, the rotor temperature is generally lower than that of the stator and is not calculated.

275. Effect of Rotor Loss on Stator Heating.—The heating of the rotor causes the air which blows on the stator to be hotter than the surrounding air. The heating curve in Fig. 250 applies to squirrel-cage motors with about 4 per cent. slip or 4 per cent.

rotor loss, and for the type of construction shown in Figs. 232 and 235. When greater rotor loss than this is required, as in the case of squirrel-cage motors for operating certain classes of cement machinery, then the air blowing on the stator will be hotter than usual and the stator temperature will be higher than the value got from Fig. 250. On account of this extra rotor loss such high torque squirrel-cage motors are built on frames that are about 20 per cent. larger than standard.

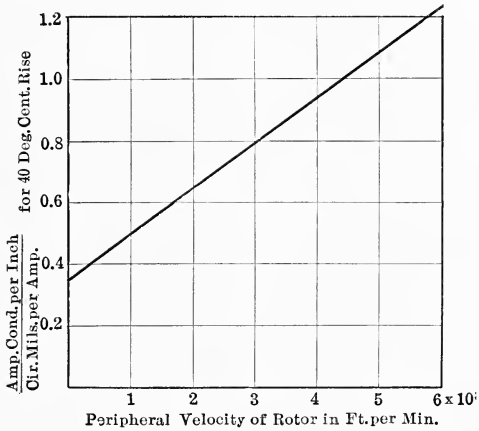


FIG. 250.—Heating of the stator end connections.

276. Effect of Construction on Heating.—Fig. 251 shows the relative proportions of a 25-cycle and of a 60-cycle motor of the same horse-power and speed and therefore with the same size of bearings. In the case of the 25-cycle machine the bearing blocks up the air inlet, and in such a case the temperature rise should be figured conservatively until the first machine has been tested and accurate data obtained. Another objection to the 25-cycle motor is that the coils stick out a considerable distance from the iron of the core and there is a tendency for the cooling air to circulate as shown by the arrows and not to pass out of the machine; in such a case it may be necessary to put in a baffle, as indicated by the dotted line A, to deflect the air stream in the proper direction.

277. Heating of Enclosed Motors.—Experiment shows that in the case of a totally enclosed motor the temperature rise of the coils and core of the machine is proportional to the total loss

(neglecting bearing friction) and is independent of the distribution of this loss, is inversely proportional to the external radiating surface and depends on the peripheral velocity of the rotor in the way shown in Fig. 94.

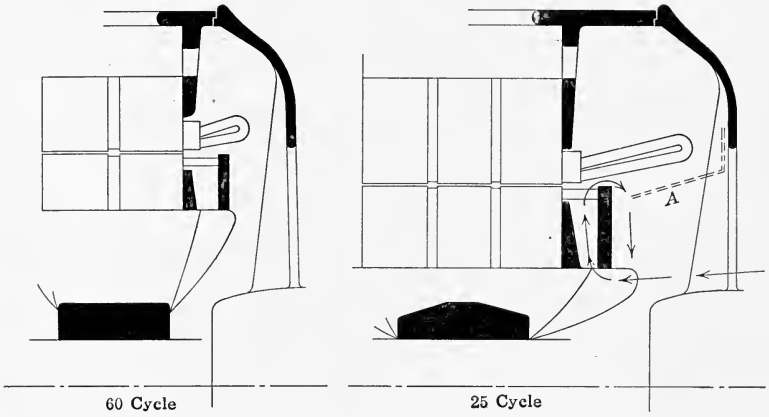


FIG. 251.—Motors built for the same output and speed but for different frequencies.

278. Heating of Semi-enclosed Motors.—When the openings in the frame of a motor are blocked up with perforated sheet metal the machine is said to be semi-enclosed. The perforated metal acts as a baffle, prevents the free circulation of air through the machine, and causes the temperature to rise, on an average, about 25 per cent. higher than it would for the same machine operating at the same load but as an open motor.

The effect of enclosing a motor may be seen from the following table which gives the results of a series of heat runs made on a motor similar to that shown in Fig. 232 and constructed as follows:

Internal dia. of stator	20 in.
Frame length	5.5 in.
Peripheral velocity of rotor	4730 ft. per minute
Stator copper loss	0.77 kw.
Rotor copper loss	0.9 kw.
Iron loss	0.77 kw.
Total loss	2.44 kw.
External radiating surface	3260 sq. in.

Parts of motor	Open motor	Opening in housings closed with perforated sheet metal $\frac{3}{8}$-in. holes on $\frac{5}{8}$-in. centers	Openings in housings closed with sheet metal	Openings in housings closed with sheet metal
		Yoke openings open	Yoke openings open	Yoke openings closed
Stator coils	18.5	22	46	74
Stator iron	18.5	20	48	71
Rotor cond	15	21	46	71
Rotor ring	14	21	44	69
Rotor iron	14	20	41	61
Oil in bearings	14	24	39	57
Outside of yoke	.....	.....	30	.....

The temperature rise is given in deg. cent.

The first machine of a new type that is built is generally very liberally designed, the copper and iron densities are low. If this machine runs cool in test it may get a higher rating than that for which it was originally built, and, based on the results of the tests on this machine, the next is designed more closely. Electrical design is not an exact science but is always changing to suit the requirements of the customer, the accumulating experience of the designer, and the competition of other manufacturers.

CHAPTER XXXVI

NOISE AND DEAD POINTS IN INDUCTION MOTORS

279. Noise due to Windage.—Fig. 252 shows the standard construction used for induction motors. As the rotor revolves air currents are set up in the direction shown by the arrows, and it may be seen from diagram A that a puff of air will pass through the stator, between the stator coils, every time a rotor tooth comes opposite a stator tooth, so that the machine acts as a siren.

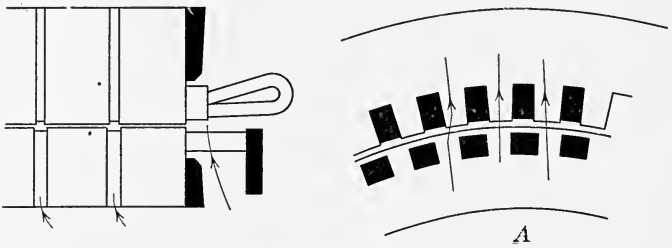


FIG. 252.—Windage in induction motors.

The intensity of the note emitted depends on the peripheral velocity of the rotor, while its pitch or frequency

$$= \text{the number of puffs per second}$$
$$= \text{the number of rotor slots} \times \text{revolutions per second}$$
$$= \frac{\text{peripheral vel. of rotor in ft. per min.}}{5 \times \text{rotor slot pitch in inches}} \qquad (54)$$

The higher the pitch of the note the more objectionable it becomes, and a note with a frequency greater than 1560 cycles per second, which is the high G of a soprano, is very objectionable if loud and long sustained. For a peripheral velocity of 8000 ft. per minute and a rotor slot pitch of 1 in., the frequency of the windage note is 1600 cycles per second.

Noise due to windage can be lowered in intensity by blocking up the rotor vent ducts; if the motor then runs hot due to poor ventilation, some other method of cooling must be adopted such as blowlng air across the external surface of the punchings,

or the motor may be totally enclosed and cooled by forced ventilation. The intensity of the note can be greatly reduced by staggering the vent ducts as shown in Fig. 253, and since the air-gap clearance is large in high-speed machines, which are the only ones that are noisy due to windage, the ventilation of such machines will not be seriously affected. It should also be noted that for high-speed machines a large number of narrow ducts will give quieter operation than a smaller number of wide ducts, because the velocity of the air through each duct will be reduced.

280. Noise due to Pulsations of the Main Flux.—In Fig. 237, page 356, A shows part of a machine which has a large number of rotor slots. The flux in a rotor tooth pulsates from a maximum when the tooth is in position x, to a minimum when the tooth is in position y, and the frequency of this pulsation is equal to

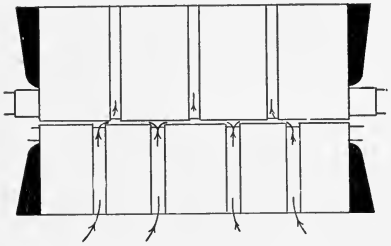


FIG. 253.—Motor with staggered vent ducts.

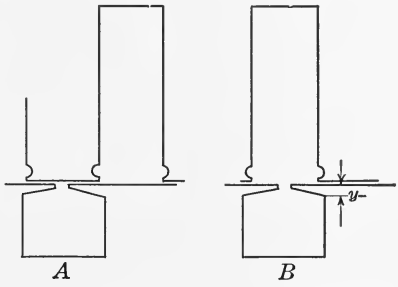


FIG. 254.—Variation of the force of magnetic attraction on the rotor tooth tips.

the number of stator teeth $\times$ revolutions per second. This pulsation of flux causes a noise which varies in intensity with the voltage. To minimize this noise the machine should be designed so as to have a minimum pulsation loss, the condition for which, as shown in Art. 257, page 356, is that the rotor tooth shall be equal to the stator slot pitch, and the rotor slit shall be narrow. The noise due to pulsation of the main flux may be minimized by stacking the rotor tightly.

281. Noise due to Vibration of the Rotor Tooth Tips.—Fig. 254 shows several of the slots of the stator and rotor of an induction motor. When the rotor tips are in the position shown at A there is a force of attraction between the stator tooth and the rotor tooth tip, while in position B this force is zero; the rotor tooth tip will therefore be set in vibration with a frequency equal

to the number of stator teeth $\times$ revolutions per second. This noise cannot be minimized by making the core tight and must be provided against by having the root y sufficiently thick to prevent bending and by the use of a moderate air-gap clearance. Noise due to this cause is rarely found in conservatively designed machines; it has been found in machines which are built with a small air-gap so as to lower the magnetizing current, and those built with a very thin rotor tooth tip so as to lower the reactance.

282. Noise due to Leakage Flux.—The principal cause of noise in induction motors is the variation in the reluctance of the zig-zag leakage path. When the stator and rotor slots are in the relative position shown in *B*, Fig. 254, the zig-zag leakage flux is a minimum, and when in the relative position shown in *A*, is a maximum, so that there is a pulsation of flux in the tooth tips and two notes are emitted which have frequencies equal to the number of rotor slots $\times$ revolutions per second and the number of stator slots $\times$ revolutions per second respectively.

To ensure that the noise thus produced will not be objectionable it is necessary to make the variation in the zig-zag leakage flux a minimum, and the most satisfactory way to do this is to make the zig-zag leakage as small as possible. This leakage flux is directly proportional to the ampere cond. per slot and inversely proportional to the air-gap clearance, and it has been found from experience that in order to prevent excessive noise up to 25 per cent. overload the ratio

$$\frac{\text{amp. cond. per slot at full-load}}{\text{air-gap clearance in inches}}$$

should not exceed 14×10^3 for machines with open stator and partially closed rotor slots, or 12×10^3 for machines with partially closed slots for both stator and rotor.

It is also found that, as the frequencies of the two notes produced by zig-zag leakage approach one another in value, the noise becomes more and more objectionable, and that, for even lower values of the ratio $\frac{\text{amp. cond. per slot at full-load}}{\text{air-gap clearance in inches}}$ than those given above, the noise will be objectionable if the number of rotor slots is within 20 per cent. of the number of stator slots.

The cause of the noise in any given case can readily be determined. Run the motor on normal voltage and no-load and, if it is noisy, the trouble is due to windage, pulsation of the main field, or weak rotor tooth tips; then open the circuit, and if the

motor is still noisy the trouble is due to windage. If the motor is quiet on no-load but noisy when loaded the trouble is due to the zig-zag leakage flux.

283. Dead Points at Starting.—It was shown in Art. 245, page 341, that the starting torque in synchronous horse-power is equal to the rotor loss at starting. If the rotor is not properly designed this torque may not all be available.

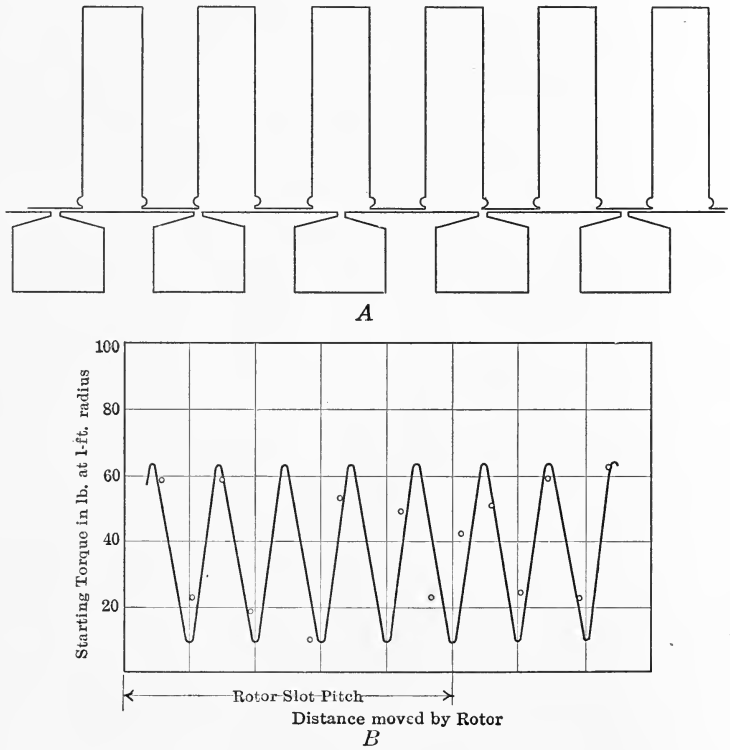


FIG. 255.—Variation in the starting torque.

In the case of a squirrel-cage motor at standstill, the applied voltage is low and the main flux therefore small, but the starting current is several times full-load current and therefore the zig-zag leakage flux is large. This flux is a maximum when the stator and rotor slots have the relative position shown in diagram A, Fig. 254, and a minimum when they have the position shown in *B*, so that there is a tendency for the rotor to lock in position *A*, the position of minimum reluctance. If the force tending to

rotate the rotor is less than that tending to hold the rotor locked, then the rotor will not start up.

A, Fig. 255, shows several slots of a machine which has five stator slots for every four rotor slots, so that when the available torque is a minimum every fourth rotor slot is in the locked position. Diagram B shows the result of a test made on this machine, the torque being measured by a brake for different positions of the rotor relative to the stator; it may be seen that there are five positions of minimum torque in the distance of a rotor slot pitch.

In order that the variation in starting torque may be a minimum it is necessary to make the number of locking points small. If the number of rotor slots is equal to the number of stator slots then, when the starting torque is a minimum, each rotor slot will be in the locked position; if there are four rotor to every five stator slots then the number of locking points will be one-fourth of the total number of rotor slots; if the number of rotor and of stator slots are prime to one another then only one rotor slot can be in the locked position at any instant and the best conditions for starting are obtained. It has been found by experience that the starting torque for a squirrel-cage motor will be practically constant, if not more than one-sixth of the total number of rotor slots are in the locked position at any instant.

Wound-rotor motors have a regular phase winding and, in order that this winding may be balanced, the number of rotor slots as well as the number of stator slots must be a multiple of the number of poles and of the number of phases; the ratio $\frac{\text{stator slots}}{\text{rotor slots}}$ must therefore $= \frac{\text{stator slots per phase per pole}}{\text{rotor slots per phase per pole}}$ and, since the number of rotor slots per phase per pole is often three or four, it might be expected that such machines would not have good starting torque because of dead points. This is not the case however because, at starting, wound-rotor motors have a large resistance in the rotor circuit, and full-load torque tending to cause rotation is obtained with full-load current, therefore the zig-zag leakage at starting is much smaller than in squirrel-cage machines and the force tending to cause locking is small.

If a wound-rotor motor be taken which has not more than 30 per cent. of the rotor slots in the locking position at any instant, and full voltage be applied to the stator while the rotor circuit is

open, then the main flux will have its normal value; it will be found that the rotor can readily be rotated by hand, which shows that dead points are not, as generally stated, due to variations in the reluctance of the air-gap to the main field. If now this same motor be short-circuited at the rotor terminals, so that it is equivalent to a squirrel-cage motor with low rotor resistance, it will be found impossible in most cases to get the motor to start up even without load, because of the locking effect of the leakage flux. If resistance be inserted in the rotor circuit it will be found that, as the rotor resistance increases, the variation in the starting torque becomes less and less, and that when this resistance is such that full-load torque is developed with normal voltage and full-load current, the variation in starting torque due to leakage flux is so small that it can be neglected.

CHAPTER XXXVII

PROCEDURE IN DESIGN

284. The Output Equation.

$$E = 2.22\, kZ\phi_a f 10^{-8} \text{ volts.} \quad \text{See page 352.}$$

$$= 2.22\, kZ(B_g \tau L_g)\left(\frac{p \times \text{r.p.m.}}{120}\right)10^{-8} \text{ volts}$$

$$= \frac{2.22\, k\, 10^{-8}}{120}\, Z B_g \pi D_a L_g \times \text{r.p.m.} \quad \text{volts}$$

and $$q = \frac{nZI}{\pi D_a}$$

therefore $$nEI = n\left[\left(\frac{2.22\, k\, 10^{-8}}{120}\right) Z B_g \pi D_a L_g \text{ r.p.m.}\right]\left(\frac{\pi D_a q}{nZ}\right)$$

and $$\text{h. p.} = \frac{nEI}{746}\cos\theta \times \eta$$

$$= 2.35 \times 10^{-12} B_g q \cos\theta \times \eta \times \text{r.p.m.} \times D_a^2 L_g$$

taking $k = 0.96$

from which $$D_a^2 L_g = \frac{\text{h. p.}}{\text{r.p.m.}} \frac{10^{12}}{2.35 \times B_g q \cos\theta \times \eta} \qquad (55)$$

The value of B_g, the apparent average gap density, is limited by the permissible value of B_t, the maximum stator tooth density, since

$$B_g = B_t \times \frac{L_n}{L_g} \times \frac{t_1}{\lambda_1} \times \frac{2}{\pi}$$

where the iron insulation factor $= \frac{L_n}{L_g} = 0.9$

the permissible value of B_t = 85,000 lines per square inch for 60 cycles

= 100,000 lines per square inch for 25 cycles

and $\frac{\lambda_1}{t_1} = 2.1$ approximately, being slightly larger for machines of small pole-pitch

therefore, for machines with a pole-pitch greater than 7 in.,

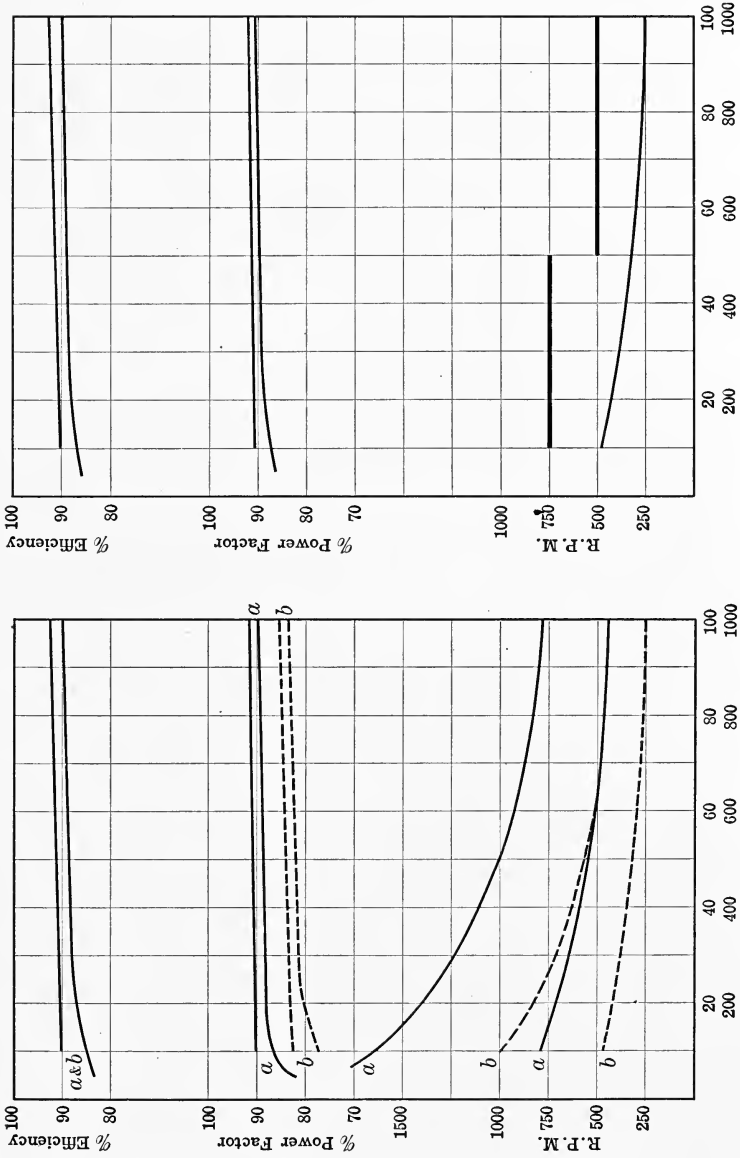


FIG. 256.—Power factor and efficiency curves for induction motors with open stator slots and partially closed rotor slots.

$B_g = 23{,}000$ lines per square inch for 60 cycles, approximately
$= 27{,}000$ lines per square inch for 25 cycles, approximately.

The Values of cos θ and η.—The values that may be expected from a line of 60-cycle motors with open stator slots and partially closed rotor slots are given in Fig. 256, diagram A, and corresponding curves for a line of 25-cycle motors are given in diagram B.

Two power factor curves *a* and *b* are given for the 60-cycle machines; these correspond to the two speed curves *a* and *b*. The power factor increases very slowly for speeds above *a* but drops very rapidly for speeds below *b*.

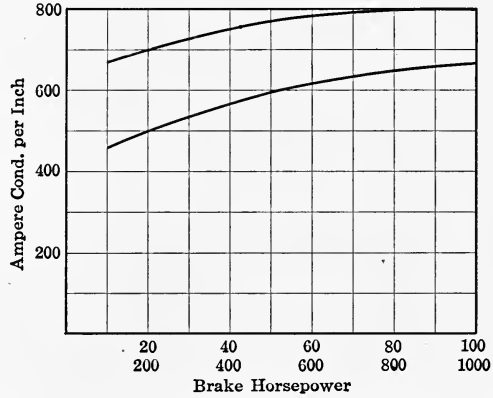


FIG. 257.—Curve to be used in preliminary design for frequencies between 25 and 60 cycles.

The power factor and efficiency can be improved by the use of partially closed slots for both stator and rotor.

285. The Relation between D_a and L_g.—There is no simple method whereby $D_a^2 L_g$ can be separated into its two components in such a way as to give the best machine, the only satisfactory method is to assume different sets of values, work out the design roughly for each case, and pick out that which will give good operation at a reasonable cost.

To simplify this work the following equations are developed.

I_o = the magnetizing current per phase

$$= \frac{1}{0.87 \text{ cond. per pole}} \times \frac{\phi_a}{\tau L_g} \times \delta \times C \times 1.2, \text{ page 354.}$$

$$= \frac{2.05}{\text{cond. per pole}} \times B_g \times \delta$$

taking C = 1.5, an average value for machines with open stator slots and partially closed rotor slots.

therefore $\frac{I_o}{I}$ = the per cent. magnetizing current

$$= \frac{2.05}{\text{cond. per pole} \times I} \times B_g \times \delta$$

$$= 2.05 \frac{B_g}{q} \times \frac{\delta}{\tau} \tag{56}$$

The minimum permissible air-gap clearance is fixed by mechanical considerations and should increase as the diameter, frame length and peripheral velocity increase; its value should not be smaller than that given by the following empirical formula:

$$\delta = 0.005 + 0.00035 D_a + 0.001 L_g + 0.003 V \tag{57}$$

where δ = the air-gap clearance in inches

D_a = the stator internal diameter in inches

L_g = the gross iron in the frame length in inches

V = the peripheral velocity of the rotor in 1000s of feet per minute.

The ratio $\frac{I_d}{I}$ is found as follows:

$$I_d = \frac{E}{X_{eq}}$$

where $E = 2.22 k Z \phi_a f 10^{-8}$ volts, formula 25, page 190.

$= 2.22 k Z (B_g \tau L_g) f 10^{-8}$ volts

and $X_{eq} = 2\pi f b^2 c^2 p n (K_1 + K_2 L_g) 10^{-8}$ ohms, formula 47, page 369.

$$= 2\pi f Z^2 n \left(\frac{K_1 + K_2 L_g}{p} \right) 10^{-8} \text{ ohms}$$

therefore $\frac{I_d}{I} = \frac{2.22 \times 0.96}{2\pi} \times \frac{B_g \tau L_g}{nZI} \times \frac{p}{(K_1 + K_2 L_g)}$

$$= 0.337 \times \frac{B_g}{q} \times \frac{L_g}{K_1 + K_2 L_g} \tag{58}$$

The Value of q.—From formulæ 55, 56, and 58 it may be seen that the larger the value of q the smaller the value of $D_a^2 L_g$, the smaller the per cent. magnetizing current, and the smaller the circle diameter and therefore the overload capacity. Since the copper heating depends on the ratio $\frac{\text{amp. cond. per inch}}{\text{cir. mils. per amp.}}$, the larger the value of q, the greater the amount of copper required to

keep the temperature rise within reasonable limits and therefore the deeper the slots and the larger the slot reactance. Fig. 257 gives average values of q for 25- and 60-cycle induction motors and this curve may be used for a first approximation.

286. Desirable Values for I_o and I_d.—Fig. 258 shows the circle diagram for a reasonably good induction motor:

The power factor at full-load	=90 per cent
The starting torque	=1.5 times full-load torque
The maximum torque	=2.7 times full-load torque
The maximum output	=2.2 times full-load

To obtain such characteristics the magnetizing current should not exceed one-third of full-load current, nor should the maximum current I_d be less than six times full-load current.

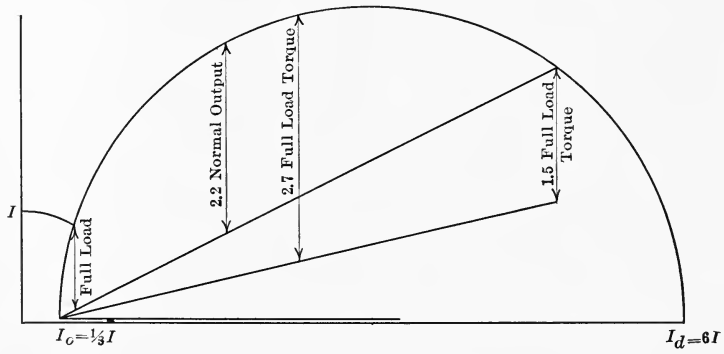


FIG. 258.—Circle diagram for an average induction motor.

287. Example of Preliminary Design.—To simplify the work the necessary formulæ are gathered together below.

$$D_a^2 L_g = \frac{\text{h. p.}}{\text{r.p.m.}} \cdot \frac{10^{12}}{2.35 B_g q \cos\theta \times \eta}$$

$$\frac{I_o}{I} = 2.05 \frac{B_g}{q} \times \frac{\delta}{\tau}$$

$$\frac{I_d}{I} = 0.337 \times \frac{B_g}{q} \times \frac{L_g}{K_1 + K_2 L_g}$$

where D_a = the internal diameter of the stator in inches
L_g = the axial length of the gross iron in inches
B_g is taken as 23,000 for 60 cycles
27,000 for 25 cycles
q is found from Fig. 257

cos θ and η are found from Fig. 256

$$\delta = 0.005 + 0.00035 D_a + 0.001 L_g + 0.003 V$$

where V = the peripheral velocity of the rotor in 1000s of feet per minute.

K_1 and $K_2 L_g$ are found by the use of the curves in Fig. 244.

The work is carried out in tabular form as shown below, where the figures are given for a 50 h. p., 60-cycle, 900 r.p.m. motor.

PRELIMINARY DESIGN SHEET

50 h. p., squirrel cage, 60 cycles, 900 r. p. m.

$B_g = 23{,}000$, $q = 600$, cos $\theta = 89$ per cent., $\eta = 89$ per cent., $D_a^2 L_g = 2150$

D_a	L_g	τ	V	δ	$K_1 + K_2 L_g$	$\frac{I_o}{I}$	$\frac{I_d}{I}$
15	9.5	5.9	3.5	0.03	3.7+14.6=18.3	0.4	6.7
17	7.5	6.7	4.0	0.03	4.0+10.8=14.8	0.35	6.5
19	6.0	7.5	4.5	0.031	4.3+ 8.1=12.4	0.32	6.2
21	5.0	8.2	4.9	0.032	4.5+ 6.4=10.9	0.31	5.9
23	4.0	9.0	5.4	0.033	4.7+ 4.8= 9.5	0.29	5.4

So far as operation is concerned the 19-in. diameter machine is probably the best all-round machine. The 15-in. machine has the largest magnetizing current and therefore the lowest power factor while the 23-in. machine has the smallest circle diameter and therefore the smallest overload capacity.

The shop conditions must be known before the costs of the above machines can be intelligently compared. The cost of yoke, spider and housings is greatest for the 23-in. machine, and the cost of assembling the cores is greatest for the 15-in. machine; the total cost is probably least for the 19-in. machine, but will not vary very much over the range of machines shown.

288. Detailed Design.—The work of completing the 50 h. p., 60-cycle, 900 r.p.m. design is carried out in tabular form for the 440-volt, three-phase rating as follows:

Stator design	
Poles $= \frac{f \times 120}{\text{r.p.m.}}$	$= 8$
Internal diameter of stator	= 19 in. from preliminary design
Gross iron	= 6 in. from preliminary design
Vent ducts	$= 1 - 0.375$ in.
Net iron = 0.9 × gross iron	= 5.4 in.
Pole-pitch	= 7.46 in.
Slots per pole	= 12, to be suitable for two and three phase

if chosen	$=6$, the machine would be noisy, see Art. 282
if chosen	$=18$, the slots would be very narrow.
Slot-pitch $=\frac{\text{pole-pitch}}{\text{slots per pole}}$	$=0.622$ in.
Slot width	$=0.311=$ half the slot pitch for a first approximation
Ampere conductors per inch	$=600$ from preliminary design
Ampere conductors per slot	$=374=$ Ampere cond. per inch $\times$ slot pitch
Full-load current	$=62$ amp. taking cos $\theta=0.89$ and $\eta=0.89$
Conductors per slot	$=6=\frac{\text{amp. cond. per slot}}{\text{full-load current}}$
Connection	$=Y$, because the current in each conductor was taken equal to the line current
$\frac{\text{Amp. cond. per inch}}{\text{Cir. mils per amp.}}$	$=1.0$ from Fig. 250
Circular mils per ampere	$=600$ required
Circular mils per conductor	$=600\times$ full-load current $=37{,}200$
Size of conductor	$=0.029$ sq. in.

Size of slot and section of conductor are worked out in tabular form thus:

0.311 in. = assumed slot width
0.064 in. = width of slot insulation, see page 203
0.04 in. = necessary clearance
0.207 in. = available width for copper and insulation on conductors

Use copper strip 0.2 in. wide insulated with double cotton covering and change the slot width to 0.32 for a second approximation, therefore the size of conductor = 0.14 in. $\times$ 0.2 in.

0.14 in. = depth of each conductor
0.015 in. = thickness of the cotton covering
0.155 in. = depth of each conductor and its insulation
0.465 in. = depth of three cond. and their insulation
0.084 in. = depth of slot insulation on each coil, see page 203.
0.549 in. = depth of insulated coil
1.098 in. = depth of two insulated coils
0.1 in. = thickness of stick in top of slot
1.198 in. = necessary depth of slot.

Before fixing the slot depth the windings for all the probable ratings to be built on this frame should be worked out and the slot made deep enough for the worst. In this case a suitable slot depth is 1.5 in.

Voltage per phase	$=\frac{440}{1.73}=254$, since the connection is Y
Flux per pole	$=1{,}040{,}000$, from formula 25, page 190
Minimum tooth width	$=0.302$ in., taking slot width $=0.32$ in.
Minimum tooth area per pole	= minimum tooth width $\times$ slots per pole $\times$ net iron

$= 19.6$ sq. in.

Maximum tooth density $= \dfrac{\text{flux per pole}}{\text{min. tooth area per pole}} \times \dfrac{\pi}{2}$

$= 83{,}000$ lines per square inch.

Had this density come out too high it would have been necessary to have increased the length of the machine or decreased the slot width.

Maximum core density $= 65{,}000$ lines per square inch, assumed

$= \dfrac{\text{flux per pole}}{2 \times \text{core depth} \times \text{net iron}}$

Core depth $= 1.48$ in.; use 1.5 to give an even figure for the stator external diameter.

The above data is now filled in on the design sheet shown on page 402.

Rotor design

Air-gap clearance $= 0.03$ in. from preliminary design

External diameter $= 18.94$ in.

Gross iron $= 6$ in.

Vent ducts $= 1 - 0.5$ in.; slightly wider than for stator

Net iron $= 5.4$ in.

Number of slots $= \dfrac{\text{stator slots}}{1.2}$ for quiet operation, Art. 282 page 387.

With this number $= 80$ there would be 5 rotor slots for every 6 stator slots and the starting torque would not be uniform, see Art. 283, page 388; use therefore 79 slots.

Rotor current per cond. at full load

$$= I_{11} \text{ (Fig. 224)} \times \frac{\text{total stator conductors}}{\text{total rotor conductors}}$$

where I_{11} is taken equal to $0.85 \times I$ for a first approximation, a value which must be checked after the data for the circle diagram has been calculated and the circle drawn.

$$= 0.85 \times 62 \times \frac{96 \times 6}{79}$$

$= 385$ amperes

Ampere conductors per inch $= \dfrac{385 \times 79}{\pi \times 19}$

$= 510$

$\dfrac{\text{Amp. cond. per inch}}{\text{Cir. mils per amp.}} = 1.0$ approx., the same as for the stator

Circular mils per ampere $= 510$ desired

Size of conductor $= 510 \times 385$

$= 195{,}000$ circular mils

$= 0.15$ sq. in. approximately.

Size of slot must be chosen so that the flux density at the bottom of the rotor tooth shall not exceed 120,000 lines per square inch and the work is carried out as follows:

Assumed copper section,	0.2 in. × 0.75 in.	0.3 in. × 0.5 in.	0.4 in. × 0.35 in.
Slot section to allow for insulation and clearance,	0.25 in. × 0.8 in.	0.35 in. × 0.55 in.	0.45 in. × 0.4 in.
Rotor diameter at bottom of slot,	17.14 in.	17.64 in.	17.94 in.
Minimum slot pitch,	0.68 in.	0.70 in.	0.72 in.
Minimum rotor tooth,	0.43 in.	0.35 in.	0.27 in.
Minimum tooth area per pole,	23 sq. in.	18.7 sq. in.	14.4 sq. in.
Flux per pole,	1.04×10^6	1.04×10^6	1.04×10^6
Maximum tooth density; lines per square inch,	71,000	87,000	114,000

The wider and shallower the slot the lower is the rotor reactance so that the last of the three is chosen, namely, 0.45 in. × 0.4 in.

The necessary data for the circle diagram is now calculated.

The magnetizing current = 21 amperes, from Art. 259, page 357.

The no-load loss = 1680 watts iron loss
+ 810 watts bearing friction, from Art. 259.

The maximum stator current = 415 amperes, from Art. 264, page 364.

The circle is then drawn to scale as in Fig. 247.

The section of the rotor ring is found as follows:

The maximum stator conductor loss = 57 kw. Art. 268, page 373.

therefore in Fig. 247, ab = 75 amperes

The maximum rotor conductor loss = 39 kw. Art. 268

therefore bc = 51 amperes

The maximum starting torque desired = 90 syn. h. p.

= 67 syn. kw.

therefore $lm = \frac{67 \times 1000}{1.73 \times 440}$

= 88 amperes

and bd = 113 amperes

For full-load torque at starting the rotor loss

= 50 syn. h. p.

= 37 syn. kw.

of which the loss in the rings $= 37 \times \frac{dc}{bd}$

= 20 kw.

The weight of the end rings = 1 lb. per kw. loss, Art. 272, page 379

= 20 lb.

The mean diameter of end ring = 17.5 in. approximately

The section of each end ring = 0.57 sq. in.

The resistance factor for the ring material is found from formula 50, namely,

loss in two rings $= 0.5 \left(\frac{N_2 b_2 I_{c_2}}{1000p} \right)^2 \times \frac{D_r k_r}{A_r}$

where I_{c_2} = the maximum rotor current $= (415 - 21) \frac{96 \times 6}{79}$

= 2880 amperes

and the corresponding ring loss $= 1.73 \times 440 \times cd$

$= 1.73 \times 440 \times 62$

= 47,000 watts

$$\text{therefore } 47{,}000 = 0.5\left(\frac{79\times 2880}{1000\times 8}\right)^2 \times \frac{17.5\times k_r}{0.57}$$

$$\text{from which } k_r = 3.8$$

Certain standard compositions are used for the end-ring material, and in this case a composition which had five times the resistance of copper was used, and the ring section increased to 0.75 sq. in. to keep the loss the same.

The design is now complete and the data should be gathered together in convenient form on a design sheet similar to that on page 402.

289. Design of a Wound-rotor Machine.—It is desired to design a rotor of the wound type for the 50-h. p., 440-volt, 3-phase, 60-cycle, 900-r.p.m. motor of which the stator data is tabulated on page 402.

The work is carried out in a similar way to that adopted for the squirrel-cage design.

Air-gap clearance	= 0.03 in. the same as for the squirrel-cage motor
External diameter	= 18.94 in.
Gross iron	= 6 in.
Vent ducts	= 1 − 0.5 in.
Net iron	= 5.4 in.
Number of slots	$= \frac{\text{stator slots}}{1.2}$ for quiet operation
	= 80; use 72 or 9 slots per pole
Conductors per slot	= 2, assumed; this number gives the simplest winding
Current per conductor at full-load	$= 0.85\times 62\times \frac{96\times 6}{72\times 2}$
	= 210 amperes
Terminal voltage at standstill	$= 440\times \frac{72\times 2}{96\times 6}$
	= 110

The brushes and slip rings will be cheaper and easier cooled if the winding is made with 4 conductors per slot, Y-connected; then

Current per conductor at full-load	= 105 amperes
Terminal voltage at standstill	= 220
Amp. cond. per inch	= 510
$\frac{\text{Amp. cond. per inch}}{\text{Cir. mils per amp.}}$	= 1.0, the same as for the stator
Circular mils per ampere desired	= 510
Size of conductor	= 510 × 105
	= 53,500 cir. mils
	= 0.042 sq. in.

Size of slot is found in the same way as for the squirrel-cage machine and that chosen in this case
$= 0.42$ in. $\times 1.0$ in.

Size of conductor $= 0.12$ in. $\times 0.35$ in. arranged 2 wide and 2 deep.

The magnetizing current $= 21$ amperes, the same as for the squirrel-cage machine

The iron loss $= 1{,}680$ watts

The bearing friction loss $= 810$ watts

The maximum stator current is worked out in a similar way to that for the squirrel-cage machine, see Art. 264, page 364; thus for a wound-rotor machine the reactance per phase

$$= 2\pi f b^2 c^2 p^2 n \left[\frac{\phi_e L_e}{2n_1 p}\times\frac{4}{3} + \left(\frac{\phi_s+\phi_z}{n_1 p}+\frac{\phi_s+\phi_z}{n_2 p}+\frac{\text{const.}\times\tau}{p\times\delta\times C_1 C_2}\right) L_g\right] 10^{-8}$$

where $\tau = 7.5$ in.

$\frac{\phi_e L_e}{2n_1} = 4.3$ from Fig. 244

$\lambda_1 = 0.622$ in.

$\delta = 0.03$ in.

$C_1 = 1.52$

$C_2 = 1.16$

$$\frac{\phi_s+\phi_z}{n_1 p} = \frac{1}{96}\left[3.2\left(\frac{1.3}{3\times.32}+\frac{0.2}{0.32}\right)+0.26\times\frac{0.622}{0.03}\left(\frac{1}{1.52}+\frac{1}{1.03}-1\right)^2\right]$$

$$= \frac{1}{96}[6.3+2.1]$$

$\lambda_2 = 0.83$

$$\frac{\phi_s+\phi_z}{n_2 p} = \frac{1}{72}\left[3.2\left(\frac{1.0}{3\times 0.42}+\frac{0.1}{0.42}+\frac{2\times 0.07}{0.42+0.1}+\frac{0.03}{0.1}\right)+ 0.26\times\frac{0.83}{0.03}\left(\frac{1}{1.52}+\frac{1}{1.03}-1\right)^2\right]$$

$$= \frac{1}{72}[5.1+2.8]$$

const. $= 1.5\times 0.00107$, from page 367.

$$X_{eq} = 2\pi\times 60\times 6^2\times 4^2\times 8^2\times 3\left[\frac{4.3}{8}\times\frac{4}{3}+\left(\frac{8.4}{96}+\frac{7.9}{72}+ 0.00107\times 1.5\times\frac{7.48}{8\times 0.03\times 1.52}\right)\times 6\right]\times 10^{-8}$$

$$= 0.415\times[0.72+(0.088+0.11+0.032)6]$$

$$= 0.87$$

$$\text{Voltage} = \frac{440}{1.73} = 254$$

$$\text{Max. current per phase} = \frac{254}{0.87} = 300 \text{ amp. as against 415 for the squirrel-cage motor.}$$

290. Induction Motor Design Sheet.—All dimensions in inch units.

	Stator	*Squirrel cage*	*Wound rotor*
External diameter,	25	18.94	18.94
Internal diameter,	19	15.5	14.5
Frame length,	6.375	6.5	6.5
End ducts,	none	none	none
Center ducts,	1–0.375	1–0.5	1–0.5
Gross iron,	6	6	6
Net iron,	5.4	5.4	5.4
Slots, number,	96	79	72
size,	0.32×1.5	0.45×0.4	0.42×1.0
Cond. per slot, number,	6	1	4
size,	0.14×0.2	0.4×0.35	0.12×0.35
Winding, type,	double-layer	squirrel-cage	double-layer
connection,	Y		Y
Minimum slot pitch,	0.622	0.72	0.73
Minimum tooth width,	0.302	0.27	0.31
Core depth,	1.5	1.22	1.12
Pole-pitch,	7.46		
Minimum tooth area per pole,	19.6	14.4	15
Core area,	8.1	6.6	6.0
Apparent gap area per pole,	45		
Flux per pole,	1.04×10^6	1.04×10^6	1.04×10^6
Maximum tooth density,	83,000	114,000	110,000
Maximum core density,	64,000	79,000	86,000
Ampere conductors per inch,	600	510	510
Circular mils per ampere,	570	460	510
Length of conductors,	20.5	10.5	19
Maximum current per conductor,	415	2,880	560
Maximum conductor loss,	57 kw.	39 kw.	32 kw.
Section of each end connector,		0.75	
Resistance factor,		5.0	
Maximum ring loss,		47 kw.	
Apparent gap density,	23,000		
Air-gap clearance,	0.03		
Carter coefficient,	1.52	1.03	1.03
Magnetizing current, gap,	17.6		
total,	21		
$K_1+K_2L_g$,	12		16.75 stator
Reactance per phase,	0.62		0.87 stator
Maximum line current I_d,	415	2,880	300 stator
Rating			
Horse-power,	50		
Terminal voltage,	440		
Amperes, full-load,	62	385	105
Phases,	3		
Frequency,	60		
Syn. r.p.m.,	900		
Poles,	8		

291. Design of a 25-cycle Motor.—It is required to design a motor of the following rating:

50 h. p., 440 volts, 3-phase, 25-cycle, 750 r.p.m., squirrel cage.
$B_g = 27{,}000$; $q = 600$; cos $\theta = 90$ per cent.; $\eta = 89$ per cent.; $D_a^2 L_g = 2200$

D_a	L_g	τ	V	δ	$K_1 + K_2 L_g$	$\frac{I_o}{I}$	$\frac{I_d}{I}$
13	13	10.2	2.55	0.30	5.0+14.4=19.4	0.27	10.2
15	9.75	11.8	2.95	0.29	5.4+10.0=15.4	0.23	9.6
17	7.6	13.3	3.34	0.28	5.7+ 7.2=12.9	0.19	9.0
19	6.1	14.9	3.74	0.29	6.0+ 5.4=11.4	0.18	8.1
21	5.0	16.5	4.12	0.30	6.4+ 4.2=10.6	0.17	7.2

Any one of these machines would be satisfactory as far as magnetizing current and overload capacity are concerned. The 13-in. machine, however, is long and difficult to ventilate properly so that it need not be considered. Of the others, the machine with the smallest diameter will generally be the cheapest so that the choice lies between the 15- and the 17-in. machine; there will be very little difference in cost between the two, but the 17-in. machine will be the easier to ventilate properly and will have the better appearance.

Detailed Design for the 17-in. Machine.—Since the per cent. magnetizing current is small it will be advisable to increase the air-gap over the minimum value of 0.029 in.; it may be taken =0.04 in. without making the magnetizing current too large or the power factor at full-load too low.

Poles,		=4	
Internal diameter of stator,		=17 in.	
Gross iron,		=7.5 in.	
Vent ducts,		=2–0.5 in.	
Net iron,		=6.8 in.	
Slots per pole,		=12 or	18
	slot pitch	=1.12 in.	0.74 in.
	amp. cond. per slot	=670	445
	$\frac{\text{amp. cond. per slot}}{\text{air-gap clearance}}$	$=17\times10^3$	11×10^3
		Noisy	Quiet, see Art. 282, page 387
Slots per pole,		=18, suitable for both two- and three-phase	
Pole-pitch,		=13.4 in.	
Slot pitch,		=0.74 in.	
Slot width,		=0.37 in.	
Ampere conductors per inch,		=600 from preliminary design	
Amperes conductors per slot,		=445	

Full-load current,	=61
Conductors per slot,	=7.3 if Y-connected =12.6 if Δ-connected
$\frac{\text{Amp. cond. per inch,}}{\text{Cir. mils per amp.}}$	=0.82
Ampere conductors per inch for 12 Δ winding.	=570
Circular mils per ampere,	=700
Amperes per conductor,	=35 for a delta-connected winding
Circular mils per conductor,	=25,000
Section of conductors,	=0.02 square inch. =0.08×0.25 in.
Size of slot,	=0.37×1.75 in.
Voltage per phase,	=440 since connection is Δ
Flux per pole,	=2,900,000
Minimum tooth width,	=0.37 in.
Minimum tooth area per pole,	=45.5 square inch.
Maximum tooth density,	=100,000 lines per square inch.

The rotor of this machine may be designed by the same method as that adopted for the 60-cycle machine in Art. 288; the probable number of slots =59.

It might seem that, since the overload capacity is more than sufficient for all ordinary purposes, a value of q higher than 600 might have been used. This would have allowed the use of a slightly shorter machine, as may be seen from formula 55, page 391, but would have necessitated a larger section of copper to keep the heating within reasonable limits, and a larger number of conductors per slot. It is probable that the small decrease in length of the machine would have been more than compensated for in price by the increased amount of copper required for both stator and rotor.

292. Variation in the Length of a Machine for a Given Diameter. —In order to save on the original outlay for the tools required to build a line of induction motors it is advisable to design at least two lengths of machine for each diameter. In the case of small factories, where the total output is not very large, three different frame lengths may be used for each diameter.

The principal dimensions of three squirrel-cage machines built on a 19-in. diameter for 8 poles, 60 cycles and 900 r.p.m. are tabulated below.

External diameter of stator,	25 in.	25 in.	25 in.
Internal diameter of stator,	19 in.	19 in.	19 in.
Frame length,	4.5 in.	6.375 in.	8.125 in.

Center ducts,	none	1–0.375 in.	1–0.375 in.
Gross iron,	4.5 in.	6 in.	7.75 in.
Net iron,	4.05 in.	5.4 in.	7.0 in.
Slots,	96	96	96
Size of slot,	.32 in. × 1.5 in.	.32 in. × 1.5 in.	.32 in. × 1.5 in.
Conductor per slot,	8	6	8
Size of conductor,	0.1 in. × 0.2 in.	0.14 in. × 0.2 in.	0.1 in. × 0.2 in.
Connection,	Y	Y	Δ
Ampere conductors per inch,	580	600	600
Circular mils per ampere,	560	580	600
Amperes per conductor,	45	62	46.5
Amperes per terminal,	45	62	80
Terminal voltage,	440	440	440
Phases,	3	3	3
Output,	35 h. p.	50 h. p.	65 h. p.
Air gap clearance,	0.03 in.	0.03 in.	0.03 in.
Magnetizing current,	16 amp.	21 amp.	27 amp.
K_1 actual, see Fig. 245, page 369,	4.30	4.30	4.30
$K_2 L_g$ actual,	5.90	7.90	9.90
Reactance per phase in ohms,	0.92	0.62	1.27
Maximum current in line,	275 amp.	410 amp.	600 amp.
Magnetizing current,	35 per cent.	34 per cent.	34 per cent.
Maximum current,	6.1 full-load	6.6 full-load	7.5 full-load.

The above machines are discussed under the following heads:

Conductors per Slot.—Since the same stator punchings are used in each case, the number of slots is fixed, and for the same flux density in the different machines the flux per pole must be directly proportional to the net iron. Now the voltage per conductor is proportional to the flux per pole, so that the number of conductors in series, for the same voltage per phase, must be inversely proportional to the flux per pole and therefore inversely proportional to the net iron in the frame length.

Size of Conductor.—This is inversely proportional to the number of conductors per slot for the same total copper section per slot.

Current Rating.—For the same current density in the conductors, the current in each conductor must be proportional to the conductor section. If the connection is Y, the current rating is the same as the current per conductor; if the connection is △, the current rating $=1.73$ times the current per conductor.

Output.—For the same total section of copper, the output of the machine is proportional to the net iron in the frame length,

because output = a const. × phases × volts per phase × current per phase
= a const. $\times n \times Z\phi_a \times I_c$
= a const. $\times nZI_c \times \phi_a$
= a const. × total copper section $\times L_n$

Magnetizing Current $= \frac{1}{0.87 \times \text{cond. per pole}} \times B_g \times \delta \times C$, and since B_g, δ and C are all constant, the magnetizing current is inversely proportional to the number of cond. per pole and therefore to the number of conductors per slot.

Maximum Current.—The leakage flux per phase is made up of two parts, the end-connection leakage which is independent of the frame length, and the slot and zig-zag leakages which are directly proportional to the frame length. The reactance per phase $= 2\pi f b^2 c^2 pn(K_1 + K_2 L_g)10^{-8}$
= a const. $\times b^2(K_1 + K_2 L_g)$
for machines built with the same punchings.

From the data in the table it may be seen that, so far as magnetizing current and overload capacity are concerned, the longest machine is the best; but a machine cannot be lengthened indefinitely because a point is finally reached at which it becomes impossible to cool the center of the core properly, without considerable modification in the type of construction. Even before this point is reached it will generally be found economical to increase the diameter rather than keep on increasing the length because, since the output is proportional to $D_a^2 L_g$, an increase in diameter of 10 per cent. is equivalent to an increase in frame length of 22 per cent.

293. Windings for Different Voltages.—The stator of a 35-h. p., 440-volt, 3-phase, 45-ampere, 60-cycle, 900-r.p.m. induction motor is constructed as follows:

Internal diameter of stator	19 in.
Frame length	4.5 in.
Slots, number	96
Slots, size	0.32 × 1.5 in.
Conductors per slot, number	8
size	0.1 × 0.2
Connection	Y

It is required to design windings for the following voltages:
220 volts, 3 phase, 60 cycles
550 volts, 3 phase, 60 cycles

250 volts, 3 phase, 60 cycles
550 volts, 2 phase, 60 cycles.

Conductors per slot.

$E = 2.22 k Z \phi_a f 10^{-8}$ volts

$$= \text{a const.} \times k \times \frac{\text{cond. per slot}}{\text{phases}}$$ for a given frame and frequency.

For the machine in question $k = 0.956$ for three-phase windings
$= 0.908$ for two-phase windings

and the constant $= \dfrac{\text{volts per phase} \times \text{phases}}{k \times \text{cond. per slot}}$

$$= \frac{\frac{440}{1.73} \times 3}{0.956 \times 8}$$

$$= 100$$

The windings for the different voltages may be tabulated thus:

Term. voltage	Phases	Volts per phase	Conductors per slot	Connection	
440	3	254	8	Y	
220	3	127	4	Y	Use 8 conductors per slot connected YY.
550	3	320	10	Y	
250	3	144	4.55	Y	An impossible winding.
.....	.....	250	8	△	Use instead of the one above.
550	2	550	12.2	Single circuit	Use 12 conductors per slot.

Size of Conductor.—This must be chosen so that the stator copper loss and copper heating are the same for each voltage; that this may be the case it is necessary to keep the ratio $\dfrac{\text{amp. cond. per inch}}{\text{cir. mils per amp.}}$ constant. The work is carried out in tabular form thus:

Terminal voltage	Phases	Current per terminal	Current per cond.	Cond. per slot	Connection	Amp. cond. per inch	Cir. mils per amp.	Conductor
440	3	45	45	8	Y	580	560	0.1×0.2 in.
220	3	90	45	8	YY	580	560	0.1×0.2 in.
550	3	36	36	10	Y	580	560	0.08×0.2 in.
250	3	80	46	8	Δ	600	600	0.11×0.2 in.
550	2	31	31	12	1 circuit	600	610	0.075×0.2 in.

The rotor winding is the same for all stator voltages and phases, because the only connection between the stator and rotor is the flux in the air-gap, and this is kept constant by the use of the proper number of stator conductors per slot. It is therefore possible to build motors for stock, complete except for the stator winding, which winding can be specified when the voltage and number of phases on which the machine will operate are known.

It must not be imagined that the designs which have been worked out in this chapter are the only ones that could have been used. Electrical design is very flexible and different values for flux density and ampere conductors per inch might have been used to give a satisfactory machine, and perhaps a cheaper one. Where labor is cheap it will often pay to use closed stator slots, fans, forced ventilation or other means to reduce the size of the machine for a given output.

When one design for a given rating is worked out completely, the designer has to go over it and try the effect of changing the different quantities until he is satisfied that, for the shop in which machines of his design will be built, the final design will give the most satisfactory machine both as regards manufacturing cost and reliability when in service.

CHAPTER XXXVIII

SPECIAL PROBLEMS IN INDUCTION MOTOR DESIGN

294. Slow Speed Motors.

Since $\frac{I_o}{I}=2.05\frac{B_g\delta}{q\tau}$ formula 56, page 394

and $\frac{I_d}{I}=0.337\frac{B_gL_g}{q(K_1+K_2L_g)}$ formula 58, page 394

therefore $\frac{I_d}{I_o}=\text{a const.}\ \frac{\tau L_g}{\delta(K_1+K_2L_g)}$ (59)

and the ratio $\frac{I_d}{I_o}$ depends largely on the ratio $\frac{\tau}{\delta}$.

In moderate speed machines $\frac{I_d}{I_o}=\frac{\text{6 full-load current}}{\text{0.33 full-load current}}=18.$ In high speed machines the dimensions are small and the peripheral velocity high, so that the ratio $\frac{\tau}{\delta}$ is generally large, since the value of τ is directly proportional to the peripheral velocity and the value of δ increases with the dimensions of the machine. Such machines therefore have a large value of I_d and a large overload capacity, they have also a small value of I_o and a high power factor.

In the case of slow speed machines the dimensions are large so as to get the necessary radiating surface, and the peripheral velocity is generally low because of the large number of poles. Because of the small pole-pitch the ratio $\frac{I_d}{I_o}$ is small, and the characteristics of the machine are small overload capacity, large magnetizing current and low power factor.

Compare for example the preliminary designs for machines of 300-h. p. output, 60 cycles and 720 and 300 r.p.m. respectively.

Horse-power,	300	300
Frequency,	60	60
r.p.m.,	720	300

B_g,	23,000	23,000
q,	730	730
cos θ, assumed,	91 per cent.	84 per cent.
η assumed	91 per cent.	90 per cent.
$D_a^2 L_g$,	12,700	33,500
D_a,	36 in.	65 in.
L_g,	10 in.	8 in.
τ,	11.3 in.	8.5 in.
V, in 1000s of ft. per min.,	6.8	5.1
δ,	0.048 in.	0.051 in.
K_1,	5.3	4.5
$K_2 L_g$,	10.5	10
$K_1 + K_2 L_g$,	15.8	14.5
$\frac{I_o}{I}$	0.275	0.39
$\frac{I_d}{I}$	6.7	5.9

These two machines are shown to scale in Fig. 259.

The above figures show that, the higher the speed for a given horse-power, the smaller is the magnetizing current and the larger the overload capacity; this is a characteristic property which cannot be changed except by the use of air-gaps on the slow speed machines which are not large enough for mechanical purposes.

For slow speed motors the use of 25 cycles offers considerable advantage over the use of 60 cycles because, for the same r.p.m., the number of poles is the smaller in the case of the 25-cycle motor and therefore the pole-pitch and the ratio $\frac{I_d}{I_o}$ are the larger. Compare for example the preliminary designs for machines of 300 h. p. at 300 r.p.m., for 25- and 60-cycle operation respectively.

Horse-power,	300	300
Frequency,	60	25
r.p.m.,	300	300
Poles,	24	10
B_g,	23,000	27,000
q,	730	730
cos θ, assumed,	84 per cent.	90 per cent.
η, assumed,	90 per cent.	90 per cent
$D_a^2 L_g$,	33,500	26,500
D_a,	65 in.	46 in.
L_g,	8 in.	12.5 in.
τ,	8.5 in.	14.5 in.
V, in 1000s of ft. per min.,	5.1	3.6

δ,	0.051 in.	0.045 in.
K_1,	4.5	5.9
$K_2 L_g$,	10	11.3
$K_1 + K_2 L_g$	14.5	17.2
$\frac{I_o}{I}$	0.39	0.235
$\frac{I_d}{I}$	5.9	9.0

When most of the motors that are to be used are slow speed machines, that is, they lie below the speed curve *b* in Fig. 256, then 25-cycle apparatus will have a comparatively high power factor and overload capacity, while for 60-cycle apparatus these will be low.

295. Closed-slot Machines.—By the use of closed slots for both stator and rotor the characteristics of slow-speed machines can be considerably improved. The only objection to the use of closed slots for the stator is that the windings are difficult to repair in case of breakdown. There is not the same objection to their use for the rotor because, in the case of the squirrel-cage machine, there is only one conductor per slot and this is put in from the end, while in the case of the wound-rotor machine the winding can be chosen with two or four conductors per slot which can be formed to shape at one end, the slot part and one end insulated, and the other end bent to shape and insulated after the conductor has been pushed into place. There is no connection between the stator applied voltage and the rotor voltage at standstill; this latter voltage can be kept low by the use of a small number of rotor conductors per slot. When the machine is up to speed and the rotor short-circuited, the voltage between the rotor windings and the core is very low and the chance of breakdown small.

By the use of closed slots the air-gap area is increased, and the magnetizing current decreased for the same winding, the slot leakage and zig-zag leakage are increased and the overload capacity reduced; as a rule the increase in leakage is not as large as the decrease in magnetizing current.

The following table shows comparative designs for a 300-h. p., 440-volt, 3-phase, 60-cycle, 300-r.p.m. motor; the one design has open stator slots and the other closed stator slots; the rotor is of the squirrel-cage type in each case and has closed slots. The designs need not be worked through carefully, but it is necessary to understand the conclusions tabulated at the end of the design sheet.

Inch units are used.	A Open slot stator	B Closed slot stator	A Rotor	B Rotor
External diameter,	72	63	64.9	56.9
Internal diameter,	65	57	61	53
Frame length,	9.5	11	9.5	11
End ducts,	2–0.5	2–0.5	none	none
Center ducts,	2–0.5	2–0.5	2–0.5	2–0.5
Gross iron,	8.5	10	8.5	10
Net iron,	7.6	9	7.6	9
Slots, number,	288	288	220	220
size,	0.35×1.5	0.31×1.6	0.55×0.45	0.45×0.45
Conductors per slot, number,	2	2	1	1
size,	0.23×0.5	0.2×0.5	0.5×0.4	0.4×0.4
Winding, type,	double layer	double layer	squirrel cage	squirrel cage
connection,	△	YY		
Minimum slot pitch,	0.71	0.62	0.91	0.8
Minimum tooth width,	0.36	0.31	0.36	0.35
Core depth,	2	1.4	1.4	1.4
Pole-pitch,	8.5	7.45		
Minimum tooth area per pole,	33	33.5	25	29
Core area,	15.2	12.6	10.6	12.6
Apparent gap area per pole,	72.5	74.5		
Flux per pole,	1.8×10^6	2.08×10^6	1.8×10^6	2.08×10^6
Maximum tooth density,	85,000	97,000	113,000	113,000
Maximum core density,	59,000	82,000	85,000	83,000
Ampere conductors per inch	660	640	520	510
Circular mils per ampere,	630	640	520	490
Apparent gap density,	25,000	28,000		
Air-gap clearance,	0.05	0.05		
Carter coefficient,	1.4	1.04	1.02	1.02
Magnetizing current, gap,	145, terminal	140, terminal		
total,	175	168		
K_1,	4.5	4.2		
K_2L_g,	10.2	16.4		
Maximum line current, I_d,	3,000	2,800		
Horse-power,	300	300		
Terminal voltage,	440	440		
Amperes, full-load,	400	400	485	415
Phases,	3	3		
Frequency,	60	60		
Syn. r.p.m.,	300	300		
Poles,	24	24		

$$\text{Reactance per phase} = 2\pi f b^2 c^2 p^2 n\left[\frac{\phi_t L_e}{2n_1 p}+\left(\frac{\phi_s+\phi_z}{n_1 p}+\frac{\phi_s+\phi_z}{n_2 p}\right)L_g\right]10^{-8}$$

where for machine A

$$\frac{\phi_e L_e}{2n_1} = 4.5$$

$$\frac{\phi_s+\phi_z}{n_1 p}=\frac{1}{288}\left[3.2\left(\frac{1.3}{3\times0.35}+\frac{0.2}{0.35}\right)+0.26\frac{0.71}{0.05}\left(\frac{1}{1.4}+\frac{1}{1.02}-1\right)^2\right]=\frac{7.56}{288}$$

$$\frac{\phi_s+\phi_z}{n_2 p}=\frac{1}{220}\left[3.2\left(\frac{0.45}{3\times0.55}+\frac{2\times0.07}{0.55+0.07}+\frac{0.03}{0.07}\right)+0.26\times\frac{0.93}{0.05}\left(\frac{1}{1.4}+\frac{1}{1.02}-1\right)^2\right]$$

$$=\frac{5.25}{220}$$

Reactance per phase $= 2\pi \times 60 \times 2^2 \times 4^2 \times 24^2 \times 3 \left[\frac{4.5}{24} + \left(\frac{7.56}{288} + \frac{5.25}{220}\right) 8.5\right] 10^{-8}$

$= 0.256$

Voltage per phase $= 440$ for Δ connection

Max. current per phase $= \frac{440}{0.256}$

$= 1720$ amp.

Max. line current $= 1720 \times 1.73$

$= 3000$ amp.

For machine B

$\frac{\phi_e L_e}{2n} = 4.2$

$\frac{\phi_s + \phi_z}{n_1 p} = \frac{1}{288}\left[3.2\left(\frac{1.5}{3 \times 0.31} + \frac{2 \times 0.07}{0.31 + 0.1} + \frac{0.03}{0.1}\right) + 0.26 \frac{0.62}{0.05}\left(\frac{1}{1.04} + \frac{1}{1.02} - 1\right)^2\right]$

$= \frac{10.5}{288}$

$\frac{\phi_s + \phi_z}{n_2 p} = \frac{1}{220}\left[3.2\left(\frac{0.45}{3 \times 0.45} + \frac{2 \times 0.07}{0.45 + 0.07} + \frac{0.03}{0.07}\right) + 0.26 \frac{0.81}{0.05}\left(\frac{1}{1.04} + \frac{1}{1.02} - 1\right)^2\right]$

$= \frac{7.0}{220}$

Reactance per phase $= 2\pi \times 60 \times 1^2 \times 4^2 \times 24^2 \times 3 \left[\frac{4.2}{24} + \left(\frac{10.5}{288} + \frac{7.0}{220}\right) 10\right] 10^{-8}$

$= 0.09$

Voltage per phase $= \frac{440}{1.73} = 254$ for a Y-connection

Max. current per phase $= \frac{254}{0.09}$

$= 2800$

$=$ maximum line current.

The points of importance in the above designs are:

	Open slot	Closed slot
Flux density, stator teeth	85,000	97,000
Flux density, stator core	59,000	82,000
Core depth	2.0 in.	1.4 in.
Carter coefficient	1.4	1.04
Conductors per slot	$2\Delta = 1.16$ Y	2YY = 1Y
Internal diameter of stator	65 in.	57 in.
Flux per pole	1.8×10^6	2.08×10^6
Magnetizing current	175 amp.	168 amp.
$K_2 L_g$	10.2	16.4
Maximum current	3,000	2,800

When closed slots are used the flux density may be high, since the pulsation loss is almost entirely eliminated.

The value of the Carter fringing coefficient is decreased and

therefore the magnetizing current is reduced for the same winding or, as in the above machines, the number of conductors is reduced for the same magnetizing current.

Because of the reduction in the number of conductors the stator internal diameter is decreased for the same value of q, the ampere conductors per inch.

Because of the reduced number of conductors the reactance of the winding is decreased, and because of the increase in the slot and the zig-zag leakage, due to the closing of the slot, the reactance is increased.

The final result is that the closed slot machine is smaller and cheaper than the open slot machine, especially if, as in Europe, the cost of winding labor is cheap; it has the same characteristics as the open slot machine.

If the closed slot machine were made on the same diameter as the open slot machine it would have the better characteristics.

296. High-speed Motors.—The number of speeds in the useful range for small belted motors is smaller for 25 than for 60 cycles, as may be seen from the following table:

Poles	Revolutions per minute	
	60 cycles	25 cycles
2	3,600	1,500
4	1,800	750
6	1,200	500
8	900	375
10	720	300

for that reason, and also because they are cheaper than 25-cycle motors of the same speed, 60-cycle motors should be used where most of the driven machines are moderate speed belted machines.

Compare for example the preliminary design for a 300 h. p., 60 cycle, 720 r.p.m., machine with that for a 300 h. p., 25 cycle, 750 r.p.m. machine.

Horse-power,	300	300
Frequency,	60	25
R.p.m.,	720	750
Poles	10	4

B_g	23,000	27,000
q,	730	730
cos θ, assumed,	91 per cent.	92 per cent.
η, assumed,	91 per cent.	91 per cent.
$D^2{}_aL_g$,	12,700	10,300
D_a	36 in.	27 in.
L_g,	10 in.	14 in.
τ,	11.3 in.	21 in.
V, in 1000s of ft. per min.	6.8	5.3
δ,	0.048 in.	0.045 in.
K_1,	5.3	7
K_2L_g,	10.5	10.2
$K_1+K_2L_g$,	15.8	17.2
$\frac{I_o}{I}$,	0.275	0.16
$\frac{I_d}{I}$,	6.7	10.2

These two machines are shown to scale in Fig. 259.

Since the magnetizing current of the 25-cycle machine is so small, it will be advisable to increase the air-gap clearance and thereby have less chance of mechanical trouble.

Because the 25-cycle machine has the smaller diameter it must not be imagined that it is the cheaper machine. It has the smaller number of poles, the larger flux per pole and, therefore, the deeper core. Due to the large pole-pitch the end connections are long, and due to the lower peripheral velocity and the greater difficulty in cooling the long machine, a large section of copper and deep slots are necessary. These facts are shown by the following table:

Horse-power,	300	300
R.p.m.,	720	750
Poles,	10	4
Pole-pitch,	11.3 in.	21 in.
Gross iron,	10 in.	14 in.
Flux per pole $=B_g\tau L_g$,	2.6×10^6	8.0×10^6
Stator core density, assumed,	65,000	85,000
Necessary core area,	20 sq. in.	47 sq. in.
Core depth,	2.25 in.	3.75 in.
Rotor core density, assumed,	85,000	85,000
Necessary core area,	15 sq. in.	47 sq. in.
Core depth,	1.7 in.	3.75 in.
Length of stator conductor	30 in.	48 in. from Fig. 84.

A section through the two machines is shown to scale in Fig. 259, from which it may be seen that the 25-cycle motor has the

larger amount of material in it, and will probably be the more expensive machine.

In the case of large high-speed motors, such as those for direct connection to centrifugal pumps or for motor generator sets, care must be taken in the design to ensure that the machines will be quiet in operation.

Consider for example, the design of a direct-connected wound-rotor motor of the following rating:

1000 h. p., 60 cycles, 900 r.p.m.

$B_g = 24{,}000$, $q = 800$, $\cos\theta = 92$ per cent., $\eta = 92$ per cent., $D_a^2 L_g = 29{,}000$

D_a	L_g	τ	V	δ	$K_1 + K_2 L_g$	$\frac{I_o}{I}$	$\frac{I_d}{I}$
32	28.5	12.6	7.55	0.067	$7.5+36 = 43.5$	0.33	6.6
36	22.5	14.1	8.45	0.065	$7.7+27 = 34.7$	0.28	6.6
40	18.0	15.7	9.40	0.065	$8.3+20.5 = 28.8$	0.26	6.3
44	15.0	17.2	10.3	0.066	$8.6+17 = 25.6$	0.24	6.0

When using the curves in Fig. 244 to determine K_1 and K_2 it should be noted that, for wound-rotor machines, the values of K_1 found from the curve must be multiplied by 4/3, and the values of K_2 must be taken from the upper curve.

In order that the noise of the machine be not objectionable at no-load, the peripheral velocity should, if possible, be less than 8000 ft. per min. and the pitch of the windage note should be kept below 1400 cycles per second. The number of rotor slots per pole to satisfy this latter condition may be found from formula 54, page 385, namely:

frequency of windage note = rotor slots × revolutions per second
= 1440 if 12 slots per pole are used.

For quiet operation at full-load, the number of slots per pole for the stator should exceed that for the rotor by about 20 per cent.; see Art. 282, page 387.

For machines with 15 slots per pole, the values of the stator slot pitch $= \frac{\pi D_a}{8 \times 15}$, of the ampere conductors per slot $= q \times \lambda$, and of the ratio $\frac{\text{amp. cond. per slot}}{\text{air-gap clearance}}$ on which the noise at full-load largely depends, are given in the following table:

The last of these machines is close to the noise limit, and the first is too long to ventilate properly, so that the choice lies

D_a	λ_1	Amp. cond. per slot	δ	$\frac{\text{amp. cond. per slot}}{\delta}$
32	0.84	670	0.067	10.0
36	0.94	750	0.065	11.5
40	1.05	840	0.065	12.8
44	1.15	920	0.066	13.8

between a machine of 36-in. and one of 40-in. diameter. If the designer has been troubled with noisy machines he will probably choose that with the smaller diameter, because it will have the lower peripheral velocity, and, therefore, the lower intensity of windage note. If he is satisfied, from experience with other high-speed machines which have been built and tested, that the 40-in. motor will not be noisy, then it will probably be chosen because it is shorter and easier to ventilate.

The rating of 1000 h. p. is about the highest that can safely be built at 900 r.p.m. with the open type of construction, since a machine of larger diameter is liable to give trouble due to noise, and a longer machine is liable to get hot at the centre of the core. A larger diameter and a higher peripheral velocity may be used if the motor is partially enclosed, so as to muffle the noise, and then cooled by forced ventilation.

297. Two-pole Motors.—Since, for induction motors operating on 25 cycles, there is no available speed between 1500 and 750 r.p.m., the former speed is largely used for motors that are direct-connected to centrifugal pumps, and would also be largely used for small belted motors were it not that the cost of a 1500-r.p.m. motor is seldom less than that of a 750-r.p.m. machine of the same horse-power.

Consider for example, comparative designs for a 300-h. p. 25-cycle induction motor at 1500 and at 750 r.p.m. respectively.

Horse-power,	300	300
R.p.m.,	750	1,500
Poles,	4	2
B_g,	27,000	27,000
q,	730	730
cos θ, assumed,	92	93

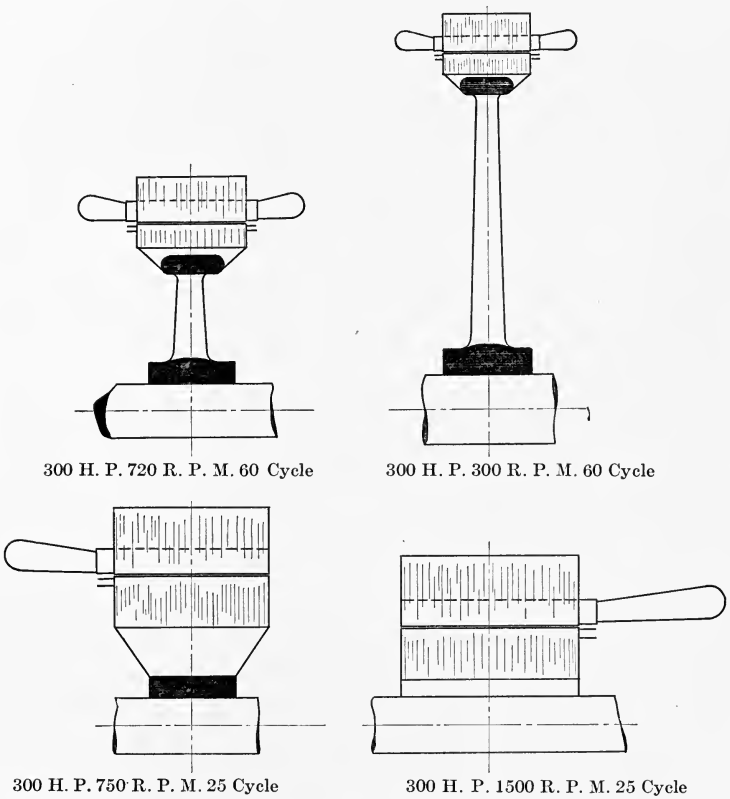
η, assumed,	91	90
$D_a^2 L_g$,	10,300	5,100
D_a,	27 in.	18 in.
L_g,	14 in.	16 in.
τ,	21 in.	28 in.
V, in 1000s of ft. per min.	5.3	7.0
δ	0.045 in.	0.048 in.
K_1,	7	8
$K_2 L_g$,	10.2	9.3
$K_1 + K_2 L_g$,	17.2	17.3
$\frac{I_o}{I}$,	0.16	0.13
$\frac{I_d}{I}$,	10.2	11.5
Flux per pole,	8.0×10^6	12.0×10^6
Core density; lines per sq. in.	85,000	85,000
Core area,	47 sq. in.	71 sq. in.
Core depth,	3.75 in.	5 in.
Length of stator conductor,	48 in.	62 in.

Some idea as to the relative proportions of these two machines may be obtained from Fig. 259.

When full-pitch windings are used for induction-motor stators, the revolving field consists of a fundamental and harmonics. If there is a pronounced nth harmonic, the resultant revolving field may be considered as the resultant of a fundamental which revolves at synchronous speed, and of a harmonic which revolves at n times synchronous speed; if these fields move in the same direction the resultant torque curve will be curve 3, Fig. 260, which is made up of curves 1 and 2, due to the fundamental and the harmonic respectively. One of the characteristics of the two-pole motor, and particularly of the two-pole 25-cycle motor, is its large pole-pitch and consequent large overload capacity, so that for such machines, when of the squirrel-cage type and designed with small slip in order to obtain high efficiency, the overload torque ab of the harmonic may become comparable with the normal torque of the fundamental, and in such a case the motor will run at a speed m, which is approximately $\frac{1}{n}$th of synchronous speed, and is called a sub-synchronous speed.

This sub-synchronous locking speed may be eliminated by the use of a high-resistance rotor, which raises the part cd of curve 1 without affecting the overload torque; an extreme case of a high-resistance rotor is a rotor of the wound type. Another way to eliminate these locking speeds is to eliminate the harmonics

by the use of a short-pitch winding in the stator. Fig. 261 shows the flux distribution curve for a machine with a large number of stator slots; in diagram A a full-pitch winding is used, while in diagram B the winding is made 2/3 of full-pitch; it may be seen that in the latter case a nearer approach to a sine wave is



300 H. P. 720 R. P. M. 60 Cycle

300 H. P. 300 R. P. M. 60 Cycle

300 H. P. 750 R. P. M. 25 Cycle

300 H. P. 1500 R. P. M. 25 Cycle

FIG. 259.—300 h.p. induction motors for different speeds and frequencies.

obtained than in the former. The short-pitch winding has the additional advantage that it reduces the length of end connections considerably and for that reason is useful for both two- and four-pole machines. Against this advantage there is the disadvantage that, since

$$E = 2.22 k Z \phi_a f 10^{-8} \cos \frac{\theta}{2}$$

where θ is the angle in electrical degrees by which the pitch is

shortened, therefore, for the same flux per pole, a larger number of conductors is required with a short than with a full pitch winding.[1]

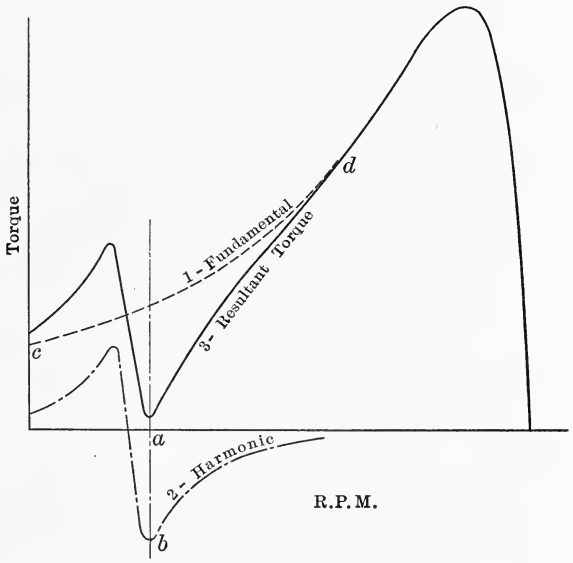


FIG. 260.—Speed torque curve for a motor with harmonics in the wave of flux distribution.

298. Effect of Variations in Voltage and Frequency on the Operation.—For a given machine

$$E = \text{a const.} \times \phi_a \times f$$
$$I_o = \text{a const.} \times \phi_a$$
$$X_{eq} = \text{a const.} \times f$$
$$I_d = \frac{E}{X_{eq}} = \text{a const.} \times \phi_a$$

If the voltage applied to the motor terminals is reduced, the value of ϕ_a, the flux per pole, will be reduced, and therefore,
I_o will be reduced and the power factor improved;
I_d will be reduced and the overload capacity decreased.
The flux density will be reduced, the current will be increased for the same output, and the result will probably be increased heating and a liability to pull out of step.

If the voltage be increased, the value of the flux per pole will be increased and therefore,

[1] For a discussion of short-pitch windings see Adams, *Transactions* of A. I. E. E., Vol. 26, page 1485.

I_o will be increased and the power factor decreased;
I_d will be increased and so also will be the overload capacity;
The flux density will be increased, but the current will be reduced for the same rating, and the result will be reduced heating.

If the frequency of the applied voltage be reduced, the voltage being unchanged, then the flux per pole will be increased and so also will be the magnetizing current and the overload capacity.

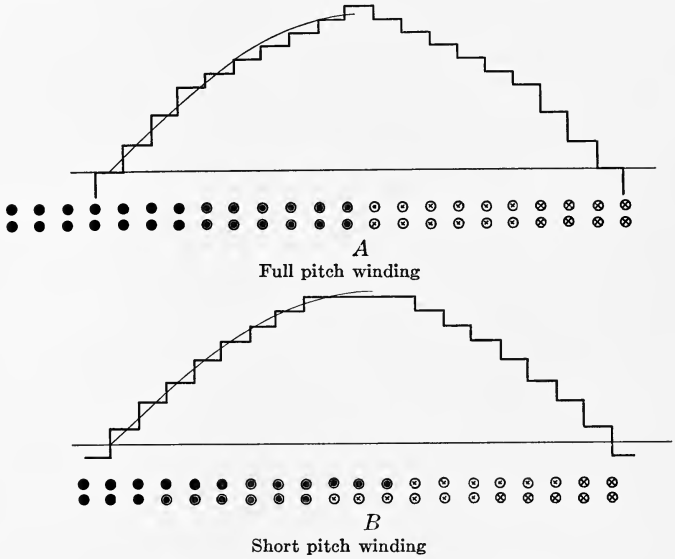


FIG. 261.—Flux distribution in the air gap.

The core loss and copper loss will be practically unchanged, but the heating will be increased due to the reduction in speed.

If the frequency be increased, the voltage being unchanged, then the flux per pole will be reduced and so also will be the magnetizing current and the overload capacity. The core loss and copper loss will be unchanged and the heating reduced because of the increase in speed, but the rating of the motor cannot be increased because of the reduced overload capacity.

If the voltage and frequency both increase or decrease together, then, for a change of 10 per cent., the characteristics of the machine are not changed; thus it is possible to sell 440-volt 60-cycle motors for operation on 400 volts and 50 cycles, with the same guarantee in each case except that of heating; the temperature will increase due to the increase in current and to the decrease in speed.

CHAPTER XXXIX

SPECIFICATIONS

299. The following is a typical specification for a large induction motor.

SPECIFICATION FOR INDUCTION MOTORS

Type.—Squirrel-cage.

Rating.

Rated capacity in horse-power	300
Normal terminal voltage	2,200
Phases	3
Amperes per terminal at full-load	70 approx.
Frequency in cycles per second	60
Speed at full-load in revolutions per minute	1,160 approx.

Construction.—The motors are for direct connection to centrifugal pumps and shall be of the open type, with split housings and bushings, and with the shaft extended at both ends to carry couplings. The type of drive is shown in the accompanying sketch.

Stator.—The stator coils must be form wound, must be completely insulated before being put into the slots, and must be readily removable for repair.

Bidders must state the type of slot to be used, whether open, partially closed or totally closed; an open-slot construction will be given preference.

A sample coil must be submitted with each bid, and will be tested for its ability to withstand moisture. The coils put into the machine must conform to the sample.

Rotor.—This shall be of the squirrel-cage type. All connections in the electrical circuit shall be made with good mechanical joints and shall then be soldered. The end connectors must be rigidly supported from the spider and not merely overhung on the ends of the rotor bars. All fans and projecting parts on the rotor must be screened in such a way that a person working around the machine is not liable to be hurt.

Workmanship and Finish.—The workmanship shall be first class. All parts shall be made to standard gauges and be inter-

changeable. All surfaces not machined are to be dressed, filled and rubbed down so as to present a smooth-finished appearance.

Potential Starter.—A suitable potential starter, complete with transformer and switches, shall be supplied. Only one starting notch is desired, but the transformer must be supplied with taps so that the starter may be adjusted to operate with the smallest possible starting current. The switches must be of ample capacity to handle the starting current. The starter must be provided with a no-voltage release.

Base and Couplings.—The base shall be supplied by the builder of the pumps. The couplings shall be supplied by the builder of the motor, who must press one-half of each coupling on the motor shaft, and turn, bore, and key-seat the other half to dimensions which will be supplied later.

General.—Bidders will furnish plans or cuts with descriptive matter from which a clear idea of the construction may be obtained. They shall also state the following:

Net weight of machine without starter;
Shipping weight of machine and starter;
Efficiency at $\frac{1}{2}$, $\frac{3}{4}$, full- and $1\frac{1}{4}$ load;
Power factor at $\frac{1}{2}$, $\frac{3}{4}$, full- and $1\frac{1}{4}$ load;
Air-gap clearance from iron to iron.

Efficiency.—This shall be taken as $\frac{\text{electrical input} - \text{losses}}{\text{electrical input}}$.

The losses to be used for the calculation of the efficiency are: Windage, friction and iron loss which shall be taken as the electrical input determined by wattmeter readings, the motor being run at normal voltage and frequency and at no-load; allowance should be made for the copper loss of the stator exciting current.

Copper loss which shall be found as follows: The loss for different stator and rotor currents shall be found from a short-circuit test, the copper temperature being 60° C., and the total input found from wattmeter readings being taken as copper loss. The hot stator resistance shall then be taken with direct-current and the stator copper loss separated out. The stator and rotor currents corresponding to the different loads shall be found from the circle diagram, and the copper losses corresponding to these currents found from the results of the short-circuit test.

Power Factor.—This shall be obtained from the no-load and short-circuit tests.

Overload Capacity and Starting Torque.—The maximum horse-power at normal voltage and frequency shall not be less than twice full-load, and the starting current in the stator windings for full-load torque shall not exceed five times full-load current.

Temperature.—The machine shall carry the rated output in horse-power, at normal voltage and frequency, for 24 hours, with a temperature rise that shall not exceed 40° C. by thermometer on any part of the machine, and, immediately after the full-load heat run, shall carry 25 per cent. overload at the same voltage and frequency for 2 hours, with a temperature rise that shall not exceed 55° C. by thermometer on any part of the machine.

The temperature rise of the oil in the bearings shall not exceed 40° C., measured by a thermometer in the oil well, either at normal load or at the overload.

The temperature rise shall be referred to a room temperature of 25° C.

No compromise heat run will be accepted. If the manufacturer cannot load the machine, a heat run will be made within 3 months after erection to determine if it meets the heating guarantee, the test to be made by the motor builder who shall supply the necessary men and instruments.

Insulation.—The machine shall withstand the puncture test recommended in the standardization rules of the American Institute of Electrical Engineers (latest edition) and the insulation resistance shall be greater than one megohm. No puncture or insulation resistance test shall be made on the rotor.

Testing Facilities.—The builder shall provide the necessary facilities and labor for testing the machine in accordance with this specification.

Notes on Induction Motor Specifications.—The characteristics of an induction motor are all more or less interdependent. A large starting torque is obtained by a sacrifice of efficiency, and a large air-gap clearance and a large overload capacity by a sacrifice of power factor.

The overload capacity of a motor should never be less than twice full-load because, since the overload capacity is approximately proportional to the square of the applied voltage, a drop in voltage of 20 per cent. would cause such a motor, operating at 50 per cent. overload, to drop out of step.

300. Effect of Voltage on the Efficiency and Power Factor.—If machines are built on the same frame, and for the same output and speed, but for different voltages, the losses are affected in the following way:

The windage and friction loss is independent of the voltage.

The iron loss is unchanged since the winding is such that the flux per pole is the same for all voltages.

The copper loss is proportional to the ratio $\frac{\text{amp. cond. per inch}}{\text{cir. mils per amp.}}$ which ratio must be kept constant for the same heating. Now the ratio

$$\frac{\text{amp. cond. per in.}}{\text{cir. mils per amp.}} = \frac{\text{amp.}^2 \times \text{total cond.}^2}{\pi D_a \times \text{cir. mils per cond.} \times \text{total cond.}}$$

$$= \text{a const.} \times \frac{\text{output}^2}{\text{total copper section}}$$

since (total cond)^2 is proportional to (voltage)^2.

If then the total copper section in the stator is the same at all voltages the efficiency is independent of the voltage, but if, as is generally the case, the total section of stator copper decreases as the voltage increases, on account of the space taken up by insulation, then in order that the copper loss and heating be not too large the output of the machine must be decreased, and for the same total loss but a reduced output, the efficiency of the machine is reduced.

$$\text{The per cent. magnetizing current} = \frac{I_o}{I}$$

$$= \text{a const.} \times \frac{B_g}{q}$$

Since the windings for different voltages are chosen so as to keep the value of B_g constant, the per cent. magnetizing current depends largely on the value of q. It was pointed out above that, if wound for high voltage, the rating of a given machine has to be reduced below the value which it would have if wound for low voltage. Now the number of conductors is directly proportional to the voltage, and, so long as the output is constant and the current inversely proportional to the voltage, the product of current $\times$ conductors is constant, and the per cent. magnetizing current and the power factor are independent of the voltage. If the current drops faster than the voltage increases, then the value of the product of conductors $\times$ current will decrease and the per cent. magnetizing current increase, thereby causing the power factor to be poorer, the higher the voltage.

301. Effect of Speed on the Characteristics.—The effect of speed on the efficiency is the same as for alternators and has been studied in Art. 228, page 316; the higher the speed for a given rating, the higher is the efficiency.

The effect of speed on the power factor and overload capacity has been discussed fully in the last chapter; the result of an increase in speed for a given horse-power is to improve the power factor and increase the overload capacity.

302. Specifications for Wound-rotor Machines.—For such motors the same style of specification may be used as for the squirrel-cage machine, with the following modifications:

No potential starter is required; in its place a resistance starter is supplied which has several starting notches. The type of service should be stated; a starter to be used for variable-speed work must be large because it absorbs a large amount of energy while in operation, for example, if a motor has to operate at half speed the amount of heat to be dissipated by the starter is approximately equal to the output of the motor; a starter that is built only for starting duty would burn up in a short time if used for variable-speed service.

The starting torque can be made anything from zero to the pull-out torque by the variation of the external resistance of the rotor.

The temperature guarantee is made for the maximum speed. A motor with a temperature rise of 40 deg. cent. at full load and maximum speed can generally operate continuously at half speed with full-load torque without injury.

When a wound-rotor motor is operating at reduced speed, but with constant stator current, the iron loss of the stator remains constant, while that of the rotor increases because of the increase in slip and, therefore, of rotor frequency; the copper losses also remain constant, so that the heating increases as the speed decreases; the horse-power is not constant but is proportional to the speed.

CHAPTER XL

OPERATION OF TRANSFORMERS

303. No-load Conditions.—Fig. 262 shows a transformer diagrammatically; the two windings, which have T_1 and T_2 turns respectively, are wound on an iron core C.

An alternating voltage E_1, applied to the primary coil T_1, causes a current I_e, called the exciting current, to flow in the coil;

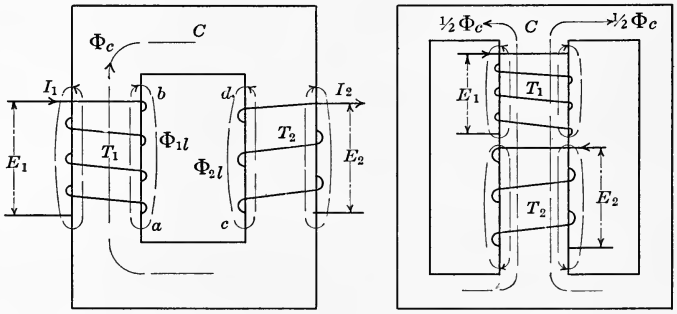


FIG. 262.—Diagrammatic representation of transformers.

this current produces in the core an alternating flux, ϕ_{co}. The flux ϕ_{co} generates an e.m.f. E_2 in the secondary winding and an e.m.f. E_{1b} in the primary winding, and to overcome this latter voltage the applied e.m.f. must have a component which is

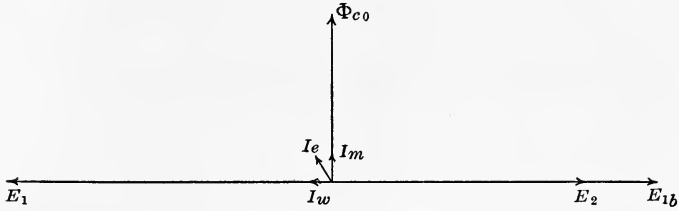


FIG. 263.—Vector diagram at no load.

equal and opposite to E_{1b} at every instant; the other component of the applied e.m.f. must be large enough to send the exciting current I_e through the impedence of the primary winding.

In Fig. 263, which shows the vector diagram at no-load,

ϕ_{co} is the flux in the core,

I_m is the magnetizing current, or that part of I_e which is in phase with ϕ_{co},

I_w, the component of I_e which is in phase with the applied e.m.f., is required to overcome the no-load losses,

E_2, the e.m.f. generated in the secondary winding, lags ϕ_{co} by 90°,

E_{1b}, the generated e.m.f. in the primary winding, also lags ϕ_{co} by 90°,

E_1, the applied e.m.f., is equal and opposite to E_{1b}, since the component required to overcome the impedence of the primary winding may be neglected at no-load.

Since E_2 and E_{1b} are produced by the same flux ϕ_{co}, and since $E_1 = E_{1b}$, therefore

$$\frac{E_2}{E_1} = \frac{T_2}{T_1} \tag{60}$$

304. Full-load Conditions.—When the secondary of a transformer is connected to a load, a current I_2 flows in the secondary winding; the phase relation between E_2 and I_2 depends on the power factor of the load.

The fluxes which are present in a transformer core at full-load are shown in Fig. 262. The m.m.f. between a and $b = T_1 I_1$ ampere-turns, where I_1 is the full-load current, and due to this m.m.f. a flux ϕ_{1l} is produced which threads the primary winding but does not thread the secondary; ϕ_{1l} is called the primary leakage flux and is in phase with the current I_1.

The m.m.f. between c and $d = T_2 I_2$ ampere-turns, where I_2 is the full-load secondary current, and due to this m.m.f. the secondary leakage flux ϕ_{2l} is produced which threads the secondary winding but does not thread the primary.

In Fig. 264

I_2 is the current in the secondary winding,

ϕ_{2l}, the secondary leakage flux, is in phase with I_2,

ϕ_c, the flux in the core, threads both primary and secondary windings,

ϕ_2 is the actual flux threading the secondary winding,

E_2 is the e.m.f. generated in the secondary winding by ϕ_2,

E_{2t}, the secondary terminal e.m.f., is less than E_2 by $I_2 R_2$, the e.m.f. required to overcome the resistance of the secondary winding,

I_1 is the current in the primary winding,
ϕ_{1l}, the primary leakage flux, is in phase with I_1,
ϕ_1 is the actual flux threading the primary winding,
E_{1b} is the e.m.f. generated in the primary winding by ϕ_1,
E_1, the primary applied e.m.f., is made up of a component equal and opposite to E_{1b}, and a component I_1R_1 to overcome the primary resistance.

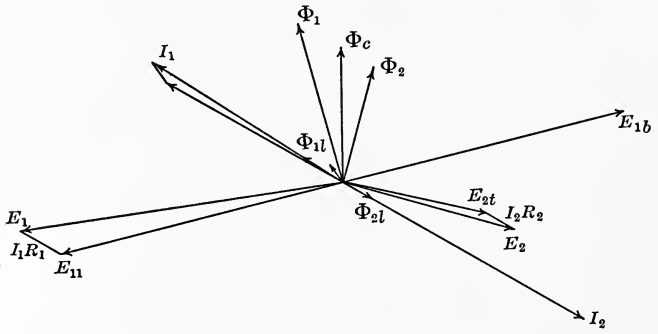


FIG. 264.—Vector diagram at full load.

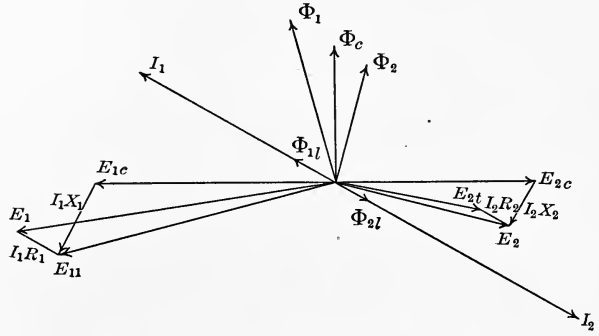


FIG. 265.—Vector diagram at full load.

The applied voltage E_1 is constant, and I_1R_1 is comparatively small even at full-load, so that it may be assumed that E_{1b} is equal to E_1, and that ϕ_1, which produces E_{1b}, is approximately constant at all loads; the resultant of the m.m.fs. of the primary and secondary windings at full-load must, therefore, as far as the primary is concerned, be equal in effect to the m.m.f. of the exciting current I_e, so that the primary current may be divided up into two components, one of which has a m.m.f.

equal and opposite to that of the secondary winding, while the other is equal to I_e.

It is usual to consider that the primary and secondary leakage fluxes have an existence separate from that of the core flux ϕ_c, so that in Fig. 265

ϕ_2 consists of two components ϕ_c and ϕ_{2l},

E_2 consists of two components; E_{2c} due to ϕ_c, and E_{2l} due to ϕ_{2l},

E_{2l} lags ϕ_{2l} and therefore I_2 by 90°, it is also proportional to I_2, so that it acts exactly like a reactance and $= I_2X_2$, where X_2 is the secondary leakage reactance,

ϕ_1 consists of two components ϕ_c and ϕ_{1l},

E_{11} consists of two components E_{1c} and E_{1l} which latter $= I_1X_1$ the primary leakage-reactance drop.

305. Conditions on Short-circuit.—On short-circuit the terminal voltage of the secondary is zero; the diagram representing

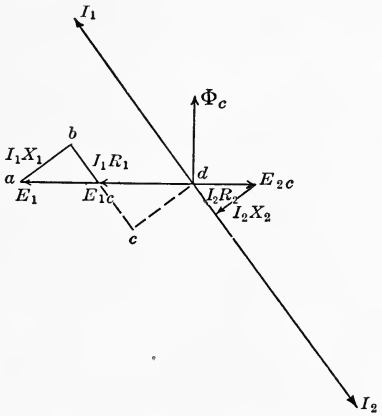


FIG. 266.—Vector diagram on short circuit.

the operation is shown in Fig. 266 when the transformer is short-circuited and a primary e.m.f. applied which is large enough to circulate full-load current through the transformer. Since the applied e.m.f., and therefore the primary flux, is small, the exciting current may be neglected and T_1I_1 and T_2I_2 will then be equal and opposite.

$$E_1 = \sqrt{(ab+dc)^2+bc^2}$$

where $ab = I_1X_1$

$$dc = \frac{T_1}{T_2}(I_2X_2) = \left(\frac{T_1}{T_2}\right)^2 I_1X_2$$

$$bc = I_1 R_1 + \left(\frac{T_1}{T_2}\right)^2 I_1 R_2$$

therefore $$E_1 = I_1 \sqrt{\left(R_1 + \left(\frac{T_1}{T_2}\right)^2 R_2\right)^2 + \left(X_1 + \left(\frac{T_1}{T_2}\right)^2 X_2\right)^2}$$

$$= I_1 \sqrt{(R_{eq})^2 + (X_{eq})^2}$$

where R_{eq} and X_{eq} are the equivalent primary resistance and reactance respectively.

306. Regulation.—Fig. 267 shows the vector diagram at full-load with the primary voltages expressed in terms of the secondary and the magnetizing current neglected.

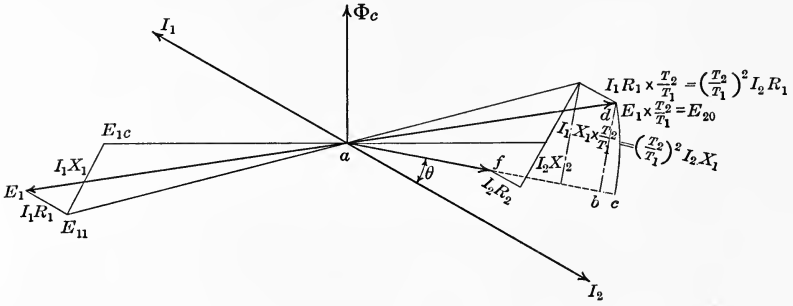


FIG. 267.—Vector diagram on full load with primary voltages expressed in terms of the secondary.

The regulation of constant potential transformers is defined as the per cent. increase of secondary terminal voltage from full-load to no-load, the primary applied voltage being constant.

In Fig. 267 the rise in secondary voltage from full-load to no-load $= fc$

$$= I_2 R_e \cos\theta + I_2 X_e \sin\theta + bc$$

where R_e and X_e are the equivalent secondary resistance and reactance respectively and

$$bc \times 2ab = (bd)^2$$

or $$bc = \frac{(bd)^2}{2ab}$$

$$= \frac{(I_2 X_e \cos\theta - I_2 R_e \sin\theta)^2}{2(E_{20} - bc)}$$

$$= \frac{(I_2 X_e \cos\theta - I_2 R_e \sin\theta)^2}{2E_{20}} \text{ approximately.}$$

The per cent. regulation $= 100\left(\frac{fc}{ac}\right)$

$$= 100\,\frac{I_2 R_e}{E_{2o}} + \tfrac{1}{2}\left(\frac{I_2 X_e}{E_{2o}}\right)^2 \text{ at 100 per cent. power factor.}$$

$$= 100\left(\frac{0.8\,I_2 R_e + 0.6\,I_2 X_e}{E_{2o}}\right) \text{ approximately at 80 per cent.}$$

power factor, lagging current. (61)

CHAPTER XLI

CONSTRUCTION OF TRANSFORMERS

307. Small Core-type Distributing Transformers.—Figs. 268 and 269 show the various parts of such a transformer. The core *A* is built of L-shaped punchings, insulated from one another by varnish, and stacked to give a circuit with only two joints;

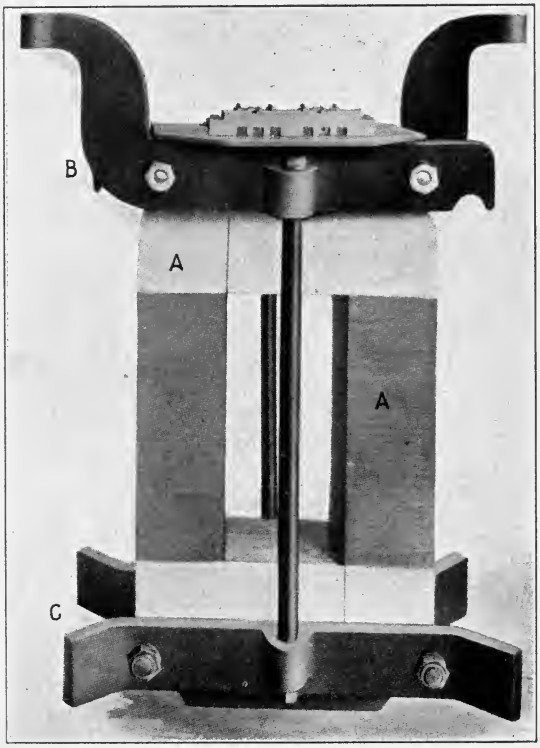


FIG. 268.—Core of a small distributing transformer.

the joints are made by interleaving the punchings so as to keep the total reluctance of the magnetic circuit small. The core is assembled through the coils and is then clamped together

at the ends by brackets *B* and *C;* the limbs of the core are held tight to prevent vibration and humming by wooden spacers *D* which are put in between the core and low-tension winding.

The coils are wound on formers and are impregnated with special compound. In the example shown the low-voltage

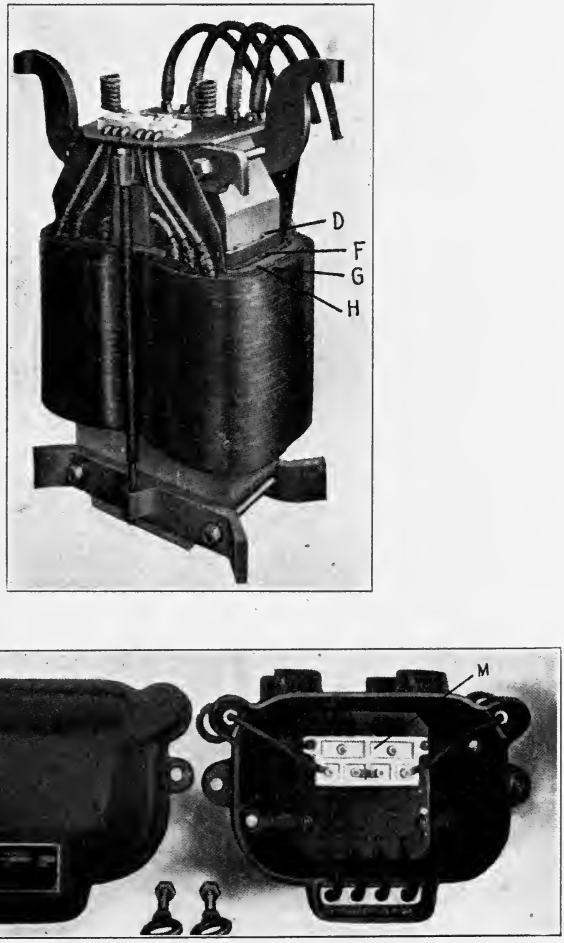


Fig. 269.—Small core-type transformer.

winding has two half coils *F* and *G* on each leg, and a high-voltage coil *H* sandwiched between them; the object of this construction is to keep the reactance low. Wooden spacers are

used to separate the windings from one another and from the core so as to allow the oil in which the transformer is placed to circulate freely and thereby keep the coils and the core cool.

There are two primary and two secondary coils, and the leads from each coil are brought up through an insulated board on which is mounted a porcelain block *M* for the high-voltage leads; by means of connectors the high-voltage coils may be put in

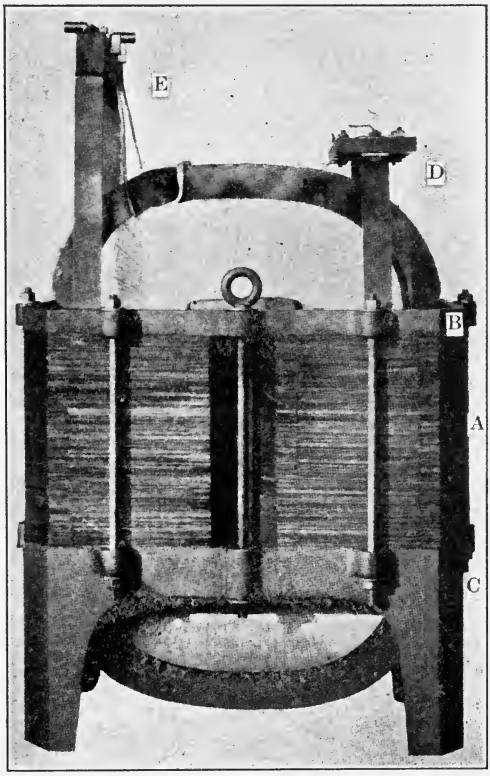


FIG. 270.—Large shell-type transformer.

series or in parallel. The four low-voltage leads and the two leads from the high-voltage terminal board are brought out of the transformer tank through porcelain bushings which are cemented in.

The tank is filled with a special mineral oil above the level of the windings, this oil acts as an insulator and also helps to keep

the transformer cool by circulating inside of the tank and carrying the heat from the transformer where it is generated, to the tank where it is dissipated.

308. Large Shell-type Power Transformers.—Figs. 270, 271 and 272 show the various parts of such a transformer. The core

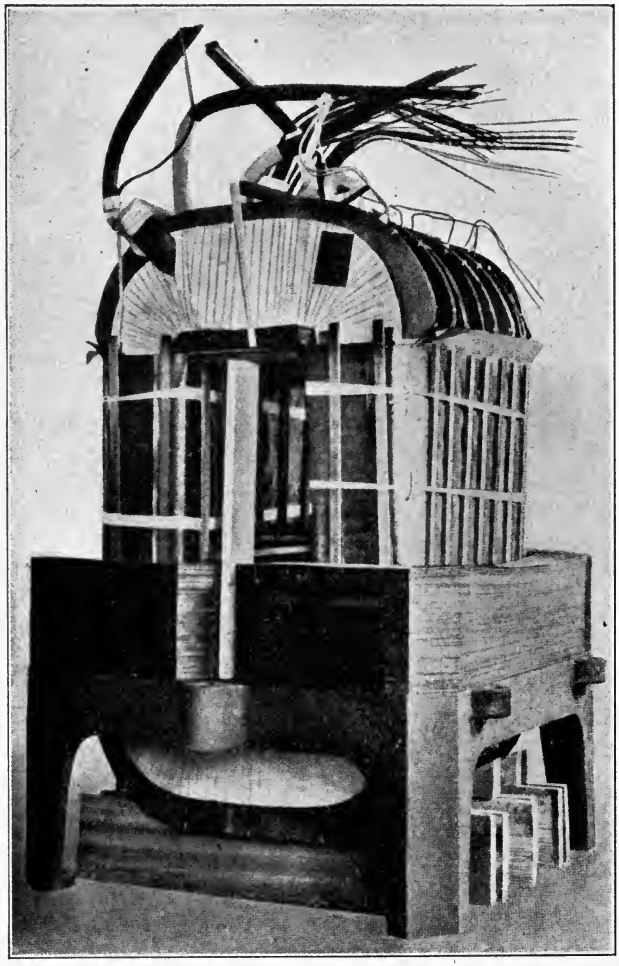


FIG. 271.—Shell-type transformer in process of construction.

A is built up of punchings which are insulated from one another by varnish and are put together as shown in Fig. 273, with the joints overlapped so as to keep the total reluctance of the mag-

netic circuit small. The core is assembled around the coils and is then clamped between two end supports *B* and *C;* the bottom one is supplied with legs to support the whole transformer, while the top one carries the two terminal boards *D* and *E*, the former for the high-voltage leads and the latter for those of the

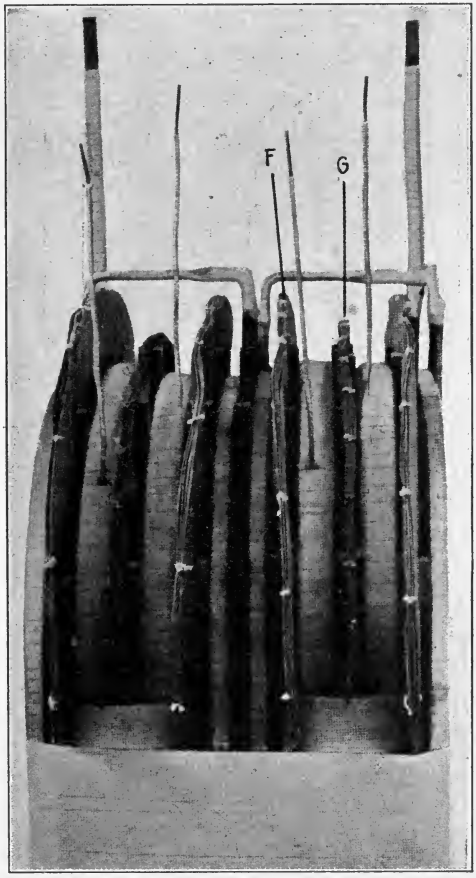


FIG. 272.—Pancake coils for a shell-type transformer.

low-voltage winding. The core is sometimes supplied with horizontal ducts which are obtained by building the core up with spacers; these ducts add to the radiating surface of the core and help to cool it but were not necessary in the transformer shown.

The coils are wound on formers and are then taped up individually, after which they are gathered together with the necessary insulation between them and insulated in a group; around these coils the core is built as shown in Fig. 271. The high- and low-voltage coils are sandwiched as shown in Fig. 272 to keep the reactance small; this latter illustration also shows the method of insulating the high- and low-voltage windings from one another by pressboard washers F, and the two adjacent high-

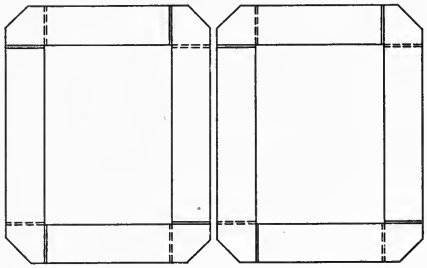


Fig. 273.—Method of stacking the punchings of a shell-type transformer.

voltage coils by a similar but smaller washer G. The winding is well supplied with ducts through which the oil can circulate.

For large power transformers a tank such as that in Fig. 269 has not sufficient radiating surface and it is necessary to use corrugated tanks or tanks with extra cooling surface such as those shown in Fig. 298 or else to cool the transformer by forced draft as shown in Fig. 299 or by water coils as shown in Fig. 300. In all cases the eye bolts which are used for lifting the tanks should extend to the bottom to prevent straining of the tank while it is being lifted.

CHAPTER XLII

MAGNETIZING CURRENT AND IRON LOSS

309. The E.M.F. Equation.—If in a transformer

ϕ_a is the maximum flux threading the windings at no-load,

T is the number of turns in the winding,

f is the frequency of the applied e.m.f., then the flux threading the windings changes from

$$\phi_a \text{ to } -\phi_a \text{ in the time of half a cycle,}$$

or the average rate of change of flux $= 2\ \phi_a \times 2f$

and the average voltage in the coil $= 4\ T\phi_a f 10^{-8}$

The effective voltage in the coil = average voltage × form factor

$$= 4 \times \text{form factor} \times T\phi_a f 10^{-8}$$

For a sine wave of e.m.f., the form factor = 1.11 and

$$E_{eff} = 4.44 T\phi_a f 10^{-8} \text{ volts} \qquad (62)$$

310. The No-load Losses.—The losses in a transformer at no-load are the hysteresis and eddy-current losses in the active iron, and the small eddy-current losses due to stray flux in the iron brackets and supports; these latter losses may be neglected if care is taken to keep the brackets away from stray fields.

The hysteresis loss $= KB^{1.6}fW$ watts, and the eddy-current loss $= K_e(Bft)^2W$ watts, where

K is the hysteresis constant and varies with the grade of iron,

K_e is a constant which is inversely proportional to the electrical resistance of the iron,

B is the maximum flux density in lines per square inch,

f is the frequency in cycles per second,

t is the thickness of the laminations in inches,

W is the weight of the iron in pounds.

The eddy-current loss $= i^2r$, where r is the resistance of the eddy-current path and i, the eddy-current $= \frac{e}{r}$, therefore the eddy-current loss $= \frac{e^2}{r}$, where e, the voltage producing the eddy-current, is proportional to the flux density, the frequency, and the thickness of the iron.

The eddy-current loss may be reduced by a reduction in t, the thickness of the laminations, but this reduction cannot be carried to extreme because, for a given volume of core, the space taken up by the insulation on the laminations depends on their thickness; as they become thinner the amount of iron in the core decreases, the flux density for a given total flux increases, and finally a value is reached at which the amount of iron in the core is so small that the increase in flux density, and therefore in iron loss, more than compensates for the reduction in the eddy-current loss due to the reduction in the value of t. The iron which is used for other electrical machinery and is 0.014 in. thick is also that in most general use for transformers up to frequencies of 60 cycles.

Special alloyed iron is largely used for 60-cycle transformers because it can be obtained with a high electrical resistance and, therefore, low eddy-current loss, it has also a small hysteresis constant; since it costs more and has lower permeability than ordinary iron, it is seldom used for 25-cycle transformers in which the core density is generally limited by magnetizing current and not by iron loss.

When iron under pressure, as in the core of a transformer or other electrical machine, is subjected to a temperature of over 80° C. for a period of the order of six months, it will be found that the hysteresis loss has increased from 10 to 20 per cent. due to what is known as ageing. This ageing causes the iron loss and the temperature of a transformer to increase and the efficiency to decrease, but is not of importance in revolving machinery, because in such machines the hysteresis loss is only a small part of the total iron loss. Special alloyed iron shows very little ageing.

The hysteresis loss in a transformer is affected by the wave form of the applied e.m.f. because, as shown by the equation

$$E = 4 \times \text{form factor} \times T\phi_a f 10^{-8}$$

for a given voltage, the higher the form factor, the lower the flux, the lower the flux density and, therefore, the lower the hysteresis loss. A wave with a high form factor is peaked, so that the advantage of low hysteresis loss is counteracted by the fact that the peaked e.m.f. has the greater tendency to puncture the insulation; a sine wave is the best for all conditions of operation.

311. The Exciting Current.—Assume first that the maximum

flux density in the transformer core is below the point of saturation, so that the magnetizing current is directly proportional to the flux; then, if the flux varies according to a sine law, the magnetizing current also follows a sine law, as shown in diagram A, Fig. 274.

For the given value of B_m, the corresponding maximum ampere-turns per inch of core may be found from Fig. 42, page 47, and the effective magnetizing current I_m

$$= \frac{\text{max. amp. turns per inch} \times \text{length of magnetic path}}{\sqrt{2} \times T}$$

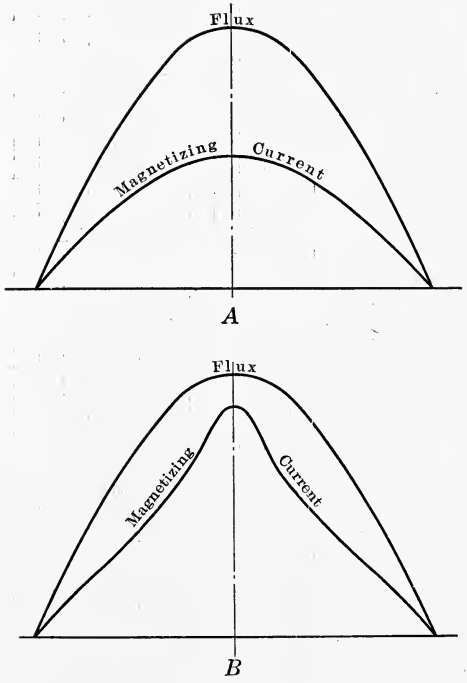


FIG. 274.—Curves of flux and magnetizing current.

or $\sqrt{2} I_m T = \text{max. amp. turns per in.} \times L_m$

now $E = 4.44\, T \phi_a f 10^{-8}$

$= 4.44\, T B_m A_c f 10^{-8}$, where A_c is the core area in square inches

therefore $E I_m = \text{a const.} \times B_m \times \text{amp.-turns per in.} \times A_c L_m \times f$

$= \text{a const.} \times \text{function of } B_m \times \text{core weight} \times f;$

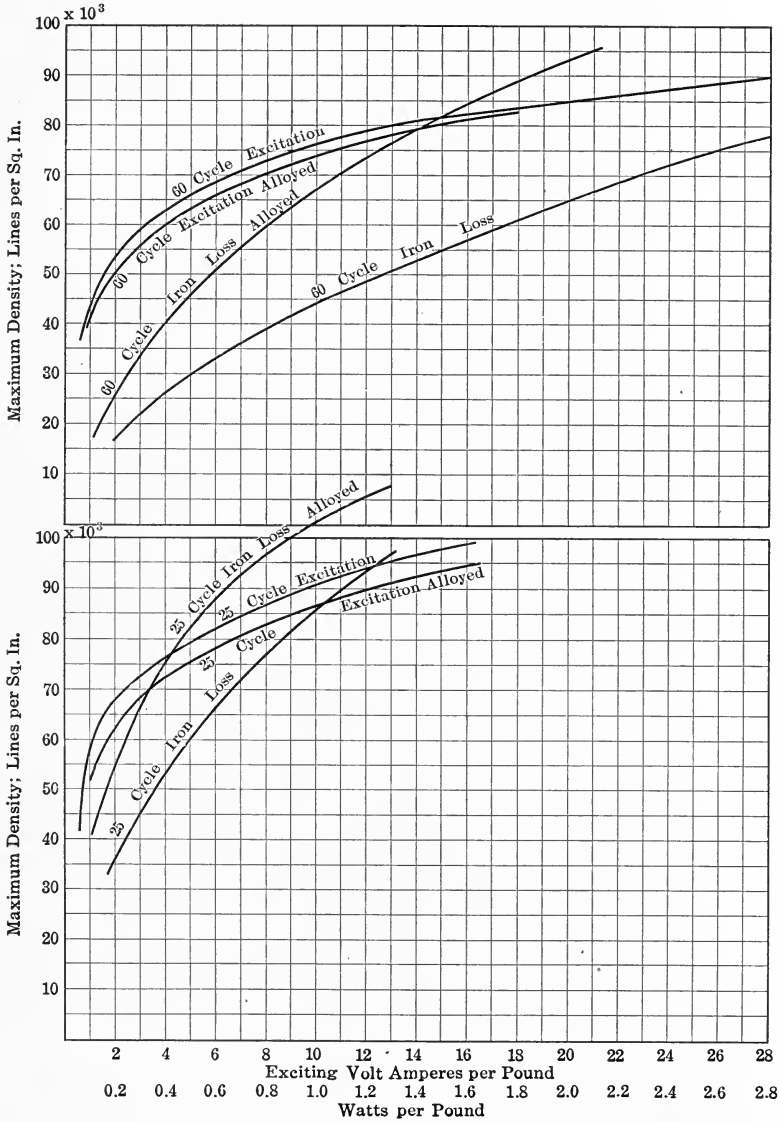


FIG. 275.—Iron loss and exciting current in transformers.

since amp.-turns per inch depends on B_m and A_cL_m, the core volume, is proportional to the core weight; therefore

$$\frac{EI_m}{\text{core weight}} = \text{the magnetizing volt amperes per pound}$$
$$= \text{a function of } B_m \times f$$

When the core is saturated at the higher densities, the curve of magnetizing current is no longer a sine curve but is peaked, as shown in diagram B, Fig. 274, because, as the points of high density are reached, the current increases faster than the flux on account of saturation, so that it is not possible to use the curves in Fig. 42 directly, a correction factor must be applied because of the form of the wave; in practice a different method is followed which allows the effect of joints, saturation and losses all to be taken into account.

It was shown that the magnetizing volt amperes per pound
$$= \text{a function of } B_m \times f$$
the iron loss per pound $= KfB^{1.6} + K_e(Btf)^2$
$$= \text{a function of } B_m \text{ for a given thickness and frequency,}$$
therefore the exciting volt amperes per pound
$$= \sqrt{(\text{magnetizing volt amp. per pound})^2 + (\text{iron loss per pound})^2}$$
$$= \text{a function of } B_m \text{ for a given thickness and frequency.}$$

If then a small test transformer be made of the material to be used for a line of transformers, and tested at no-load for exciting current and iron loss, the results may be plotted against maximum flux density as shown in Fig. 275, where test results are given for ordinary iron and for special alloyed iron at 25 and at 60 cycles; these curves may then be used for other transformers.

Example.—A transformer is constructed as follows:

Weight of core,	1,760 lb. alloyed iron
Turns of primary winding,	526
Output,	300 k.v.a.
Primary voltage,	12,000
Frequency,	60 cycles
Core section,	122 sq. in.

$\phi_a = 8.6 \times 10^6$ from formula 62, page 439

$B_m = \dfrac{8.6 \times 10^6}{122}$ = 70,000 lines per square inch

Apparent volt amperes = 8×1760; from Fig. 275
= 4.7 per cent. of output

Iron loss = 1.05×1760; from Fig. 275
= 0.62 per cent. of output.

In the case of transformers which have a butt joint in the magnetic circuit, it is necessary to insulate each joint with a layer of tough paper 0.005 in. thick to keep down the eddy-current loss, otherwise the flux will send currents across the face of the joint, as shown in diagram A, Fig. 276, because the lamina-

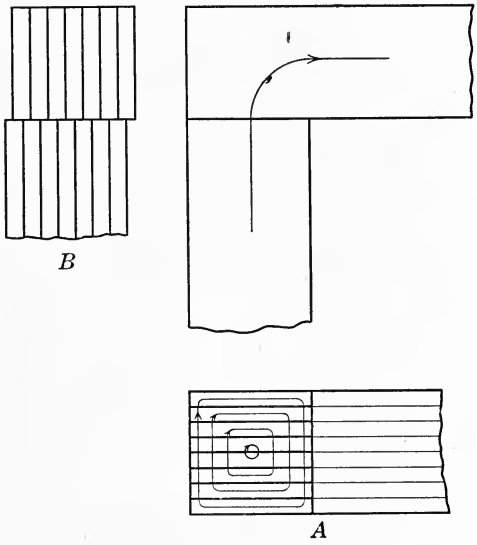


Fig. 276.—Eddy currents in transformer joints.

tions generally lie staggered as shown in diagram B. In such transformers these joints must be figured as air-gaps which have a thickness of 0.005 in. and

$$\text{the effective amp.-turns for each joint} = \frac{\text{max. amp.-turns}}{\sqrt{2}}$$

$$= \frac{B_m \times \text{air-gap.}}{\sqrt{2} \times 3.2}$$

CHAPTER XLIII

LEAKAGE REACTANCE

312. Core Type with Two Coils per Leg.—Fig. 277 shows the leakage paths for a core type transformer which has one primary and one secondary coil on each leg; the secondary is wound next to the core.

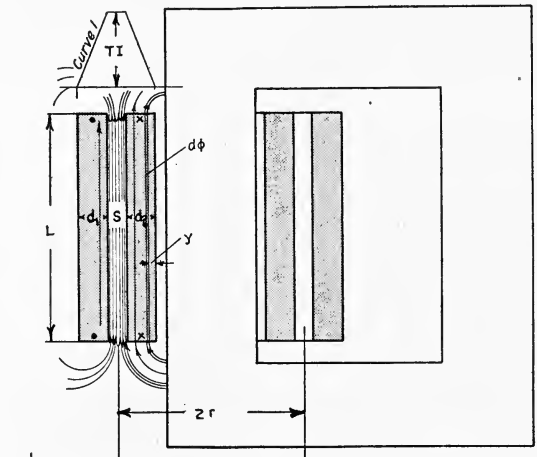


FIG. 277.

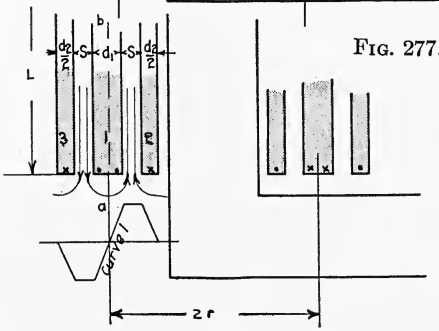


FIG. 278.

FIGS. 277 and 278.—Leakage paths in core-type transformers.

The m.m.fs. of the two coils, as shown in Fig. 264, are equal and opposite, and the currents are shown at one instant by crosses and dots. The leakage flux passes between the coils

and returns by way of the core on one side, and through the air on the other side; the space S is like the center of a long thin solenoid, and the return path of the leakage flux that links the primary coil has an infinite section so that its reluctance may be neglected; the return path for the flux that links the secondary coil is through the iron core, and its reluctance may also be neglected.

The m.m.f. between the top and bottom of the coils for different depths y is given by curve 1,

the m.m.f. at depth $y = T_2 I_2 \times \frac{y}{d_2}$ ampere-turns,

therefore the flux $d\phi = 3.2\, T_2 I_2 \frac{y}{d_2} \times \frac{2\pi r \times dy}{\mathrm{L}}$ all in inch units.

This flux links $T_2 \frac{y}{d_2}$ turns of the secondary coil, and the

interlinkages per unit current $= 3.2 \left(T_2 \frac{y}{d_2} \right)^2 \times \frac{2\pi r}{L} \times dy;$

the coefficient of self-induction of the secondary coil due to the leakage flux which passes through the coil

$$= 3.2 \times \frac{2\pi r}{L} \times T_2^2 \int_0^{d_2} \frac{y^2 dy}{d_2^2} \times 10^{-8}$$

$$= 3.2 \times \frac{2\pi r}{L} \times T_2^2 \times \frac{d_2}{3} \times 10^{-8} \text{ henry}$$

The m.m.f. along the space between coils

$= T_1 I_1 = T_2 I_2$ ampere-turns, therefore the flux which passes between the coils

$= 3.2 \times \frac{2\pi r \times S}{L} \times T_2 I_2$; since half of this flux may be assumed to link the primary and the other half the secondary coil, the coefficient of self-induction of the secondary coil due to this flux

$$= 3.2 \times \frac{2\pi r}{L} \times T_2^2 \times \frac{S}{2} \times 10^{-8} \text{ henry};$$

the total coefficient of self-induction of the secondary coil

$$= 3.2 \times \frac{2\pi r}{L} \times T_2^2 \times 10^{-8} \left(\frac{d_2}{3} + \frac{S}{2} \right) \text{ henry};$$

the leakage reactance of each secondary coil

$$= 2\pi f \times 3.2 \times \frac{2\pi r}{L} \times T_2^2 \times 10^{-8} \left(\frac{d_2}{3} + \frac{S}{2} \right)$$

similarly the leakage reactance of each primary coil

$$=2\pi f\times 3.2T_1^2\times\frac{2\pi r}{L}\left(\frac{S}{2}+\frac{d_1}{3}\right)10^{-8}$$

and X_e, the equivalent secondary reactance

$$=X_2+X_1\left(\frac{T_2}{T_1}\right)^2$$

$$=2\pi f\times 3.2\times T_2^2\times\frac{2\pi r}{L}\left(S+\frac{d_1}{3}+\frac{d_2}{3}\right)10^{-8} \qquad (63)$$

also the equivalent primary reactance

$$=2\pi f\times 3.2\times T_1^2\times\frac{2\pi r}{L}\left(S+\frac{d_1}{3}+\frac{d_2}{3}\right)10^{-8}$$

If the coils on the two legs are connected in series the total reactance will have double the above value, and if connected in parallel will have half the value.

313. Core-type Transformers with Split Secondary Coils.—Fig. 278 shows the leakage paths for one leg of such a transformer.

The m.m.f. between the top and bottom of the coils for different depths of winding space is given by curve 1, and the distribution of leakage flux is symmetrical about the line *ab*.

Consider coil 2 and half of coil 1, the equivalent primary reactance

$$=2\pi f\times 3.2\times\left(\frac{T_1}{2}\right)^2\frac{2\pi r}{L}\left(S+\frac{d_1}{6}+\frac{d_2}{6}\right)10^{-8}$$

and, similarly, for coil 3 and the other half of coil 1, the equivalent primary reactance has the same value, therefore the total equivalent primary reactance

$$=4\pi f\times 3.2\times\frac{T_1^2}{4}\times\frac{2\pi r}{L}\left(S+\frac{d_1}{6}+\frac{d_2}{6}\right)10^{-8}$$

$$=\pi f\times 3.2\times T_1^2\times\frac{2\pi r}{L}\left(S+\frac{d_1}{6}+\frac{d_2}{6}\right)10^{-8} \qquad (64)$$

which is between half and quarter of the value given in formula 63, for the case where the coils are not split up.

314. Shell-type Transformers.—The coils of such a transformer are generally arranged as shown in Fig. 272; the same arrangement is shown diagrammatically in Fig. 279, which also shows the distribution of leakage flux.

The distribution of leakage flux is symmetrical about the

lines a, b, and c, and for one primary and one secondary coil the equivalent primary reactance

$$=2\pi f\times 3.2\times T_1^2\times\frac{MT}{L}\left(S+\frac{d_1}{3}+\frac{d_2}{3}\right)10^{-8}\text{ ohms} \quad (65)$$

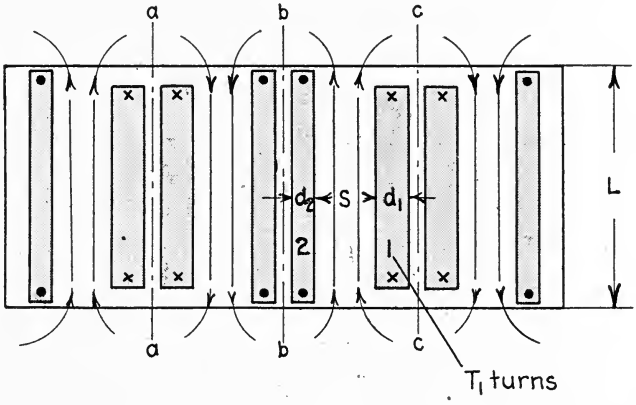


FIG. 279.—Leakage paths in shell-type transformers.

where MT is the mean turn of coil; the total equivalent primary reactance when the primary coils are all in series

$$=2\pi f\times 3.2\times T_1^2\times\frac{MT}{L}\left(S+\frac{d_1}{3}+\frac{d_2}{3}\right)10^{-8}\times\text{ number of primary coils.}$$

This gives a value which is low; the error, however, seldom exceeds 6 per cent.

CHAPTER XLIV

TRANSFORMER INSULATION

The insulation of transformers differs from that of the machines previously discussed in that it is submerged in oil.

315. Transformer Oil.—Oil has the following properties which make it valuable for high-voltage insulation: It fills up all the spaces in the windings; it is a better insulator than air at normal pressure; it can be set in rapid circulation so as to carry heat from the small surface of the transformer to the large surface of the tank; it has a fairly high specific heat, and will allow the transformer immersed in it to carry a heavy overload for a short time without excessive temperature rise; it will quench an arc.

To be suitable for transformer insulation and cooling the oil should be light over the range of temperature through which the transformer may have to operate, because the ability of the oil to carry heat readily from the transformer to the case or cooling coils depends on its viscosity; it must be free from moisture, acid, alkali, sulphur, or other materials which might impair the insulation of the transformer; it must have as high a flash point as is consistent with low viscosity and must evaporate very slowly up to temperatures of 100° C.

The oil largely used is a mineral oil which begins to burn at 149° C., has a flash point of 139° C., a specific gravity of 0.83, and a dielectric strength greater than 40,000 volts when tested between 1/2-in. spheres spaced 0.2 in. apart.

The dielectric strength is greatly reduced by the addition of moisture; 0.04 per cent. of moisture will reduce the dielectric strength about 50 per cent. This moisture may be removed by filtering the oil under pressure through dry blotting paper, or, if a filter press is not available, by boiling it at 110° C. until the dielectric strength has reached its proper value. When a transformer is in operation the oil may be sampled by drawing some off from the bottom of the tank through a tap provided for the purpose; the bulk of the moisture settles to the bottom.

Only materials which are not attacked by nor are soluble in transformer oil should be used for transformer insulation; the

materials generally used are cotton, paper, wood and special varnishes.

Wood, which is largely used for spacers, must be free from knots, and, in the case of maple, must also be free from sugar. Maple and ash are largely used, and, to ensure that they are free from moisture, they are baked and then impregnated with compound, or are boiled for about 24 hours in transformer oil at 110° C.

Fullerboard and pressboard are largely used for spacers in transformers; the latter material has a laminated structure, so that impurities seldom go through the total thickness of the piece, it also bends readily, so that there is no objection to its use in sheets 1/16 in. thick. When baked and then allowed to soak in transformer oil 1/16-in. pressboard will withstand about 30,000 volts.

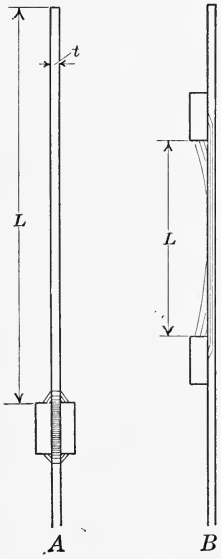


FIG. 280.—Surface leakage.

Only varnishes which are specially made for transformer work should be used for impregnating coils, other varnishes may be soluble in hot transformer oil.

316. Surface Leakage.—Fig. 280 shows two electrodes in air with pressboard between them. When the difference of potential between the two electrodes is increased, streamers will creep along the surface of the pressboard and finally form a short-circuit between the electrodes.

The distribution of the dielectric flux is shown in the two cases and in diagram A the stress around the electrodes is much larger than in diagram B. The mechanism of the breakdown due to surface leakage is not definitely known; creepage takes place under oil as well as in air, but the creepage distance under oil is only about one-third of that in air for the same test conditions.

The following tests[1] were made on a sheet of pressboard which was dried and then boiled in transformer oil:

Tested as in diagram A: $L=6$ in.; $t=0.095$ in.; creepage distance $=12.095$ in.; voltage to cause arcing across the surface $=$ 40,000 volts.

[1] A. B. Hendricks, *Transactions* of A. I. E. E., Vol. 30, page 295.

Tested as in diagram B: $L=3$ in. = the creepage distance; voltage to cause arcing across the surface = 50,000 volts.

Fig 281 shows the results of similar tests made under oil, different thicknesses of pressboard being obtained by increasing the number of sheets; it may be seen from the curves that an increase in the thickness of the insulation has little effect on the creepage distance between two electrodes that are on the same side of the sheet, but has a large effect on the creepage distance around the end, the reason being that in the former case, shown in diagram B, the distribution of stress around the electrode

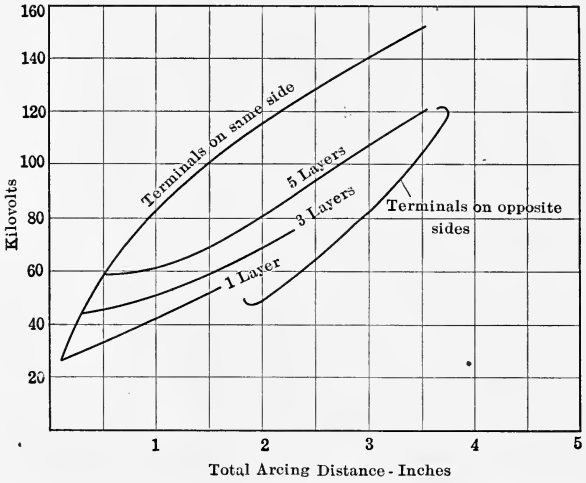


Fig. 281.—Creepage voltage on oiled pressboard. Conditions of test; Pressboard 0.095 in. thick, boiled in transformer oil, then tested under oil for creepage voltage with the electrodes first on the same side of the pressboard and then on opposite sides.

is almost independent of the thickness of the material on which the electrodes rest, while in diagram A the dielectric flux passing through the material is inversely proportional to the thickness between electrodes, so that the greater the thickness the lower the stress around the electrodes. Trouble due to surface leakage is often eliminated more economically by an increase in the thickness of the dielectric than by an increase in the creepage length.

317. Transformer Bushings.[1]—The terminals of the high- and low-voltage windings have to be brought out of the tank through

[1] A. B. Reynders, *Transactions* of A. I. E. E , Vol. 28, page 209.

bushings. For voltages up to about 40,000 above ground, bushings made of porcelain or of composition are used. These bushings extend below the surface of the oil at one end; the other end is carried above the tank to a height sufficient to prevent breakdown due to surface leakage.

Fig. 282 shows a bushing built to withstand a puncture test of 200,000 volts to ground for one minute. Diagram B shows the distribution of dielectric flux across a section of the bushing at xy, and since the strain in the material is proportional to the dielectric flux density, this strain is a maximum at the surface

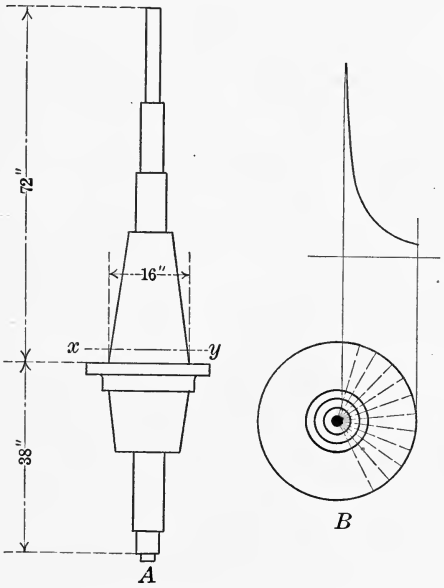


Fig. 282.—Solid bushing to withstand a puncture test of 200,000 volts for one minute.

of the conductor and a minimum at the outer surface of the bushing; the strain at any point is proportional to the potential gradient at the point, the curve of which is shown.

To reduce the maximum value of the potential gradient it is necessary to reduce the dielectric flux density at the surface of the conductor, which may be done by increasing the diameter of the conductor without reducing the thickness of the bushing, or by increasing the thickness of the bushing.

In order to make the outer layers of the bushing carry their

proper share of the voltage, the condenser type of bushing shown in Fig. 283 was designed. It consists of a number of concentric condensers of tin-foil and paper in series between the center conductor and the cover of the tank. When condensers are put in series, the voltage drop across each is inversely proportional to its electric capacity so that, in the condenser type of bushing, if each concentric condenser have the same thickness and the

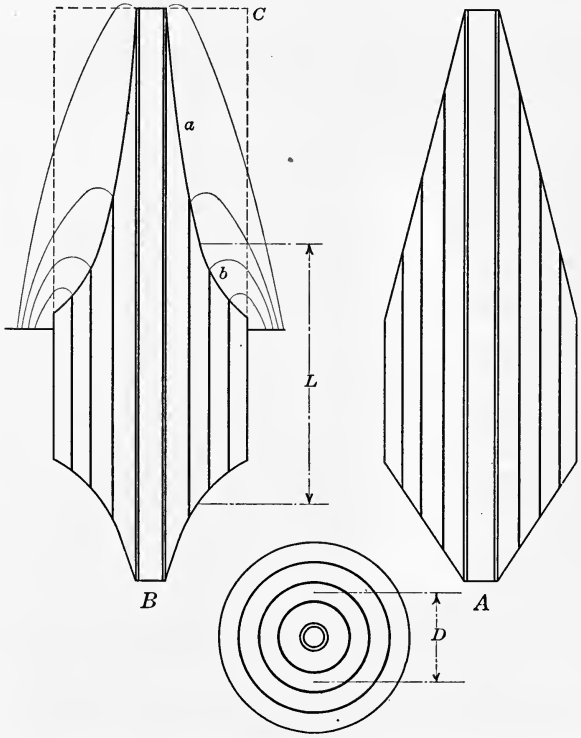


FIG. 283.—Diagrammatic representation of a condenser bushing.

same capacity, the voltages across the different layers will all be equal and the strain will be uniform through the thickness of the bushing.

The capacity of such condensers is proportional to

$$\frac{\text{area of plates}}{\text{thickness of dielectric}}$$

so that, if the thickness of the different layers of dielectric, and also the area πDL of the different layers of tin-foil, be kept con-

stant, the voltage across each thickness of insulation will be equal to the total voltage divided by the number of layers.

A bushing built with equal areas of plate and equal thicknesses of dielectric has the shape shown in diagram B; in such a case the voltages across the different layers are all equal, but the distance between two adjacent layers of tin-foil at *a* is greater than at *b*, so that the bushing is not economically designed for surface leakage. Condenser bushings are generally constructed as shown in diagram A; the surface distance between layers of tin-

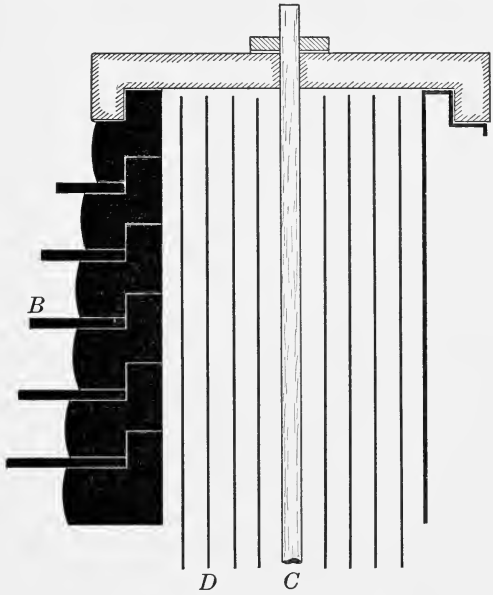


FIG. 284.—Oil filled bushing.

foil is constant, and the creepage distance under oil is considerably less than in air.

When such a bushing is in operation, lines of dielectric flux pass through the air as shown in diagram B, and since the edge of the tin-foil is thin, the stress in the air at that edge is large and the air breaks down there, forming a corona. The corona contains ozone and oxides of nitrogen which attack the adjoining insulation and finally cause breakdown of the bushing. To prevent the corona from forming it is necessary to eliminate the air from around the bushing, and for this purpose

the bushing is surrounded by a cylinder of fiber shown at *C*, which is then filled up with compound.

Another type of bushing which is largely used for high-voltage work is the compound filled bushing of which a section is shown in Fig. 284. This type of bushing is built up of composition rings which are carefully fitted into one another and then clamped between two metal heads by a bolt *C* which acts as the conductor.

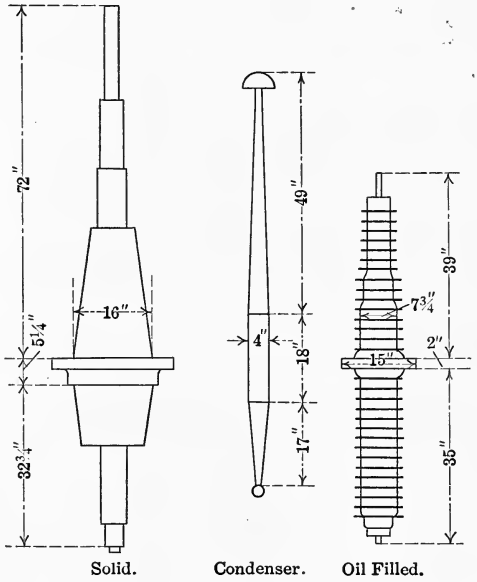


FIG. 285.—Bushings built to withstand 200,000 volts for one minute.

The creepage distance is obtained by building rings *B* of composition into the bushing. The whole center of the bushing is filled with compound or thick oil into which baffles of pressboard are placed as shown at *D* to prevent lining up of impurities in the compound along the lines of stress.

Fig. 285 shows a solid bushing, a condenser bushing, and an oil filled bushing, all able to withstand a puncture test of 200,000 volts for 1 minute.

318. Insulation of Coils.—In most high-voltage shell-type transformers of moderate output, and in all core-type transformers, the high-voltage coils have several turns per layer, and the voltage between end turns of adjacent layers, which is equal to the volts per turn multiplied by twice the turns per layer, may

become high, so that, while the cotton covering on the wire is generally sufficient for the insulation of adjacent conductors in the same layer, it is necessary to supply additional insulation between adjacent layers of the coils. It is also necessary to provide sufficient creepage distance, which is done, where the space is available, by making the insulation between layers extend beyond the winding as shown in Fig. 288. Where the space is not available, as in the coils for shell-type transformers, the construction shown in diagram A, Fig. 287, is often adopted; one turn of cord is placed at each end of each layer and the insulation between layers is carried out to cover the cord; this construction lessens the chance of damage to the coil while being handled.

For high-voltage transformers it is advisable to make the high-voltage winding of a number of coils in series with ample creepage distance and ample insulation between them, and to limit the voltage per coil to about 5000. The voltage between layers should not, if possible, exceed 350 volts.

319. Extra Insulation on the End Turns of the High-voltage Winding.—[1]A condenser consists of two conductors with dielectric between, so that the high- and low-voltage windings of a transformer, with the oil between, form a condenser; so also is there an electrostatic capacity between the high-voltage winding and the tank and between the low-voltage winding and the tank. A transformer may therefore be represented diagrammatically by a distributed inductance and capacity as shown in Fig. 286.

Suppose that the transformer is disconnected from the line and all at the ground potential. If the potential of one end A of the high-voltage winding be suddenly raised to a value E, the potential of the whole high-voltage winding will gradually rise to the same value. The potential cannot rise instantaneously; the voltage at B for example, cannot reach the value E until the condensers at that point have been charged to a value $Q = \int i dt = CE$, where C is the electro-static capacity between point B and the ground. The charging current i has to flow through the winding to B, and this takes a definite, though very short, time. The condensers near A will be charged first and the potential above ground of each point in the high-voltage winding is given by curves 1, 2 and 3 at successive instants.

At the instant after switching represented by curve 1, the difference of potential between two points A and D, that is,

[1] Walter S. Moody, *Transactions* of A. I. E. E., Vol. 26, page 1173.

across the first few turns of the winding, $=E_1$, which is almost equal to the full potential E; because of this high voltage between turns, it is necessary to insulate the end turns from one

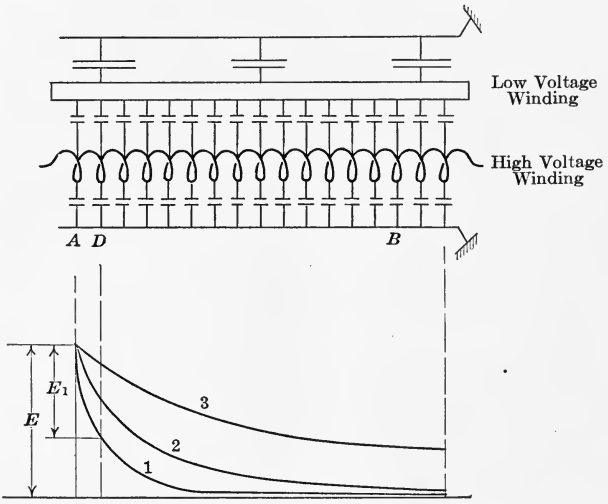


FIG. 286.—The potential of transformer coils immediately after switching.

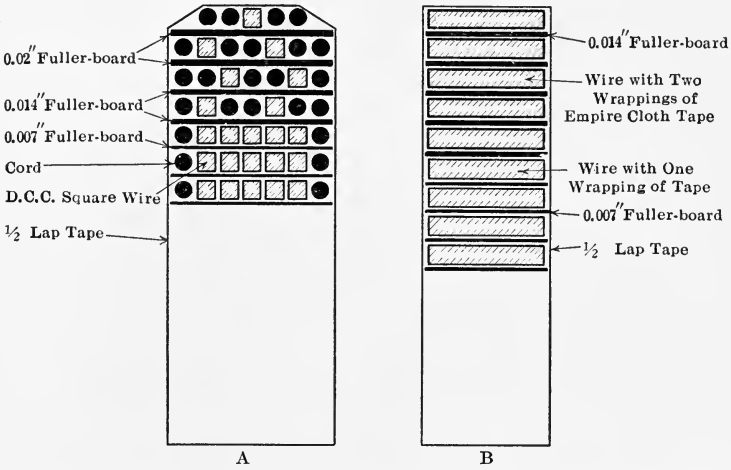


FIG. 287.—Insulation of the end turns of a 12,000 volt transformer coil.

another for a voltage between turns of many times the normal value.

Any sudden change in the potential of a transformer terminal,

such as that due to switching or to grounding of a line, will produce this high voltage between end turns.

Diagram A, Fig. 287, shows a method of insulating the end turns of a coil for a shell-type transformer, when the coil is of wire wound in layers, and diagram B, for a coil wound with strip

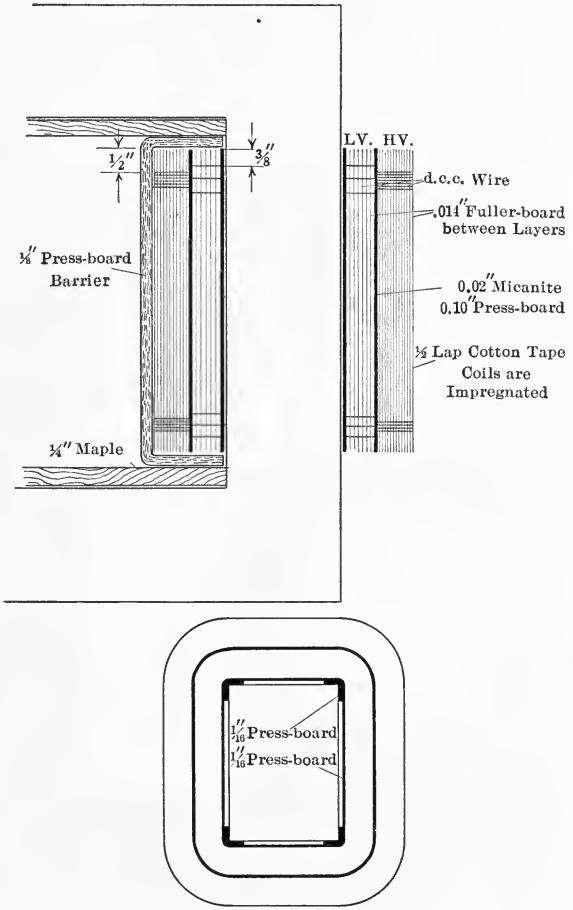


Fig. 288.—Insulation for a 2200/220 volt transformer.

copper. This extra insulation is put on each end of the high-voltage winding for a distance of about 75 ft., and then any taps that are required for the purpose of changing the transformer ratio are connected to the inside of the winding, so that the extra insulation is always on the end turns.

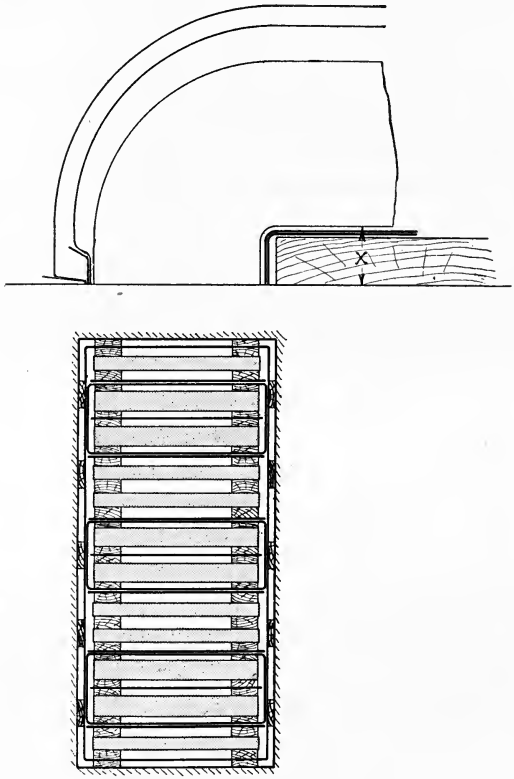


FIG. 289.—Insulation for a shell-type transformer.

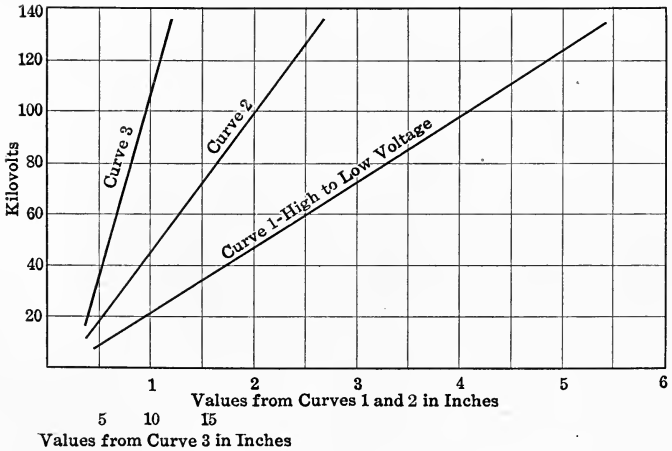


FIG. 290.—Spacings for the coils of shell-type transformers.

320. Insulation between the Windings and Core.—Fig. 288 shows the method of insulating a small core-type distributing transformer wound for 2200/220 volts.

Fig. 289 shows an example of a shell-type transformer wound for moderate voltage, an illustration of such a winding and insulation is shown in Figs. 271 and 272. The various spacings used for different voltages are given in Fig. 290, where curve 1 gives the distance between high- and low-voltage windings and also between the high-voltage winding and the core; curve 2 gives

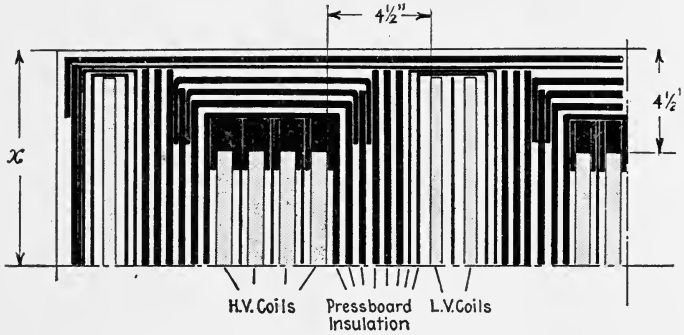


FIG. 291.—Insulation for 110,000 volts.

the total thickness of the pressboard in this distance; curve 3 gives the distance X between the high-voltage coil and the iron at the top and bottom of the core.

Fig. 291 shows an example of a shell-type transformer wound for 63,500/13,200 volts for operation in a Y-connected bank on 110,000 volts; the insulation to ground, and also from high- to low-voltage winding, is the same as that for a 110,000-volt transformer. One-quarter of the total winding is shown in plan; there are 12 high-voltage coils, so that the voltage per coil $= 5300$.

CHAPTER XLV

LOSSES, EFFICIENCY AND HEATING

321. The Losses.—The losses in a transformer are: the iron loss, the loss in the dielectric, and the copper loss.

The iron loss has already been discussed in Art. 310, page 439.

The loss in the dielectric, about which very little is known, causes the material to heat up and its dielectric strength to decrease. The loss is kept small by the use of ample distances between points at different potential, and the heating is kept small by a liberal supply of oil ducts through the insulation; the layers of solid insulation should not be thicker than 0.25 in.

322. The Copper Loss.

If MT is the mean turn of a transformer coil in inches,

M is the section of the wire in the coil in circular mils,

I is the effective current in each turn of the coil,

then the resistance of each turn $= \frac{MT}{M}$ ohms

and the loss per coil in watts $= \frac{MT \times I^2}{M} \times$ turns (65)

In addition to the above copper loss there is the eddy-current loss in the conductors, which may be large if the conductors are not properly laminated or arranged so that the leakage flux cuts their narrow sides. Diagram B, Fig. 292, shows part of the winding of a core-type transformer and the direction of the alternating leakage flux at one instant. If the coils are wound of strip copper on edge as shown at E, then eddy currents will flow in the direction shown by the crosses and dots, and the loss will be much larger than if the coils are wound with flat strip as shown at A.

Diagram D, Fig 292, shows part of the winding of a shell-type transformer, and the direction of the leakage flux and the eddy currents at one instant. If such a coil is laminated in the direction of the leakage lines it will be weakened mechanically; for that reason it is not advisable to laminate the conductors unless

they are wider than 0.5 in. for 60 cycles, or 0.75 in. for 25 cycles, for which values, and for ordinary current densities, the eddy-current loss will be about 20 per cent. of the calculated I^2R loss.

Even after the conductors of a core-type transformer have been laminated, considerable eddy-current loss may under certain circumstances be found in the windings; for example, *A*, Fig. 292, shows part of the winding of a low-voltage large-current transformer where the coil is made up of four wires in parallel; eddy currents tend to flow in the direction represented at one

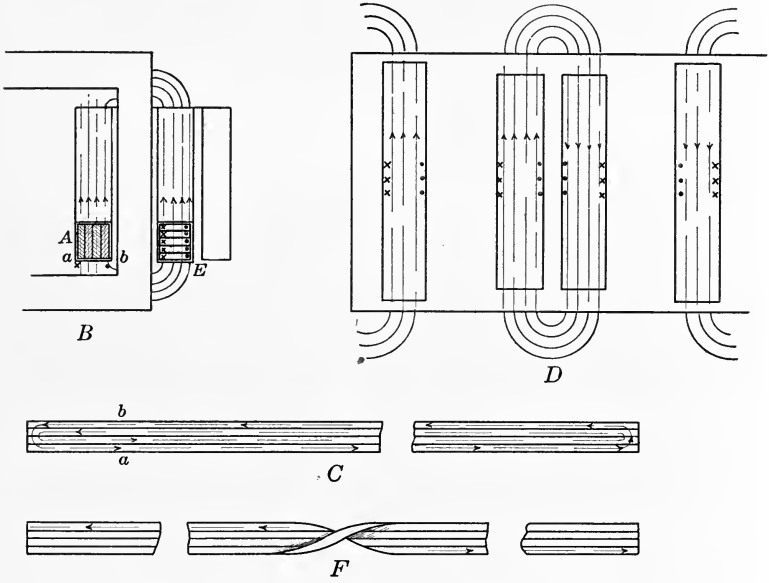


FIG. 292.—Eddy currents in transformer coils.

instant by the crosses and dots, and since the parallel wires are all connected together at the ends, the current will flow down one wire *a*, cross through the soldered joint at the end to wire *b*, up which it will pass and then return by the other soldered joint to conductor *a*, so that if the coil is developed on to a plane the currents will flow as shown in diagram C. To eliminate this circulating current the bunch of wires is given a half twist, as shown at *F*, so that in any one strip there are two e.m.fs., produced by the leakage flux, which are equal and opposite, the resultant e.m.f. is therefore zero and no circulating current will flow.

323. The Efficiency.

If $C.L.$ is the iron or core loss in watts,

I^2R the copper loss in watts,

EI the output of the transformer in watts,

then, η, the efficiency $= \dfrac{\text{output}}{\text{output} + \text{losses}}$

$$= \frac{EI}{EI + I^2R + C.L.}$$

The efficiency is a maximum when

$$\frac{d\eta}{dI} = 0$$

or $$(EI + I^2R + C.L.)E - EI(E + 2IR) = 0$$

or $$I^2R = C.L.$$

that is, when the copper loss is equal to the core loss.

The all-day efficiency, which is of importance in distributing transformers, is defined by the following equation:

$$\text{all-day efficiency} = \frac{EI \times X}{EI \times X + C.L. \times 24 + I^2R \times X}$$

where X is the number of hours during which the transformer is loaded each day, and 24 is the number of hours during which the iron loss is supplied. Distributing transformers should therefore be designed to have as small a core loss as possible, because this loss has to be supplied continuously.

324. Heating of Transformers.—Since nearly all except instrument transformers are oil immersed, the subject of cooling by natural draft is not of importance and shall not be discussed.

When a transformer is in operation under oil the heat generated in the core and windings has to be carried to the tank, from the external surface of which it is dissipated to the air. When the oil in contact with the transformer surface is heated, it becomes lighter and rises, and cool oil flows in from the bottom of the tank to take its place, so that a circulation of oil is set up, as shown in Fig. 293. That it may circulate freely the oil should have a low viscosity, and the lighter the oil the better it is as a cooling medium for transformers.

325. The Temperature Gradient in the Oil.—Fig. 293 shows a core-type transformer and also shows the temperature of the oil at different points along its surface. At the bottom of the tank the oil temperature is T_b; the oil moves along the circulation path and its temperature rises and reaches a maximum value T_t

at the top of the transformer, it then passes to the tank and, as it moves downward, the heat is gradually given up to the tank. After a number of hours, during which the whole body of the transformer and the oil are absorbing heat and being raised in temperature, conditions become fixed, the oil circulation and its temperature cycle become definite, and the different points of the transformer have their maximum temperature.

326. The Temperature Gradient in a Core-type Transformer.— Consider the core of the transformer shown in Fig. 293. Iron is a good conductor of heat along the laminations, so that the difference in temperature between the points A and B of the

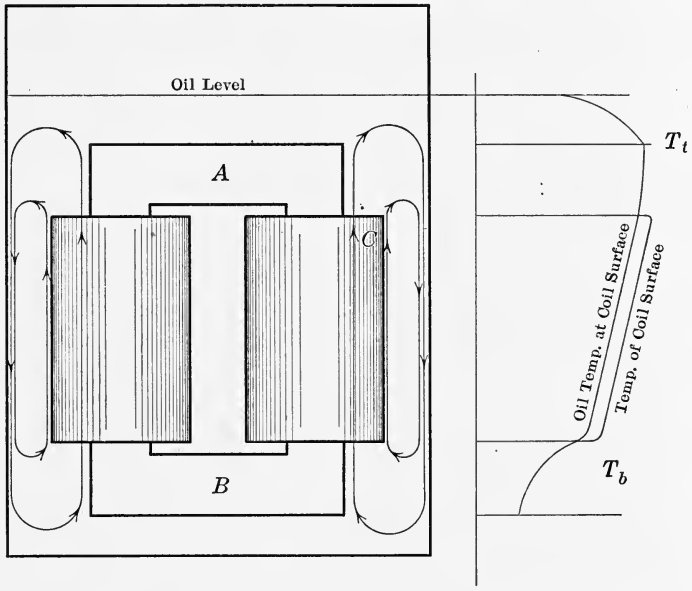


FIG. 293.—Temperature gradient in a core-type transformer.

core cannot be great, and the difference in temperature between the core and the adjacent oil must be greater at the bottom of the core than at the top; because of this, much of the heat generated in the top part of the core is conducted downward and dissipated from the surface at the bottom, that is, the bottom part of the core surface is more active than the top part in dissipating the heat due to the iron losses.

The conditions are different for the windings. These are formed of insulated wire wound in layers, and, because of the

number of layers of insulation in the length of the coil, very little of the heat generated in the top turns will be conducted downward through the winding. Most of the heat generated at any point in the winding will be conducted to and dissipated from the nearest coil surface, so that the watts dissipated per unit area of coil surface, and therefore the temperature difference between the coil surface and the adjacent oil, will be approxi-

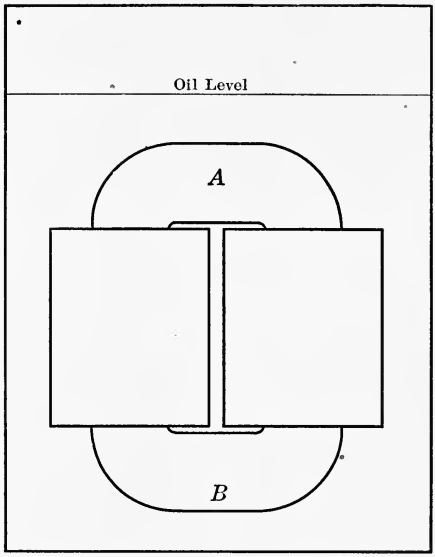


FIG. 294.—Shell-type transformer in tank.

mately constant at all points, and the temperature of the surface of the coil will be a maximum at the top of the transformer. The hottest part of the whole winding will be at C.

Measurement of the temperature rise by resistance gives little information as to the temperature of the hottest part of the coil of a core-type transformer, because the temperature so found is the average temperature and may be less than that of the oil measured at the top of the transformer.

327. The Temperature Gradient in a Shell-type Transformer.—Fig. 294 shows such a transformer. Since the core is laminated horizontally, and since iron is a poor conductor of heat across the laminations, most of the heat generated at any point in the core is conducted to and dissipated from the nearest core surface, so that each part of the core surface is equally active in dissipating

heat, and the temperature of the core is a maximum at the top and a minimum at the bottom.

The conditions are different for the windings. These are made up of what are known as pancake coils, which are thin and have a large radiating surface. Since the top layers of the winding at A are connected directly to the bottom layers at B by a short length of copper, the temperature difference between

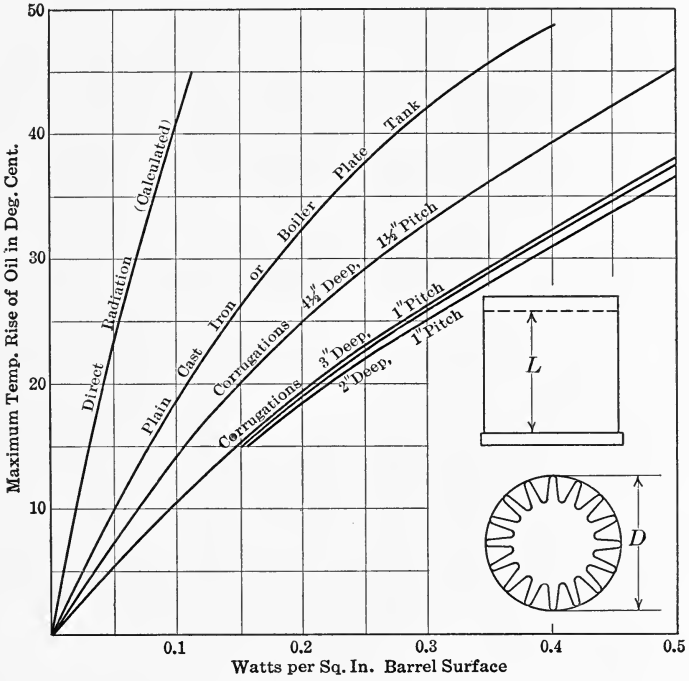


Fig. 295.—Heating curves for transformer tanks.

A and B cannot be very great, and the difference in temperature between the winding and the adjacent oil must be greater at the bottom than at the top. Because of this much of the heat generated in the top part of the winding is conducted downward and dissipated from the winding surface at the bottom of the coil. The temperature of the winding is more uniform throughout than in a core type transformer, and resistance measurements are of more value.

328. The Temperature of the Oil.—The rise in temperature of

the oil over that of the external air depends principally on the loss to be dissipated, and on the external surface of the tank.

The heat in the oil is transmitted through the tank and dissipated from its external surface. Part of this heat is dissipated by direct radiation, and part by convection currents which flow up the sides of the tank. For a plain boiler-plate tank, without ribs or corrugations, the highest temperature rise of the

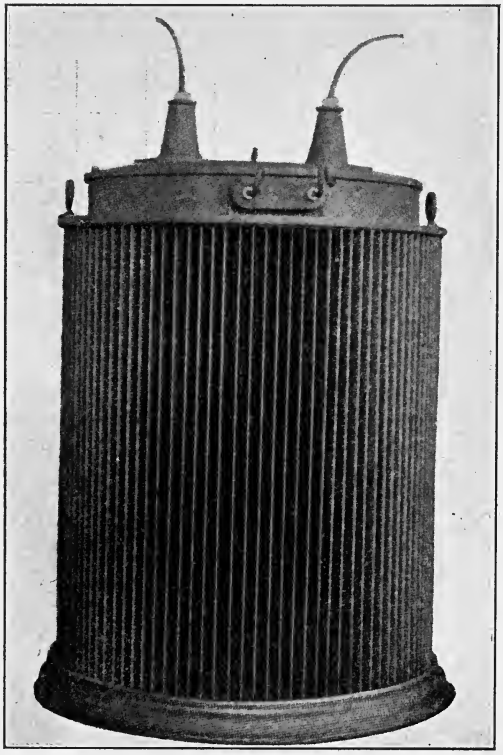


Fig. 296.—Corrugated tank.

oil is plotted against watts per square inch external surface in Fig. 295, for a tank which is round in section, and which has a height of approximately 1.5 times the diameter. This temperature rise is made up principally of the temperature difference between the air and the tank, and that between the tank and the oil; the former is about three times as large as the latter.

The temperature rise of the oil may be reduced by increasing

the surface of the tank which is readily done by making it corrugated, as shown in Fig. 296. This increase in surface does not increase the direct radiation from the tank, because only that component of surface which is perpendicular to a radius is effective; for this reason the watts per square inch for a given temperature rise does not increase directly as the increase in surface. Consider the curve in Fig. 295 for corrugations 3 in. deep and spaced 1 in. apart; the external surface of the tank is

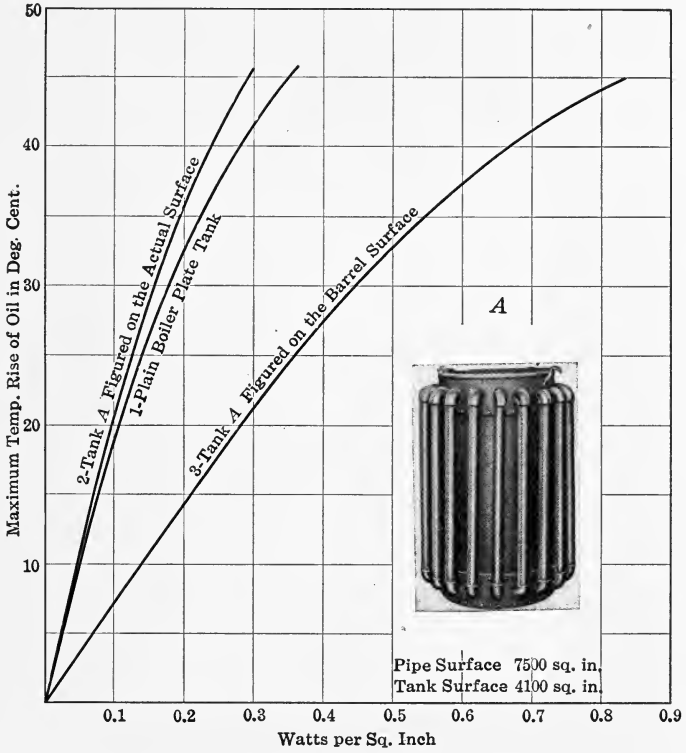


FIG. 297.—Heating curves for transformer tanks.

increased about six and one-half times, while the watts per square inch for a given temperature rise is only increased about 60 per cent. over the value for a plain boiler-plate tank.

Fig. 297 shows the results of tests on a boiler-plate tank with external pipes added to improve the circulation. It will be seen from curves 1 and 2 that the watts per square inch of total surface for a given temperature rise is almost as large in

the special tank as in the plain boiler-plate tank or, comparing curves 1 and 3, the surface of the special tank is 2.8 times that of the plain boiler-plate tank, while the watts per square inch for a given temperature rise is increased about 2.5 times.

Fig. 298 shows a tank built so as to present three cooling surfaces to the air.

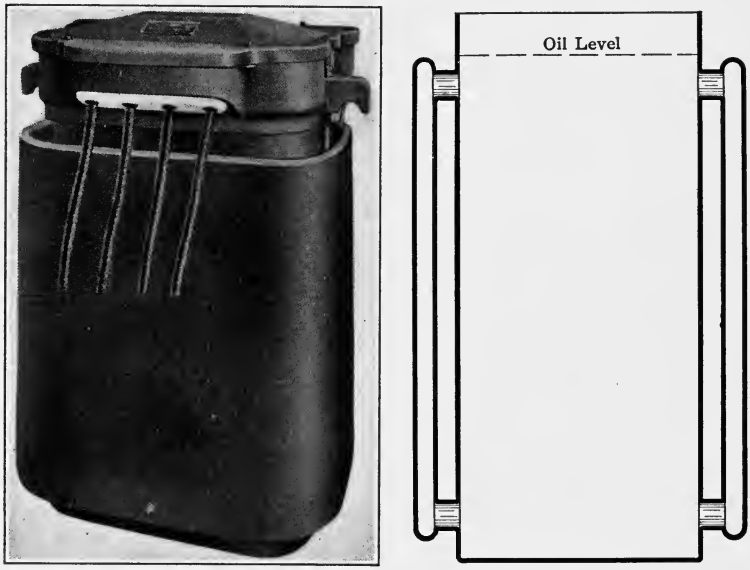


FIG. 298.—Tank with large cooling surface.

329. Air-blast Transformers.—Fig. 299 shows such a transformer. The problem in this case is like that discussed fully on page 281 on the heating of turbo generators; 150 cu. ft. of air is supplied per minute per k.w. loss, and the average temperature of the air increases about 12 deg. cent. between the inlet and outlet. The temperature of the coils and core is kept within reasonable values by providing the necessary radiating surface, using the formula,

$$\text{watts per square inch for 1 deg. cent. rise} = 0.0245(1 + 0.00127\,V),$$

where the temperature rise is measured on the surface and V is the velocity of the air across the surface in feet per minute. In the case of vent ducts, and surfaces which are facing one another, it must be noted that there can be no radiation term

because the surfaces are at the same temperature and in such cases,

watts per square inch for 1 deg. cent. rise $= 0.0245(0.00127\ V)$.

The air used should be filtered, otherwise the ducts will become clogged up with dust and the transformer get hot. Dampers are usually supplied at the top of the case so that the distribution of the air through the core and coils may be controlled.

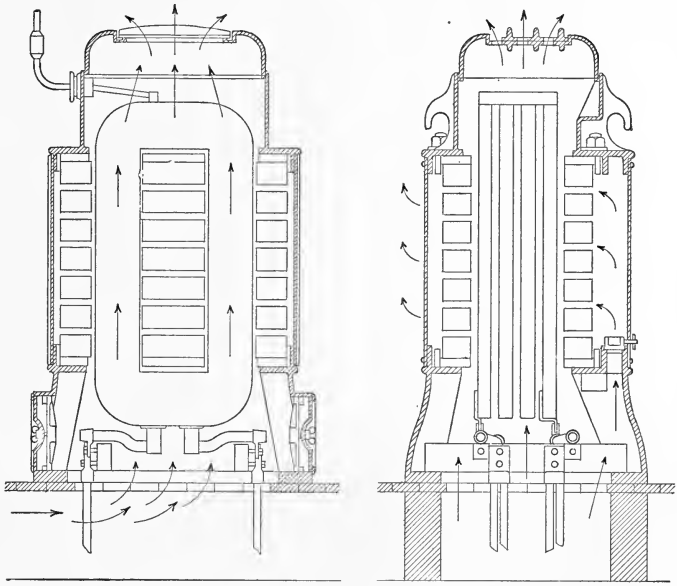


FIG. 299.—Air blast transformer.

330. Water-cooled Transformers.—If coils of copper pipe carrying water be placed at the top of the case as shown in Fig. 300, then the oil which is heated by contact with the transformer will rise and carry the heat to the cooling coils.

If t_i is the inlet temperature of the water,

t_o is that at the outlet,

then each pound of water passing through the coils per minute takes with it $(t_o - t_i)$ lb. calories per minute

or $32(t_o - t_i)$ watts.

With 2.5 lb. of water per minute per kilowatt loss the average temperature rise of the water will be 12.5 deg. cent.

It is advisable in water-cooled transformers to immerse the whole of the cooling coil, otherwise, due to the low temperature of the water passing through, moisture will deposit on the coil and get into the oil. The coil should be of seamless copper tube about 1 1/4 in. external diameter, and the drain tap should be

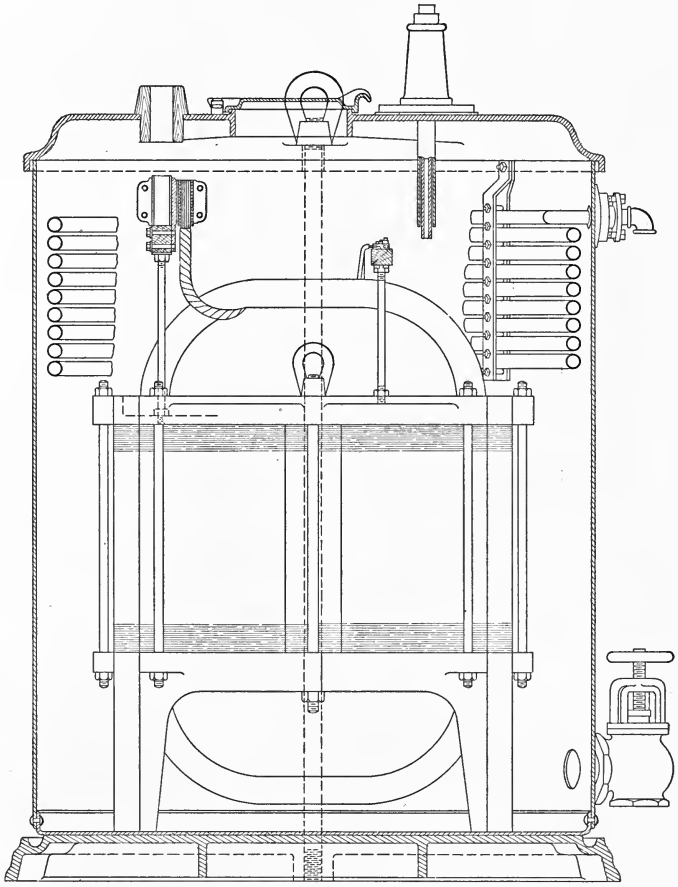


FIG. 300.—Water cooled transformer.

at the bottom of the spiral so that, when not in operation, the spiral will be empty and therefore will not burst in frosty weather.

331. Heating Constants used in Practice.—The calculation of the temperature rise of a transformer is so complicated by the oil circulation, and by the temperature gradient in the oil, coils and

core that, until the results of a complete investigation of the subject are available, empirical constants will have to be used.

The necessary tank surface for a given loss is found from Fig. 295.

The watts per square inch coil surface

= 0.35 for self-cooled shell-type coils wound with small wire

= 0.4 for self-cooled shell-type coils wound with strip copper

= 0.35 for small core-type transformers; these figures may be increased 20 per cent. for transformers which are water cooled or cooled by forced draft.

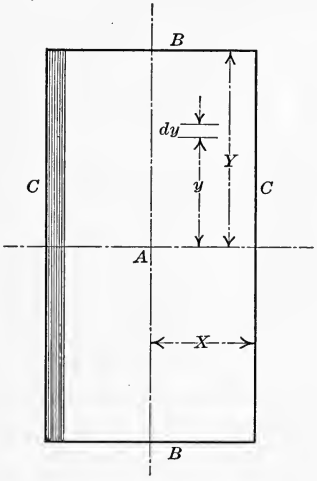


FIG. 301.—Heat paths in a transformer core.

The watts per square inch iron surface

= 1.0 for both core and shell type; the area of cooling surface is taken as the edge surface and half of that part of the flat surface which is exposed to the oil circulation, see Art. 332.

The watts per square inch water pipe surface

= 1.0, for a 1.25-in. pipe, the supply being 2.5 lb. per minute per k.w. loss.

332. Effect of Ducts.—It is difficult to determine how effective the ducts are in keeping a transformer core cool. Fig. 301 shows a block of iron which is laminated vertically. The hottest part of the iron is at A and the temperature difference from A to B.

$$= T_{ab}$$

$$= (\text{watts per cu. in.})\frac{Y^2}{3} \text{ deg. cent; page 105}$$

that between A and C

$$= T_{ac}$$

$$= (\text{watts per cu. in.})\frac{56X^2}{3} \text{ deg. cent.}$$

The temperature difference between surface B and the oil

$$= (\text{watts per cu. in.})\, Y \times 16$$

since the temperature difference between the iron and the adjoin-

ing oil is 16 deg. cent. per watt per square inch. The temperature difference between surface C and the oil

$$= (\text{watts per cu. in.}) X \times 16$$

The relative heat resistance of the two paths may be taken approximately as

$$\frac{\text{path } B}{\text{path } C} = \frac{Y\left(\frac{Y}{3}+16\right)}{X\left(\frac{56X}{3}+16\right)}$$

If the ducts are spaced 2 in. apart, so that $X=1.0$ in., then for different values of Y the relative heat resistance may be found from the following table:

RELATIVE HEAT RESISTANCE

X	Y	Along laminations	Across laminations
1 in.	1 in.	0.48	1.0
1 in.	2 in.	1.0	1.0
1 in.	3 in.	1.5	1.0
1 in.	4 in.	2.0	1.0

that is, for the particular values taken, the duct surface is half as effective as that of the edge, if it is as well supplied with cool oil, that is, if the ducts are vertical and of sufficient width to allow free circulation.

333. The Maximum Temperature in the Coils.—Although the maximum temperature in the coils of a transformer cannot readily be determined, it is necessary to find out on what it depends and what its probable value may be.

In Fig. 302, which shows part of the coil of a transformer, let the thickness of the coil be small compared with the mean turn MT, and assume that the heat passes in both directions from the center line L.

If, of the thickness x, the part kx is insulation and $(1-k)\ x$ is copper then the current in the section xy

$$= xy(1-k) \times \text{amperes per square inch}$$

$$= \frac{xy\ (1-k)}{\text{sq. in. per amp.}}$$

$$= \frac{xy\ (1-k) \times 1.27 \times 10^6}{\text{cir. mils per amp.}}$$

The resistance of a ring of length MT and section $xy(1-k)$ sq. in.

$$= \frac{MT.}{xy\ (1-k) \times 1.27 \times 10^6}$$

the loss in this ring = current^2 × resistance

$$= \frac{(xy\ (1-k) \times 1.27 \times 10^6)^2 \times MT.}{xy\ (1-k) \times 1.27 \times 10^6\ (\text{cir. mils per amp.})^2}$$

$$= \frac{MT \times xy\ (1-k) \times 1.27 \times 10^6}{(\text{cir. mils per amp.})^2}$$

The heat due to this loss crosses the section of thickness dx, of which $k \times dx$ is insulation, and since the specific conductivity of

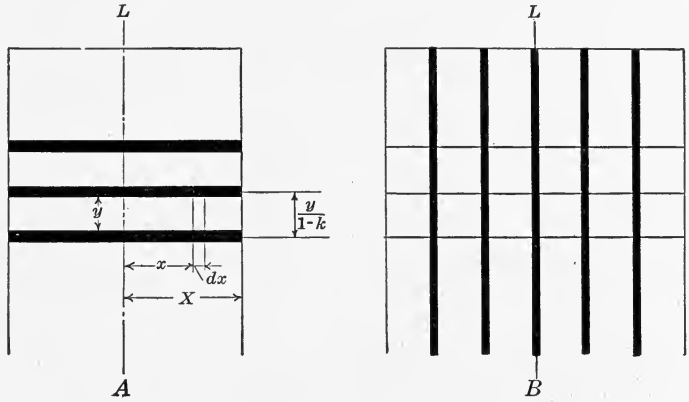


FIG. 302.—Part of a transformer coil showing insulation between layers.

insulating material = 0.003, in watts per inch cube per deg. cent. difference in temperature, therefore the difference in temperature between the center and the surface

$$= \theta = \int_o^\theta d\theta = \int_o^x \frac{MT \times xy(1-k) \times 1.27 \times 10^6}{(\text{cir. mils per amp.})^2} \times \frac{k \times dx}{MT \times y} \times \frac{1}{0.003}$$

$$= \frac{2.1 \times 10^8\ k(1-k)X^2}{(\text{cir. mils per amp.})^2} \qquad (66)$$

Consider the following example: A core-type transformer with the windings insulated as in Fig. 288, has the high-tension winding made with No. 12 square d. c. c. wire. The high-voltage winding is 1 in. thick, the current density is 1600 cir. mils per ampere, and there is one thickness of 0.007 in. fullerboard between layers;

it is required to find the maximum difference in temperature between the inner and outer layers of the winding.

Thickness of wire	$=0.0808$
Thickness of cotton covering	$=0.01$
Thickness of fullerboard	$=0.007$
Value of k	$=\frac{0.017}{0.0978}=0.174$
Value of $1-k$	$=0.89$
Temperature difference	$=\frac{2.1\times10^8\times0.174\times0.89}{1600^2}$
	$=13$ deg. cent.

If round wire is used instead of square, then the contact area between adjacent layers is greatly reduced, and the temperature difference increased. It is advisable for such coils as that discussed above to use square or rectangular wire and to limit the thickness of the coil to 1 in., and the current density to about 1600 cir. mils per ampere.

334. The Section of the Wire in the Coils.—Diagram A, Fig. 302, shows part of a coil of a shell-type transformer, and B shows part of a coil of a core type transformer.

The loss in one layer of the winding, as may be seen from the last Art.

$$=\frac{MT\times2Xy(1-k)\times1.27\times10^6}{(\text{cir. mils per amp.})^2}\text{ watts}$$

the corresponding radiating surface $=MT\times\frac{2y}{1-k}$ sq. in.

therefore the watts per sq. in. $=\frac{2X(1-k)^2\times1.27\times10^6}{2\times(\text{cir. mils per amp.})^2}$

and cir. mils per amp. $=8\times100\sqrt{\frac{2X(1-k)^2}{\text{watts per sq. in.}}}$ (67)

$=1350\sqrt{2X(1-k)^2}$ when watts per sq. in. $=0.35$

$=1260\sqrt{2X(1-k)^2}$ when watts per sq. in. $=0.4$

CHAPTER XLVI

PROCEDURE IN DESIGN

335. The Output Equation.

$E = 4.44 T \phi_a f 10^{-8}$, formula 62, page 439;

and EI = the watts output

$$= 4.44 TI \times \phi_a \times f \times 10^{-8}$$

$$= 4.44 \frac{\phi_a^2}{k_t} \times f \times 10^{-8} \text{ where } k_t = \frac{\phi_a}{TI} = \frac{\text{magnetic loading}}{\text{electric loading}}$$

The volts per turn of coil $= V_t$

$$= 4.44 \phi_a f 10^{-8}$$

$$= 4.44 f \times 10^{-8} \sqrt{\frac{\text{watts} \times k_t}{4.44 f 10^{-8}}}$$

$$= \sqrt{\text{watts}} \times \sqrt{4.44 k_t f 10^{-8}}$$

Now $E = 4.44 T \phi_a f 10^{-8}$

so that, for a given voltage, the lower the frequency the larger the product $\phi_a \times T$. If a transformer is built with a large number of turns, so that $k_t = \frac{\phi_a}{TI}$ is small, then the copper loss is large because of the large number of turns, and the core loss is small because of the low frequency; such a transformer would therefore have its maximum efficiency at a fraction of full-load; see page 463. In order that the efficiency may be a maximum at or near full-load, the full load copper and the core loss must be approximately equal and the flux must increase as the frequency decreases; it is found in practice that the value of $k_t = \frac{\phi_a}{TI}$ is approximately inversely proportional to the frequency, or that $k_t f$ is approximately constant, therefore

volts per turn, V_t = a const. $\times \sqrt{\text{watts}}$ (68)

where the following average values of the constant are found in practice;

$\frac{1}{80}$ for core-type distributing transformers

$\frac{1}{50}$ for core-type power transformers

$\frac{1}{25}$ for shell-type power transformers.

The constant for the distributing transformer is less than that

for the power transformer because, while in the latter the highest efficiency is desired around full-load, in the former a small core loss and a high all day efficiency is desired. To obtain a small core loss it is necessary to keep the value of $k_t = \frac{\phi_a}{TI}$ small, and therefore the constant in formula 68 must be small.

The constant is different for core- and for shell-type transformers because of the difference in construction. Fig. 303 shows the ordinary proportions of a core-type transformer; the

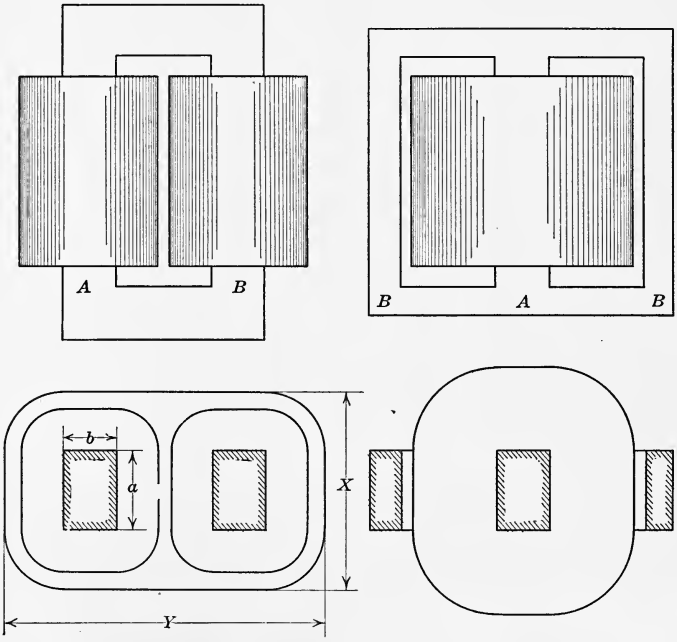


FIG. 303.—Core-type transformer. FIG. 304.—Shell-type transformer.

distance a is generally about $1.5 \times b$ so as to keep the ratio of X to Y within reasonable limits, and prevent the use of a thin wide tank.

If the coil on limb B of the transformer in Fig. 303 be placed on limb A, and limb B then split up the center and one-half bent over to give Fig. 304, a shell-type transformer is produced which has the same amount of copper and iron as the corresponding core-type transformer. The resulting shell-type transformer is flat and low, so that the tank required to hold it takes up con-

siderable floor space; the proportions are therefore changed so as to give the ordinary shape shown in Fig. 305, and for a given rating it will be found that the ratio $\frac{\phi_a}{TI}$ is about four times as large for the transformer in Fig. 305 as it is for that in Figs. 303 or 304; that is, the shell-type transformer has generally twice the flux and half the number of turns that the core-type transformer has for the same rating. The distance a is generally about $6 \times b$ to give a reasonable shape of core.

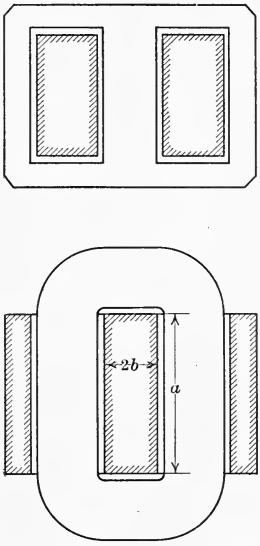


FIG. 305.—Shell-type transformer.

336. Procedure in the Design of Core-type Transformers.

The volts per turn $= \frac{1}{50}\sqrt{\text{watts}}$ for power transformers

$= \frac{1}{80}\sqrt{\text{watts}}$ for distributing transformers.

The number of coils is chosen so as to keep the voltage per coil less than 5000, but there should not be less than two high-voltage and two low-voltage coils; the number of turns per coil is equal to

$$\frac{\text{terminal voltage}}{\text{volts per turn} \times \text{number of coils}}.$$

The depth of the coil measured from the nearest oil duct should not be greater than 1.0 in., except in the case of small distributing

transformers insulated as shown in Fig. 288 which have no oil duct between the low voltage winding and the core. In such a case the depth from the core to the oil may be 2 in., the reason being that the heat in the inner layers of the winding is conducted through the insulation into the core and dissipated by the core surface.

The section of the wire in circular mils is found from formula 67, page 475, namely,

cir. mils per amp. $= 1350\sqrt{(1-k)^2 \times 2X}$ where

$X =$ the greatest depth from the inside of the winding to the nearest oil or core surface,

$(1-k)^2 =$ per cent. copper in the vertical layers of the winding $\times$ that in the horizontal layers

and the area of the wire is the product of the full-load current in the winding and the circular mils per ampere. The section of wire should be chosen so that it is not thicker than 0.125 in., and the wire should be wound flat as shown at *A*, Fig. 292.

The number of layers in the winding is the same as the number of wires in the assumed depth of the coil; the number of conductors per layer is the total number of turns per coil divided by the number of layers, and the height of the winding is the number of turns per layer multiplied by the width of the wire in the direction parallel to the limbs of the core.

The flux in the core is found from formula 62, page 439; namely,

$E = 4.44 \times$ turns $\times$ flux $\times$ frequency $\times 10^{-8}$

and the core area $= \dfrac{\text{flux in core}}{\text{core density}}$, where the value of the core density is taken for a first approximation,

$= 65{,}000$ lines per square inch 60-cycle distributing transformers

$= 75{,}000$ lines per square inch 25-cycle distributing transformers

$= 90{,}000$ lines per square inch 60-cycle power transformers

$= 80{,}000$ lines per square inch 25-cycle power transformers

alloyed iron being used for 60-cycle transformers, to keep down the loss in the distributing transformer so as to have a high all-day efficiency, and to keep down the heating in the power transformers. Ordinary iron is used for 25-cycle transformers, because, due to the low frequency, the loss is generally small, while the densities have to be kept low in order that the magnetizing current will not be too large a per cent. of the full-load current.

The core and windings are drawn in to scale; the losses, magnetizing current, resistance and reactance drops are deter-

mined, and the size of the tank is fixed. If the transformer as designed does not meet the guarantees then certain changes must be made.

If the core loss is too high, either the core density must be reduced, which will increase the size of the transformer, or the total flux must be reduced, which will require an increase in the number of turns for the same voltage, and, therefore, an increase in the copper loss.

The resistance and the reactance drops may both be reduced by a reduction in the number of turns, because the resistance is directly proportional to the number of turns, while the reactance is proportional to the square of the number of turns.

Example.—Design and determine the characteristics of a 15-k. v. a., 2200- to 220-volt, 60-cycle distributing transformer.

Design of the High-voltage Winding

Volts per turn	$=1.52$
Total number of turns	$=1440$
Coils	$=2$; one on each leg
Turns per coil	$=720$
Total depth of winding	$=2.0$ in., assumed
Cir. mils per ampere	$=1350\times\sqrt{2}\times 0.9\times 0.8$ $=1620$
Full-load current	$=6.8$ amp.
Section of wire	$=11{,}000$ cir. mils; use no. 10 square B. & S. $=0.1019$ in. $\times 0.1019$ in. which has a section of 13,000 cir. mils, not allowing for rounding of the corners
Insulation	$=0.01$ in. double cotton-covering, 0.014 in. fullerboard between layers $(1-k)$ vertical $=\frac{0.1019}{0.1119}=0.91$ $(1-k)$ horizontal $=\frac{0.1019}{0.1259}=0.81$
Number of layers	$=\frac{1.0}{0.1019+0.01+0.014}$ $=8$
Turns per layer	$=90$
Height of winding	$=90\times(0.1019+0.01)$ $=10$ in.
Height of opening	$=11.75$ in.; see Fig. 288

Design of the Low-voltage Winding

Total number of turns	$=1440\times$ ratio of transformation $=144$

Turns per coil $=72$

Circular mils per amperes $=1620$; the same as for the high-voltage coil

Section of wire $=110{,}000$ circular mils
$=0.087$ sq. in.
$=0.11$ in. $\times 0.8$ in.; this will be difficult to wind on a small transformer because of its width, therefore, change the winding as follows:

Turns per coil $=72$; number of coils $=4$ connected two in parallel; size of wire $=0.11$ in. $\times 0.4$ in.

Insulation $=0.015$ in. cotton covering
0.014 in. fullerboard between layers

Turns per layer $=\dfrac{\text{winding height}}{\text{width of wire}}$
$=\dfrac{10.0}{0.415}$
$=24$

Number of layers per coil $=\dfrac{72}{24}$
$=3$

Number of layers per leg $=6$, because there are two coils per leg

Depth of winding $=(0.11 \text{ in.} + 0.015 \text{ in.} + 0.014 \text{ in.})\ 6$
$=0.83$ in.

Thickness of insulation between the high- and low-voltage coils $= 0.12$ in.

Design of Core

Flux $=5.72 \times 10^5$

Core density, assumed $=65{,}000$ lines per square inch

Necessary core section $=8.8$ sq. in

Actual section adopted $=2.5 \times 4$ and stacking factor $=0.9$

Width of opening $=4.5$ in., to allow a little clearance between coils on different legs; see Fig. 288.

Calculation of the Losses and the Magnetizing Current

Mean turn of low-voltage coil $=16.6$ in.

Resistance of secondary winding $=\dfrac{144 \times 16.6}{2 \times 0.11 \times 0.4 \times 1.27 \times 10^6}$
$=0.021$ ohms

Mean turn of high-voltage coil $=24$ in.

Resistance of primary winding $=\dfrac{1440 \times 24}{13000}$
$=2.6$ ohms

Loss in primary winding $=2.6 \times 6.8^2$
$=120$ watts

Loss in secondary winding	$=0.021\times68^2$
	$=97$ watts
Weight of core	$=110$ lb.
Actual core density	$=63{,}500$ lines per square inch
Core loss in watts (alloyed iron)	$=110\times0.9$; from Fig. 275, page 442.
	$=99$ watts
Volt amperes excitation	$=110\times5$
	$=550$
Per cent. exciting current	$=3.3$

Calculation of the Regulation

$$\text{Equivalent primary reactance} = 2\pi f\times3.2\ T_1^2\times\frac{MT.}{L}\left(\frac{d_1}{3}+\frac{d_2}{3}+S\right)10^{-8}\times2$$

where $f = 60$

$T_1 = 720$

$$MT = \frac{16.6+24}{2}$$

$= 20.3$ in.

$L = 10$ in.

$d_1 = 1.0$ in.

$d_2 = 0.83$ in.

$S = 0.12$ in.

coils $= 2$

$$X_{eq}\text{ of primary} = 2\pi\times60\times3.2\times720^2\times\frac{20.4}{10}\left(\frac{1.0+0.83}{3}+0.12\right)\times10^{-8}\times2$$

$= 18.6$ ohms

The reactance drop referred to the primary	$=18.6\times6.8$
	$=126$ volts
	$=5.8$ per cent.
The primary resistance drop	$=2.6\times6.8$
	$=17.6$ volts
The secondary resistance drop	$=0.021\times68$
	$=1.43$ volts
The resistance drop referred to the primary	$=17.6+1.43\times\frac{2200}{220}$
	$=31.9$ volts
	$=1.44$ per cent.

Design of the Tank

The total losses at full-load are:

Iron loss,	99 watts
Primary copper loss,	120 watts
Secondary copper loss,	97 watts
Total loss,	316 watts

The watts per square inch for 35 deg. cent. rise of the oil = 0.225, therefore the tank surface in contact with the oil

$$=\frac{316}{0.225}$$

$= 1400$ sq. in.

337. Procedure in the Design of a Shell-type Transformer.—The work is carried out in exactly the same way as for a core-type transformer except that the width of the coil is seldom made greater than about 0.5 in., so as to provide ample radiating surface and allow the use of comparatively high copper densities.

The number of coils is again chosen so as to keep the voltage per coil less than 5000; still further subdivision of the winding may be required in some cases to reduce the reactance.

Example.—Design a 1500 - k.v.a., 63,500- to 13,200-volts, 25-cycle power transformer for operation in a three-phase bank on a 110,000 volt line.

Design of the High-voltage Winding

Volts per turn $=49$

Total number of turns $=1300$

Coils $=12$

Turns per coil $=108$ average; use 10 coils with 113 turns and 2 end coils with 85 turns

Width of coil, assumed $=0.4$ in.

Because of the high voltage the distance x, Fig. 291, will be large, and there will be little iron saved by making the strip wider than 0.4 in. with the idea of keeping the distance x small.

Circular mils per ampere $=1260\times\sqrt{0.4\times0.6}$

where $(1-k)^2$ is assumed to be $= 0.6$

$=615.$

Section of wire $=14{,}500$ circular mils

$=0.0285\times0.4$ in.

Insulation $=0.015$ in. cotton covering

0.014 in. fullerboard

$$(1-k)=\frac{0.0285}{0.0575}=0.5$$

Under ordinary circumstances the calculation for the size of conductor would be repeated using the correct value for $(1-k)$ to find the value of the circular mils per ampere. In a very high voltage transformer, however, the amount of copper is small compared with the amount of iron and insulation, so that it is not advisable to run the chance of high temperature rise for a small gain in the amount of copper used. The conductor chosen for this transformer is therefore 0.03×0.4 in.

Height of winding $=$ turns $\times$ thickness of insulated wire

$=113\times(0.03+0.015+0.014)$

$=6.7$ in.

$=7$ in. to allow for bulging

Space between winding and core $=4.5$ in., from Fig. 290.

Width of opening $=7+(2\times4.5)$

$=16$ in.

The arrangement of the winding must now be decided on and several possible methods are shown in Fig. 306. *A* would give a very long core and take up a large floor space, it would also have a large core loss and magnetizing current because of the large amount of iron in the magnetic circuit, but of the three shown it would have the lowest reactance.

B would have a lower magnetizing current and a lower core loss than *A*, it would also be considerably cheaper, but would have a larger reactance.

C will have values of core loss, magnetizing current and reactance between these of *A* and *B*, and the design will be completed for this arrangement to find its characteristics.

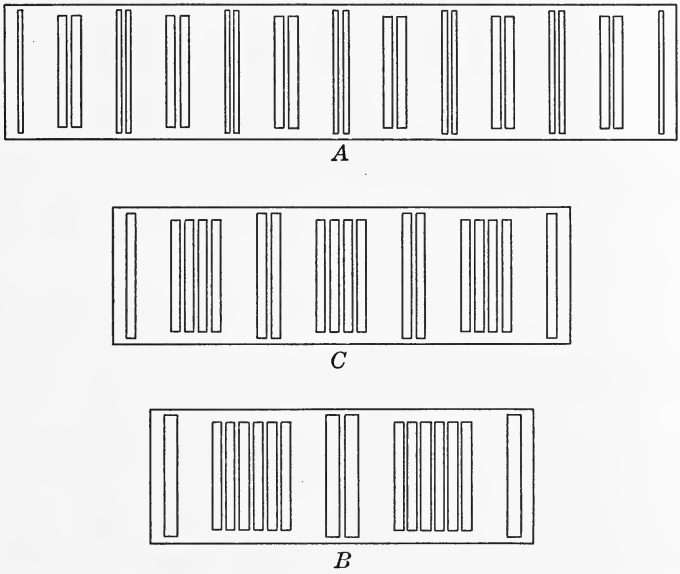


FIG. 306.—Arrangement of coils in a 110,000-volt shell-type transformer.

Design of the Low-voltage Winding

Total number of turns	$=270$
Coils	$=6$
Turns per coil	$=45$
Width of copper assumed	$=0.4$ in.
Circular mils per ampere	$=1260\times\sqrt{0.4\times0.9}$ where $(1-k)^2$ is assumed to be $=0.9$ $=755$
Section of wire	$=86{,}000$ circular mills $=0.17$ in. $\times 0.4$ in. use $2\times(0.085$ in. $\times 0.4$ in.) with a strip of fuller-board between

Insulation	=0.007 fullerboard between wires 0.024 half lap cotton tape 0.014 fullerboard between layers
Thickness of insulated strip	=0.215 in.
Height of winding	=0.215×45 =9.7 in. =10.2 in. to allow for bulging.

This allows ample clearance between the winding and the core, in fact the winding could be made narrower and higher because 0.75 in. would be ample spacing for 13,200 volts, but it is advisable to use the larger spacing, where it is available without any sacrifice of space or material.

The coils with the insulation are now drawn to scale as shown in Fig. 291, and the length of the opening determined; this value is 43 in.

Design of the Core

Flux	$=44\times10^6$
Core density, assumed	=80,000 lines per square inch
Necessary core section	=550 sq. in.
Actual section adopted	=14×43.5 in.

Calculation of the Losses and the Magnetizing Current

Mean turn of the low-voltage winding	=170 in.
Resistance of the low-voltage winding	$=\frac{6\times45\times170}{0.17\times0.4\times1.27\times10^6}$ =0.53 ohms
Mean turn of high-voltage winding	=190 in.
Resistance of high-voltage winding	$=\frac{1300\times190}{0.03\times0.4\times1.27\times10^6}$ =16.2 ohms
Loss in the high-voltage winding	$=16.2\times23.6^2$ =9000 watts
Loss in the low-voltage winding	$=0.53\times114^2$ =6900 watts
Weight of core	=22,000 lb.
Core loss in watts (ordinary iron)	=22000×0.9; from Fig. 275. =20,000 watts
Total loss at full-load	=35,900 watts
Efficiency	=97.7 per cent.
Volt amperes excitation	=22000×5.4 =119,000 =8 per cent.

The equivalent primary reactance $=2\pi f\times3.2\times T_1^2\frac{MT}{L}\left(\frac{d_1}{3}+\frac{d_2}{3}+S\right)\times10^{-8}$ × coil groups

where $MT.$ =180 in.
L=16 in.
d_1=1.3 in. See Figs. 279 and 291.
d_2=0.4 in.
S=4.5 in.

coil groups $=6$; diagram C, Fig. 306

Equivalent primary reactance $=2\pi\times25\times3.2\times216^2\times\frac{180}{16}\left(\frac{1.7}{3}+4.5\right)10^{-8}\times6$

$=80$ ohms

The reactance drop referred to the primary $=80\times23.6$

$=1900$ volts

$=3.0$ per cent.

The primary resistance drop $=380$

The secondary resistance drop referred to the primary $=60\times\frac{63500}{13200}=290$

The resistance drop referred to the primary $=670$

$=1.05$ per cent.

The tank is made round in section and clears the core by 2 in. at the corners.

The total loss $=35{,}900$ watts

The water-pipe surface required $=35{,}000$ sq. in.

$=750$ ft. of 1 1/4-in. pipe.

CHAPTER XLVII

SPECIAL PROBLEMS IN TRANSFORMER DESIGN

338. Comparison between Core- and Shell-type Transformers.—Figs. 303 and 305 show the two types built for the same output and drawn to the same scale. It was shown in Art. 335, page 476, that, due to the difference in the construction, the volts per turn has twice the value for shell-type that it has for core-type transformers, in fact

$$\text{volts per turn} = \frac{1}{25}\sqrt{\text{watts}} \text{ for shell-type power transformers}$$

$$= \frac{1}{50}\sqrt{\text{watts}} \text{ for core-type power transformers}$$

so that the number of turns for a given voltage is the smaller for transformers of the shell type.

The essential differences between the two types are: For the same voltage and output the shell type has half as many turns as the core type, it has also the longer mean turn of coil, but the smaller total amount of copper.

The shell type, since it has half as many turns, must have twice the flux of the core type and, because of that, must have twice the core section; but the mean length of magnetic path is shorter and the total weight of iron only slightly greater.

Since, as shown on page 443, the magnetizing current and the iron loss are proportional to the weight of the core for a given frequency and density, transformers of the core type can more readily be designed for small iron loss and small exciting current than can those of the shell type, and for that reason are generally used for distributing transformers, since these are built with a small iron loss so as to have a high all-day efficiency. It must, of course, be understood that by changing the proportions somewhat, shell-type transformers can also be built suitable for distributing service, but the fact that the core type is the one generally used would indicate that for such service it is the cheaper to construct.

For very high-voltage service the losses are not of so much

importance as reliability in operation. Fig. 307 shows the relative proportions of a high-voltage core and of a high voltage shell type transformer; the coils of such transformers are not easily braced and supported if the core-type of construction is used, and the forces due to short-circuits are more liable to destroy the winding than in the case of shell-type transformers, in which the coils are braced in such a way that these forces cannot bend them out of shape.

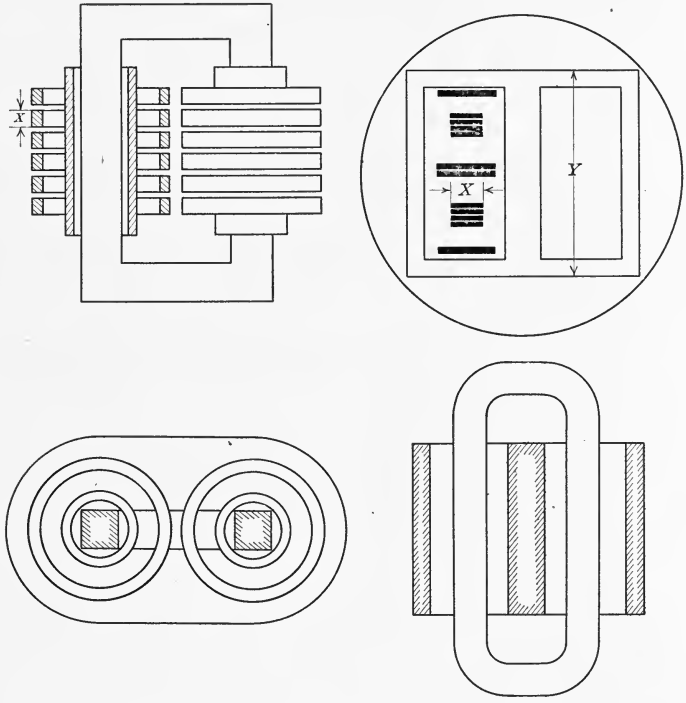


Fig. 307.—High voltage core, and shell-type transformers.

The current on a dead short-circuit $= \frac{\text{terminal voltage}}{\text{impedence of winding}}$ and may reach, in power transformers, a value of 50 times full-load current. In the case of an instantaneous short-circuit the current may reach still greater values depending, as shown in Art. 209, page 284, on the point of the voltage wave at which the short-circuit takes place. Since the forces tending to separate the high- and low-voltage windings, and to pull together the turns of windings of the same coil, are proportional to the square of the

current, they may reach very large values on short-circuit and may destroy the winding. Of the two types of transformer, the shell-type is the better able to resist such forces.

The winding of a transformer is divided up into a number of coils in series, and the voltage per coil should not exceed about 5000 volts. It will generally be found that the distance X, over which this voltage is acting, is less in the core- than in the shell-type transformer.

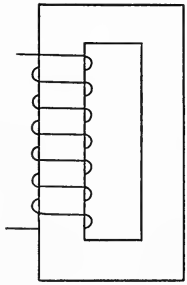
The reactance of a transformer depends on the square of the number of turns, and on the way in which the winding is subdivided. Due to the large space required between the high- and low-voltage coils it is not practicable to subdivide the winding of a core-type transformer more than is shown in Fig. 307, while the shell type winding can be well subdivided before the length Y is greater than that required for a cylindrical tank. Because of this, and also because it has the smaller total number of turns, the reactance of a shell-type transformer can readily be kept within reasonable limits while that of a core type cannot, indeed, it is principally because certain regulation guarantees are demanded that the shell type has to be used, although it is doubtful if good regulation and the consequent large short-circuit current is as desirable for power transformers as poorer regulation and a smaller current on short-circuit.

So far as the coils themselves are concerned the shell type of transformer, in large sizes, has the advantage that strip copper wound in one turn per layer may be used; this gives a stiff coil and one not liable to break down.

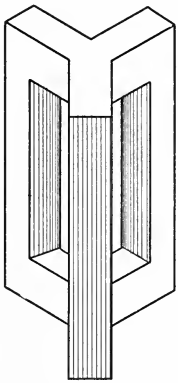
For outputs up to 500 k. v.a. at 110,000 volts the core type will probably be the cheaper, and for outputs greater than 1000 k.v.a. the shell type must be used to keep the reactance drop within reasonable limits; between these two outputs the type to be used depends largely on the previous experience of the designer.

The core-type transformer has the great advantage that it can be easily repaired, especially if built with butt joints so that the top limb may be removed and the windings and insulation lifted off.

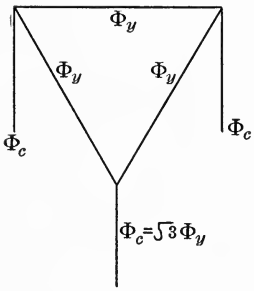
339. Three-phase Transformers.—Diagram A, Fig. 308, shows a single-phase transformer with all the windings gathered together on one leg, and diagram B shows three such transformers with the idle legs gathered together to form a resultant magnetic return path. The flux in that return path is the sum of three



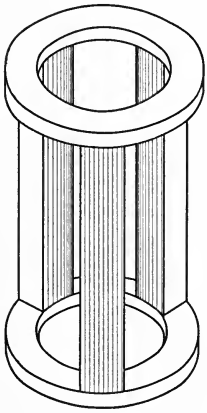
A



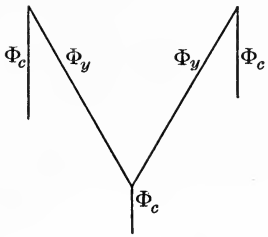
B



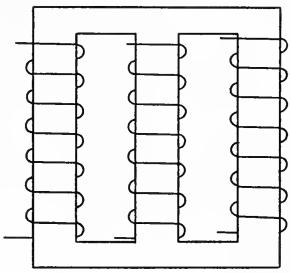
D



C



E



F

FIG. 308.—Development of the three-phase core-type transformer.

fluxes which are 120 electrical degrees out of phase with one another and is therefore zero, the center path may therefore be dispensed with.

In diagram B the flux in a yoke is the same as that in a core, the yoke and core have therefore the same area. If the three cores are connected together at the top and bottom as shown in diagram C, then the three yokes form a delta connection, and, as shown by the vector diagram D, the flux in the yoke is less than that in the core in the ratio $\frac{1}{\sqrt{3}}$, and the yoke section may be less than that of the core. The bank of transformers may be still further simplified by operating it with a *V* or open delta-connected magnetic circuit as shown in diagram E, and this latter transformer, when developed on to a plane as shown in diagram F, gives the three-phase core type that is generally used. There is

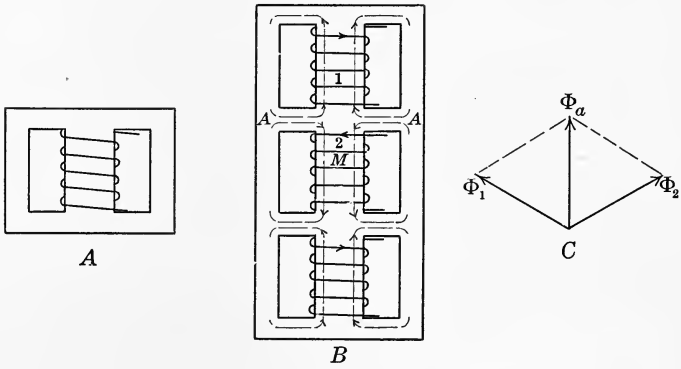


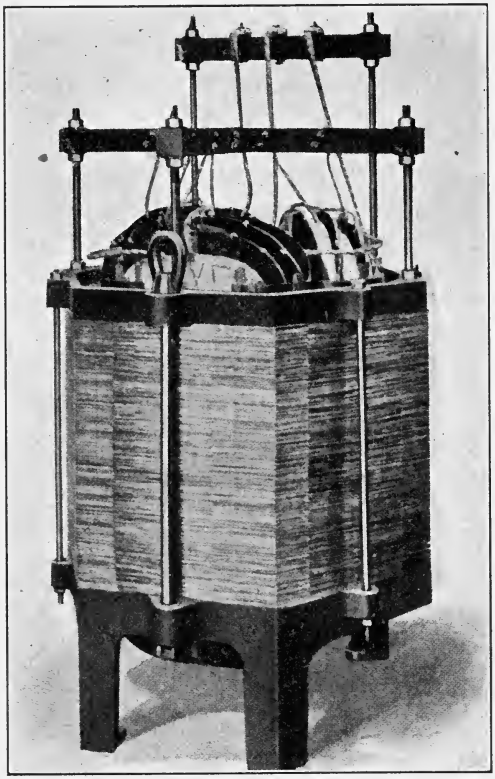
FIG. 309.—Three phase shell-type transformer.

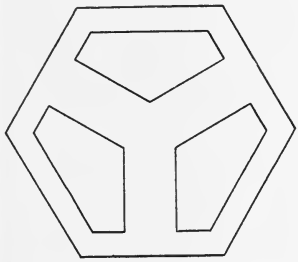
a considerable reduction in material between three transformers, such as that in diagram A, and the single transformer in diagram F, so that the three-phase transformer is cheaper, has less material, greater efficiency, and takes up less room than three single-phase transformers of the same total output.

So far as the design is concerned there is no new problem; each leg is treated as if it belonged to a separate single-phase transformer.

Diagram A, Fig. 309, shows a single-phase shell-type transformer, and diagram B shows three such transformers set one above the other to form a three-phase bank. If the three coils

were all wound and connected in the same direction, as in the case of the three-phase core-type transformer, then the flux in the core at M would be produced by three m.m.fs., 120 degrees out of phase with one another, and would be zero, because at any instant the m.m.f. of one phase would be equal and opposite to the sum of the m.m.fs. of the other two phases. The center coil is connected backward and the direction of the flux in the





Section of the iron core.

FIG. 310.—Three-phase shell-type transformer.

cores and cross pieces at one instant is shown by the arrows. The total flux in one of the cross pieces, say A, is the sum of the fluxes in cores 1 and 2, and the value may be found from diagram C.

A very economical type of three-phase transformer is shown in Fig. 310; it is almost circular in section and may be used in a cylindrical tank.

340. Operation of a Transformer at Different Frequencies.—

The e.m.f. $= E$

$= 4.44\ T\phi_a f 10^{-8}$

$= 4.44\ TB_m A_c f 10^{-8}$

$=$ a const. $\times B_m \times f$ for a given transformer;

the iron losses $=$ hysteresis loss $+$ eddy-current loss

$= KB_m{}^{1.6} f + K_e B_m{}^2 f^2$ for a given transformer

$=$ a const. $\times \dfrac{E^{1.6}}{f^{0.6}} +$ a const. $\times E^2$

so that, for a given voltage, as the frequency increases the hysteresis loss decreases, while the eddy-current loss remains constant. A standard transformer, designed for a definite frequency, may operate at frequencies which are considerably higher or lower than that for which the transformer was designed; if the transformer is designed so that at normal frequency and full-load the copper and iron losses are equal and the efficiency a maximum, then at lower frequencies the iron loss will be larger than the copper loss, and at higher frequencies the copper loss will be the greater.

In order that a low-frequency transformer may have approximately the same core loss and copper loss at full-load, the section of the iron in the core must be increased over that required for a transformer of the same rating but of higher frequency, in order to lower the densites and reduce the core loss, so that, for the same rating and efficiency, the lower the frequency, the larger the amount of iron, and the heavier the transformer.

CHAPTER XLVIII

SPECIFICATIONS

341. The following specification is intended to cover a line of transformers for lighting and power service and for sizes up to 50 k.v.a.

STANDARD SPECIFICATION FOR OIL-IMMERSED, SELF-COOLED TRANSFORMERS

Rating.—Rated capacity in kilovolt amperes	1 to 50
Normal high voltage	2200 and 1100
Normal low voltage at no-load	220 and 110
Phases	1
Frequency in cycles per second	60

Construction.—The transformers are for combined lighting and power service. The windings shall be so arranged that with either 2200 or 1100 volts on the primary side the secondary voltage may be either 110 or 220; taps must also be supplied so that if desired the secondary voltage may be raised 5 or 10 per cent. with normal applied high voltage. The tanks must be of cast iron or sheet iron of approved shape and construction, must be impervious to oil and water, and have covers supplied with gaskets. Eye bolts or hooks must be supplied for the lifting of the tank with the transformer and oil. With transformers larger than 5 k.v.a. output, oil plugs must be supplied at the bottom of the tank for the removal of the oil. Four secondary leads and two primary leads shall be brought out of the tank through porcelain bushings properly cemented in. The transformer itself shall be properly anchored in the tank to prevent movement.

Core.—This shall be of non-ageing steel assembled tightly so as to prevent buzzing during operation.

Windings.—The high- and low-voltage coils must be insulated from one another by a shield of ample dielectric strength. The coils, when wound, shall be baked in a vacuum and then impregnated with a compound which is waterproof and is not acted on by transformer oil over the range of temperature through which the transformer may have to operate.

Oil.—The oil supplied with the transformer shall be a mineral oil suitable for transformer insulation, and must be free from moisture, acid, alkali, sulphur or other materials which might impair the insulation of the transformer. It should have a flash-point higher than 139 deg. cent. and a dielectric strength greater than 40,000 volts when tested between half inch spheres spaced 0.2 in. apart. Sufficient oil must be contained in the tank to submerge the windings and the core at all temperatures from 0 to 80 deg. cent.

Cut Outs.—Two plug cut outs shall be supplied for the high-voltage side of the transformer for operation up to 2500 volts.

Workmanship and Finish.—The workmanship shall be first class and the materials used in the construction of the highest grade. The tank shall be thoroughly painted with a black waterproof paint.

General.—Bidders shall furnish cuts with descriptive matter from which a clear idea of the construction may be obtained.

They shall also state the following:

Net weight of transformer.
Net weight of case.
Net weight of oil.
Shipping weight.
Dimensions of the tank.
Core loss.
Copper loss at full-load.
Regulation at 100 per cent. and at 80 per cent. power factor.
Exciting current at normal voltage and at 20 per cent. over normal voltage.

The core loss shall be taken as the electrical input determined by wattmeter readings, the transformer operating at no-load, on normal voltage and frequency, and with a sine wave of applied e.m.f.

The copper loss shall be taken as the electrical input determined by wattmeter readings, the transformer being short-circuited and full-load current flowing in the high-voltage coils; measurements to be made on the high-voltage side.

The regulation shall be calculated by the use of the following formulæ; the per cent. regulation

$$= 100\left[\frac{IR}{E}+\frac{1}{2}\left(\frac{IX}{E}\right)^2\right] \text{ at 100 per cent. power factor}$$

$$= 100\left(\frac{0.8IR+0.6IX}{E}\right) \text{ at 80 per cent. power factor}$$

where E is the normal secondary voltage at no-load
I is the full-load secondary current
IR is the resistance drop in terms of the secondary
IX is the reactance drop in terms of the secondary.

These two latter values are to be found by short-circuiting the secondary winding and measuring the applied voltage and the watts input required to send full-load current through the primary winding.

$$IR = \frac{\text{watts input}}{\text{secondary full-load current}}$$

IZ = the impedence drop in terms of the secondary

$$= \text{the short-circuit voltage} \times \frac{220}{2200}$$

$$IX = \sqrt{(IZ)^2 - (IR)^2}$$

Temperature.—The transformer shall carry the rated capacity at normal voltage and frequency and with a sine wave of applied e.m.f. for 24 hours, with a temperature rise that shall not exceed 45 deg. cent. by resistance or thermometer on any part of the windings, core or oil, and, immediately after the full-load run, shall carry 25 per cent. overload at the same voltage and frequency for two hours, with a temperature rise that shall not exceed 55 deg. cent. by resistance or thermometer on any part. The temperature rise shall be referred to a room temperature of 25. deg. cent.

Insulation.—The transformers shall withstand a puncture test of 10,000 volts for one minute between the high-voltage winding and the core, tank, and low-voltage winding; also a puncture test of 2500 volts for one minute between the low-voltage winding and the core and tank.

The transformer shall operate without trouble for five minutes at double normal voltage and no-load; the frequency may be increased during this test if desired.

These insulation tests shall be made immediately after the heat run.

342. Effect of Voltage on the Characteristics.—The effect of high voltage in a transformer is that large spacings are required between the high- and low-voltage coils and also between the high-voltage coils and the core, so that the higher the voltage the longer the magnetic path and also the mean turn of the coils. Because of the large spacings, the reactance of high-voltage transformers is greater than of those of lower voltage, and the

DATA BASED ON 60 CYCLES

2200-1100 Volts Primary — 220-110 Volts Secondary

Capacity K. V. A.	Weights				Losses-watts		Efficiency					Regulation	
							Proportion of rated K. V. A.						
	Trans.	Case	Oil	Ship.	Core	Copper	1¼	1	¾	½	¼	P. F. 1.0	P. F. 0.8
.6	32	40	10	120	15	16	94.9	95.1	94.9	94.0	90.3	2.7	3.1
1.	45	53	15	155	20	25	95.5	95.7	95.6	95.0	92.0	2.6	3.0
1.5	55	53	15	165	22	38	95.8	96.1	96.3	96.0	93.9	2.5	3.0
2.	70	65	20	205	27	42	96.4	96.7	96.7	96.4	94.4	2.3	3.0
2.5	75	65	20	210	30	55	96.4	96.7	96.8	96.6	94.9	2.2	3.0
3.	85	70	30	245	33	63	96.6	96.9	97.0	96.8	95.3	2.1	3.0
4.	105	70	30	265	39	75	97.0	97.2	97.4	97.2	95.8	1.9	3.0
5.	125	90	35	320	44	91	97.1	97.4	97.5	97.4	96.2	1.8	2.9
7.5	170	120	40	410	60	123	97.4	97.6	97.8	97.6	96.5	1.7	2.9
10.	205	140	65	500	74	158	97.5	97.7	97.9	97.8	96.7	1.6	2.7
15.	285	170	75	650	97	212	97.8	98.0	98.1	98.0	97.2	1.5	2.7
20.	360	340	110	925	124	264	97.9	98.1	98.2	98.1	97.2	1.4	2.6
25.	435	360	120	1,025	145	314	98.0	98.2	98.3	98.2	97.4	1.3	2.5
30.	515	360	120	1,125	163	349	98.1	98.3	98.4	98.3	97.6	1.2	2.5
40.	725	560	200	1,550	202	456	98.2	98.4	98.5	98.4	97.7	1.2	2.3
50.	890	600	230	1,840	242	532	98.3	98.5	98.6	98.5	97.8	1.1	2.3

regulation is poorer. Because of the longer mean turn of the coils and of the magnetic circuit the losses are greater and the efficiency poorer than for a transformer of the same output but of lower voltage.

The table on page 497 gives the characteristics of a line of 60-cycle distributing transformers.

CHAPTER XLIX

MECHANICAL DESIGN

343. A complete discussion of the mechanical design of electrical machinery is beyond the scope of this work, but there are several points which the student is liable to overlook unless his attention is drawn to them.

The fundamental principle in the design of revolving machinery is that the frame shall be as heavy as possible, the revolving part as light as possible, and the shaft as stiff as possible.

The yokes, housings, spiders and bases should be designed so that the work in moulding them shall be a minimum, and the smaller parts, when used in large quantities, should be designed for machine moulding.

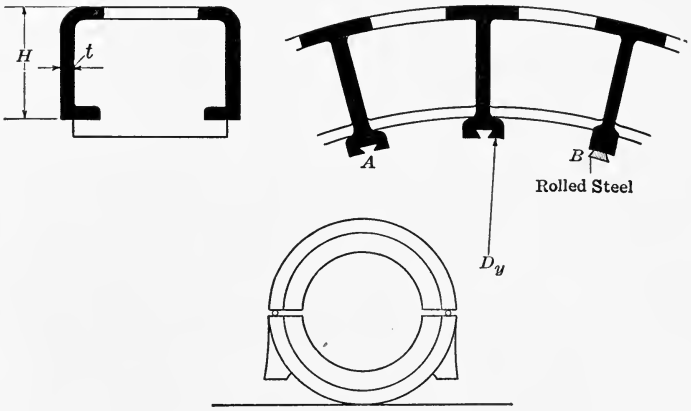


FIG. 311.—Alternator yoke.

The yoke must be made stiff enough to prevent sagging, when built up with its poles as in the case of a D.-C. machine, or with its stator punchings as in the case of an alternator or induction motor. It must also be strong enough to stand handling in the shop as, for example, when supported as shown in Fig. 311. If the yoke will stand this latter treatment it will generally be stiff enough for operating conditions.

The following rough rule may be used for checking a new yoke design. For yokes of the box type shown in Fig. 311,

$$H = 1.5 + \frac{D_y}{12} \text{ in.}$$

$t = 0.25 + 0.01 D_y$ in., for values of D_y from 5 in. to 100 in.; above 100-in. bore increase the thickness by 0.375 in. for every 50 in. increase in diameter.

The distance between dovetails should be about 12 in., for which value the length of the segment used to build up the core will be 24 in. It is at present standard practice to dovetail the laminations into the yoke as shown at A, but the method shown at B has been found satisfactory and has the advantage over A that it is cheaper and does not block up the ventilation so much. The distance between dovetails is determined finally by the slot pitch and the number of slots per segment.

344. Rotors and Spiders.—These must be designed so that they will stand handling in the shop.

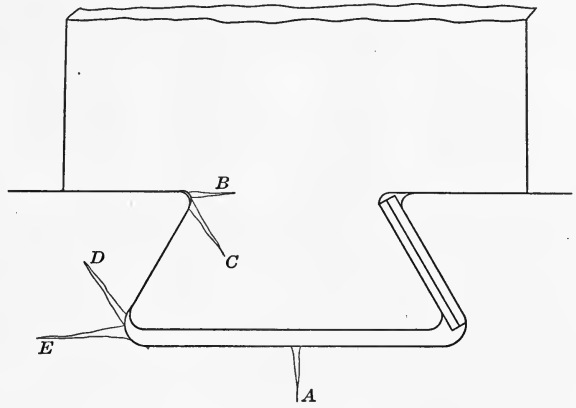


FIG. 312.—Stresses in pole dovetails.

The rotor of a revolving field alternator is usually built with the poles dovetailed in, as shown in Fig. 312, and care must be taken that it is strong enough at A to prevent overstress due to its own centrifugal force and to the bursting action of the pole dovetails, and also strong enough at B, C, D, and E. The stresses to be allowed at the maximum speed are

2500 lb. per square inch for cast iron.
12,000 lb. per square inch for cast steel.
14,000 lb. per square inch for sheet steel.
18,000 lb. per square inch for nickel steel.

The arms of the spiders should be able to transmit the full-load torque with a factor of safety of 12, then they will be strong enough to withstand sudden overloads and short-circuits.

345. Commutators.—Fig. 313 shows a typical commutator. The mica between segments has parallel sides, and the segments themselves are tapered to suit. The clamping of the segments is done on the inner surface of the V so that the segments will be pulled tightly together, and the design should be strong enough

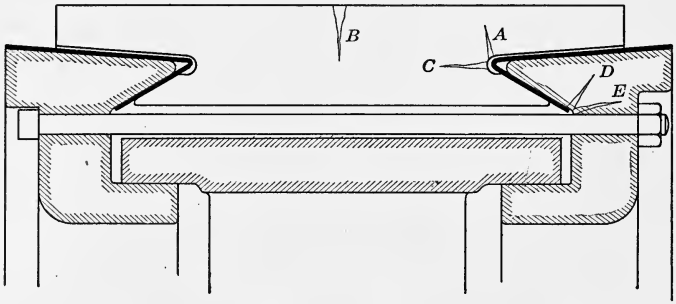


FIG. 313.—Construction of a long commutator.

to prevent the stresses at A, B, C, D, and E from becoming dangerous. In a long commutator considerable trouble is experienced due to the expansion and contraction of the bars as the commutator is heated or cooled, and the type of construction shown is designed to take care of such expansion by the extension of the bolts. If the type of construction shown in Fig. 27 were used for a long commutator then, when the commutator is heated and expands, the shell has to burst, the bolts break, or the commutator bars bend.

346. Unbalanced Magnetic Pull.[1]—If two surfaces of area S sq. cm. have a magnetic flux crossing the gap between them, and the flux density in this gap $= B$ lines per square centimeter, there will be a force of attraction between the two surfaces $= \frac{1}{8\pi} SB^2$ dynes.

Fig. 314 shows part of an electrical machine of which the revolving part is out of center by a distance Δ cm. The flux density in the air-gap at C is greater than at B, so that the magnetic force acting downward is greater than that acting upward.

[1] B. A. Behrend. Transactions of A. I. E. E., Vol. 17, page 617.

The resultant downward pull is called the unbalanced magnetic pull of the machine.

In the following discussion it is assumed that the flux density in the air-gap at any point is inversely proportional to the air-gap clearance at that point, and also that the eccentricity is not more than 10 per cent. of the average air-gap clearance.

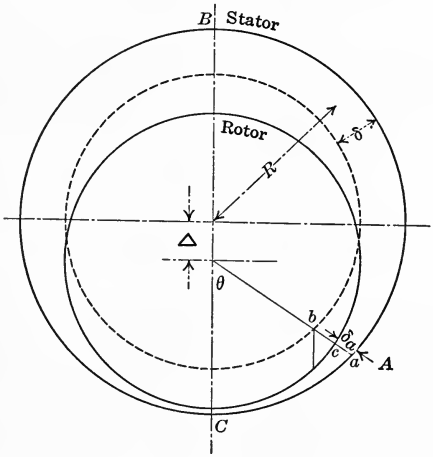


FIG. 314.—Machine with an eccentric rotor.

δ_a = the air-gap clearance at any point A

$= ab - bc$

$= \delta - \Delta \cos \theta$

and B_a = the flux density at point A

$$= B \frac{\delta}{\delta_a}$$

$$= B \left(\frac{\delta}{\delta - \Delta \cos \theta} \right)$$

$$= B \left(\frac{1}{1 - \frac{\Delta}{\delta} \cos \theta} \right)$$

df = the force at A over a small arc $Rd\theta$

$$= \frac{1}{8\pi} \times B_a^2 \times Rd\theta \times L_c \text{ dynes;}$$

and the vertical component of this force

$$= df \times \cos \theta$$

$$= \frac{1}{8\pi} B^2 \left(\frac{1}{1 - \frac{\Delta}{\delta} \cos \theta} \right)^2 RL_c \cos \theta d\theta$$

$$= \frac{1}{8\pi} RL_c B^2 \left(1 + 2\frac{\Delta}{\delta}\cos\theta\right)\cos\theta d\theta;$$ higher powers of $\frac{\Delta}{\delta}\cos\theta$ than the first being neglected since $\frac{\Delta}{\delta}$ is small, being less than 0.1.

The total downward force

$$= \frac{2}{8\pi} RL_c B^2 \int_0^{\frac{\pi}{2}} \left(1 + 2\frac{\Delta}{\delta}\cos\theta\right)\cos\theta d\theta$$

$$= \frac{2}{8\pi} RL_c B^2 \left(1 + \frac{2\pi\Delta}{4\delta}\right) \text{dynes}$$

similarly the total upward force

$$= \frac{2}{8\pi} RL_c B^2 \left(1 - \frac{2\pi\Delta}{4\delta}\right) \text{ dynes}$$

The total unbalanced magnetic pull is the difference between the total downward and the total upward force and

$$= \frac{2}{8\pi} RL_c B^2 \times \frac{4\pi\Delta}{4\delta}$$

$$= (2\pi RL_c)\frac{1}{8\pi} B^2 \frac{\Delta}{\delta} \text{ dynes}$$

In inch-pound units the unbalanced magnetic pull is given by the following formula:

$$\text{Pull in pounds} = \frac{1}{72} S \left(\frac{B}{1000}\right)^2 \frac{\Delta}{\delta} \qquad (69)$$

where B is the effective gap density in lines per square inch.

Δ is the displacement in inches

δ is the average air-gap clearance in inches

$S = 2\pi RL_c$, is the total rotor surface in square inches.

Since the flux density in the air-gap at B is less than that at C, the e.m.f. generated in the conductors at C will be greater than that generated in conductors at B. If a multiple winding is used for the rotor, such as the D.-C. multiple winding or the squirrel-cage winding of the induction motor, then the current in each conductor will be proportional to the e.m.f. generated therein and will be greater in conductors at C than in those at B. Now the armature current in a D.-C. machine tends to demagnetize the field which produces it and so also does the current in the rotor bars of an induction motor, with such multiple windings therefore the demagnetizing effect will be greatest at C and least at B; the flux in the air-gap will not be inversely proportional to the air-gap; and the magnetic pull will be less than that given by the above formulæ.

347. Bearings, Journals and Pulleys.—The subject of bearing friction has already been discussed in Art. 79, page 97.

A useful rule for the design of self-cooled bearings with ring lubrication is that

$P = 60$ to 100; PV less than 90,000 for bearings up to 5 in. diameter

$P = 40$ to 60; PV less than 60,000 for bearings above 5 in. diameter

where P = the bearing pressure in pounds per square inch projected area

V = the rubbing velocity of the journal in feet per minute.

Bearings having values within these limits will carry 50 per

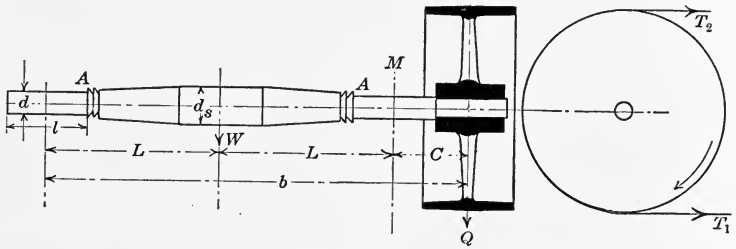


FIG. 315.—Shaft for a belted machine.

cent. overload without injury; if the values at normal load are greater water-cooled bearings should be used.

The length of the bearing will generally be from 3 to 4 times the diameter, but the stress in the neck of the journal at A, Fig. 315, must be checked to see that it is not too high;

if M_b = the bending moment at A

M_t = the twisting moment at A

then M_e, the equivalent bending moment at A

$$= \frac{M_b + \sqrt{M_b + M_t}}{2}$$

$$= \text{stress} \times \frac{\pi}{32} d^3$$

this stress should not exceed 5000 lb. per square inch.

The shaft at the center is designed principally for stiffness. The size of shaft necessary to transmit the torque is comparatively small; the actual diameter is chosen so that the rotor deflection shall not exceed 10 per cent. of the air-gap clearance; this diameter should, however, be checked for strength to resist the combined effect of torsion and bending.

Fig. 315 shows a loaded shaft. W, the weight between the bearings, includes the unbalanced magnetic pull, and Q is the total pull of the belt.

If, as is generally the case, the belt is in contact with the pulley over half the circumference, then

$$\begin{aligned} T_2 &= 2T_1 \text{ approximately} \\ \text{and } Q &= T_2 + T_1 \\ &= 3T_1 \text{ approximately} \\ T_2 - T_1 &= \frac{\text{h. p.} \times 33000}{\text{belt speed in ft. per min.}} \\ &= T_1 \end{aligned}$$

The total force on the bearing next the pulley, when the belt pull is vertically downward $= 1/2\ W + Q \times \frac{b}{2L}$

The lower the speed of the belt the greater is the value of $T_2 - T_1$, the effective force on the belt, and the greater the value of Q, so that, with a given bearing, there is a minimum diameter and a maximum width of pulley which may be used without overloading the bearing or overstressing the shaft.

Consider the following example. Design the shaft for a 50-h. p. 900-r.p.m. induction motor.

Air-gap clearance $= 0.03$ in.
Deflection allowed $= 0.003$ in.
Rotor weight $= 500$ lb.
Rotor diameter $= 19$ in.
Frame length $= 6.5$ in.
Carter coefficients $= 1.52$

S, in formula 69 $= \pi \times 19 \times 6.5 \times \frac{1}{1.52} = 255$ sq. in.

Max. flux per pole $= 1.04 \times 10^6$

$$\text{Max. flux density} = \frac{\text{flux per pole} \times \text{poles}}{S} \times \frac{\pi}{2}$$
$$= \frac{1.04 \times 10^6 \times 8}{255} \times \frac{\pi}{2}$$
$$= 51{,}000 \text{ lines per square inch.}$$

$$\text{Average value of } \left(\frac{B}{1000}\right)^2 = \frac{51^2}{2} = 1300$$

$$\text{Unbalanced pull} = \frac{1}{72} \times 255 \times 1300 \times 0.1$$
$$= 460 \text{ lb.}$$
$$= 460 + 500 = 960 \text{ lb.}$$

The deflection is limited to $0.1 \times 0.03 = 0.003$ in.

$$\text{and deflection} = \frac{W \times (2L)^3}{48 \times E \times I}$$

where $2L$ = the distance between bearings in inches, see Fig. 315
E = Young's modulus = 30,000,000 in inch-pound units
I = the moment of inertia to bending

$$= \frac{\pi}{64} d_s^4$$

therefore d_s, the necessary shaft diameter $= \sqrt[4]{\frac{W \times (2L)^3}{7 \times 10^6 \times \delta}}$

$$= \sqrt[4]{\frac{960 \times 19^3}{7 \times 10^6 \times 0.03}}$$

$$= 2.4 \text{ in. approximately}$$

Assume now that the belt speed = 4000 ft. per minute, then the pulley diameter = 17 in.

$$T_2 - T_1 = \frac{50 \times 33000}{4000} \text{ lb.}$$

$$= 414 \text{ lb.}$$

the maximum pull in the belt $= T_2$

$$= 2T_1$$

$$= 828 \text{ lb.}$$

the belt width = 10 in. and the belt stress = 83 lb. per inch width.

For a single belt the stress should be less than 45 lb. per inch width and for a double belt should be less than 90 lb. per inch; a double belt should not be used on a pulley smaller than 12 in. in diameter. With a 10-in. belt and the pulley overhung, the smallest possible value for C, Fig. 315 = 10-in. The value of $Q = 3 \times 414 = 1240$ lb.

The belt pull on the bearing $= 1240 \times \frac{29}{19}$

$$= 1900 \text{ lb.}$$

$1/2W = 480$ lb.

Total bearing pressure $= 2380$ lb.

Size of bearing $= 3 \times 9$ and the value of $P = 88$ lb. per square inch and $PV = 62{,}000$

The stress in the shaft must now be determined.

The bending moment at $M = Q \times 10$

$$= 1240 \times 10$$

$$= \text{stress} \times \frac{\pi}{32} d^3$$

therefore the stress at the neck of the journal = 4700 lb. per square inch.

The shaft will be cut out of 3 1/2 in. stock and the journals turned down to 3 in.; the deflection will be less than 10 per cent. of the air-gap clearance.

348. Brush Holders.—The more important of the conditions to be fulfilled by the brush holder are that it should be rigid enough not to be set in vibration; that it must bear on the brush with a tension which may be adjusted; that it must be adjustable for wear of the brushes; that the brush and that part of the holder which is attached to it must not have too great an inertia, because

the brush has to move with the commutator if the latter does not run perfectly true.

Two holders much used in practice are shown in Figs. 316 and 317. In the former the brush is clamped to the holder, while in

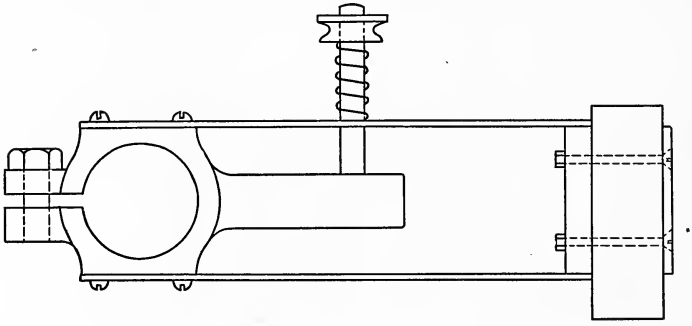


Fig. 316.

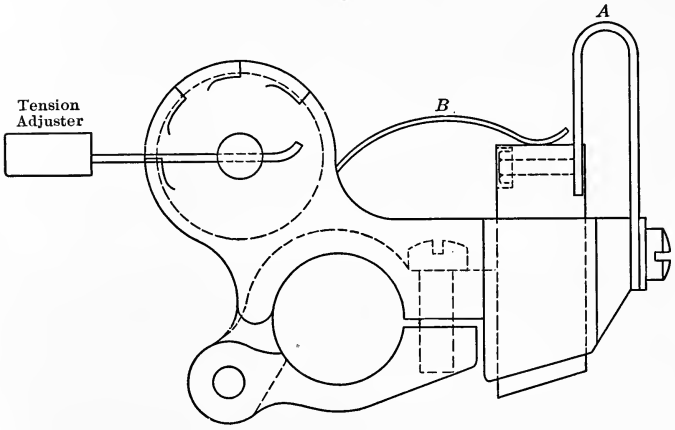


Fig. 317.—Brush holders.

the latter it is free to slide in a box and therefore requires a pig-tail *A* of stranded copper to carry the current to the holder, otherwise this current would pass along the spring *B* and draw its temper.

BROWN AND SHARP GAUGE; COPPER WIRE

Gauge number B. and S.	Diameter		Cross-sectional area			Diameter double cotton covered	Insulation (mils on diameter)	Ohms per 1000 ft.	
	Inches	Millimeters	Square inches	Circular mils	Square mms.			15° C.	60° C.
0000	.46000	11.684	.16619	211,600	107.19	.472	12	.048119	.056969
000	.40964	10.404	.13179	167,810	85.013	.421	12	.060677	.071838
00	.36480	9.266	.10452	133,080	67.433	.377	12	.076511	.090583
0	.32495	8.252	.082931	105,590	53.482	.337	12	.096428	.11416
1	.28930	7.348	.065733	83,694	42.406	.301	12	.12166	.14403
2	.25763	6.543	.052129	66,373	33.623	.269	12	.15341	.18162
3	.22942	5.827	.041338	52,633	26.667	.241	12	.19345	.22903
4	.20431	5.189	.032784	41,743	21.147	.216	12	.24392	.28879
5	.18194	4.620	.025998	33,102	16.764	.194	12	.30759	.36417
6	.16202	4.115	.020617	26,250	13.299	.174	12	.38789	.45923
7	.14428	3.665	.016349	20,817	10.549	.156	12	.48913	.57909
8	.12849	3.264	.012967	16,510	8.3673	.140	12	.61673	.73016
9	.11443	2.906	.010284	13,094	6.6325	.126	12	.77760	.92061
10	.10189	2.588	.0081536	10,382	5.2603	.114	12	.98078	1.1612

BROWN AND SHARP GAUGE; COPPER WIRE—*Continued*

Gauge number B. and S.	Diameter		Cross-sectional area			Diameter double cotton covered	Insulation (mils on diameter)	Ohms per 1000 ft.	
	Inches	Millimeters	Square inches	Circular mils	Square mms.			15° C.	60° C.
11	.09074	2.304	.0064637	8,233.7	4.1692	.103	12	1.2366	1.4640
12	.08081	2.052	.0051288	6,530.3	3.3070	.093	12	1.5592	1.8460
13	.07196	1.829	.0040669	5,178.2	2.6273	.084	12	1.9663	2.3279
14	.06408	1.628	.0032250	4,106.2	2.0816	.076	12	2.4796	2.9357
15	.05706	1.450	.0025571	3,255.8	1.6513	.068	11	3.1273	3.7025
16	.05082	1.290	.0020284	2,582.7	1.3070	.062	11	3.9424	4.6675
17	.04525	1.150	.0016081	2,047.6	1.0387	.056	11	4.9727	5.8873
18	.04030	1.024	.0012755	1,624.9	.82354	.051	11	6.2694	7.4224
19	.03589	.900	.0010117	1,288.1	.63617	.046	10	7.9047	9.3586
20	.03196	.813	.00080223	1,021.4	.51912	.042	10	9.9682	11.802
21	.02846	.724	.00063614	809.97	.41168	.038	10	12.571	14.883
22	.02534	.643	.00050431	642.12	.32472	.035	10	15.857	18.773
23	.02257	.574	.00040008	509.41	.25877	.032	10	19.988	23.664
24	.02010	.511	.00031738	404.01	.20508	.030	10	25.202	29.838

STANDARD WIRE GAUGE; COPPER WIRE

Gauge number S. W. G.	Diameter		Cross-sectional area			Diameter double cotton covered	Insulation (mils on diameter)	Ohms per 1,000 ft.	
	Inches	Millimeters	Square inches	Circular mils	Square mms.			15° C.	60° C.
0000	.400	10.160	.125663	160,000	81.070	.412	12	.063633	.075345
000	.372	9.4487	.108686	138,384	70.117	.384	12	.073575	.087117
00	.348	8.839	.095114	121,104	61.362	.360	12	.084068	.099543
0	.324	8.229	.082447	104,976	53.190	.336	12	.096991	.11484
1	.300	7.620	.070685	90,000	45.603	.312	12	.11313	.13394
2	.276	7.010	.059828	76,176	38.597	.288	12	.13368	.15827
3	.252	6.400	.049876	63,504	32.176	.264	12	.16035	.18985
4	.232	5.893	.042273	53,824	27.272	.244	12	.18918	.22399
5	.212	5.285	.035299	44,944	22.772	.224	12	.22600	.26829
6	.192	4.877	.028953	36,864	18.678	.204	12	.27670	.32761
7	.176	4.470	.024328	30,976	15.659	.188	12	.32908	.38963
8	.160	4.064	.020106	25,600	13.035	.172	12	.39784	.47104
9	.144	3.658	.016286	20,736	10.507	.156	12	.49059	.58085
10	.128	3.251	.012868	16,384	8.301	.140	12	.62473	.73968

STANDARD WIRE GAUGE; COPPER WIRE—*Continued*

Gauge number S. W. G.	Diameter		Cross-sectional area			Diameter double cotton covered	Insulation (mils on diameter)	Ohms per 1,000 ft.	
	Inches	Millimeters	Square inches	Circular mils	Square mms.			15° C.	60° C.
11	.116	2.946	.010568	13,456	6.818	.128	12	.75878	.90170
12	.104	2.642	.008595	10,816	5.480	.116	12	.94078	1.11387
13	.092	2.337	.006647	8,464	4.288	.104	12	1.2029	1.4242
14	.080	2.032	.005026	6,400	3.243	.092	12	1.5908	1.8836
15	.072	1.829	.004071	5,184	2.627	.084	12	1.9639	2.3254
16	.064	1.626	.003217	4,096	2.075	.076	12	2.4856	2.9431
17	.056	1.422	.002463	3,136	1.254	.067	11	3.2465	3.8441
18	.048	1.219	.001809	2,304	1.167	.059	11	4.4202	5.2339
19	.040	1.016	.001256	1,600	0.8107	.051	11	6.3664	7.5383
20	.036	0.914	.001018	1,296	0.6567	.046	10	7.8548	9.3006
21	.032	0.813	.0008042	1,024	0.5189	.042	10	9.9429	11.773
22	.028	0.711	.0006157	784	0.3972	.038	10	12.987	15.377
23	.024	0.609	.0004524	576	0.2918	.034	10	17.675	20.928
24	.022	0.559	.0003801	484	0.2452	.032	10	21.085	24.909

LIST OF SYMBOLS

Used along with these symbols the suffix $_1$ stands for stator or primary and the suffix $_2$ for rotor or secondary.

Unless otherwise stated the dimensions are in inch units.

		Page
A	= brush contact area	98
A_{ag}	= actual gap area per pole	43
A_b	= projected area of bearing	97
A_c	= core area	43
A_g	= apparent gap area per pole	43
A_p	= pole area	43
A_r	= section of end connector	372
A_t	= tooth area per pole	43
A_y	= yoke area	43
AT	= ampere turns.	
AT_c	= ampere turns per pole for the core	49
AT_g	= ampere turns per pole for the gap	48
AT_{g+t}	= ampere turns per pole for gap and tooth	51
AT_{max}	= ampere turns per pole, maximum excitation	238
AT_p	= ampere turns per pole for the pole core	48
AT_t	= ampere turns per pole for the teeth	48
AT_y	= ampere turns per pole for the yoke	46
B	= flux density	6
B_{ag}	= actual gap density	45
B_{at}	= actual tooth density	45
B_c	= core density	45
B_g	= apparent gap density	45
B_i	= interpole gap density	94
B_m	= maximum flux density in transformer cores	441
B_p	= pole density	45
B_t	= apparent tooth density	45
B_y	= yoke density	45
$B.\ \&\ S.$	= Brown and Sharp gauge.	
C	= Carter fringing constant	43
D_a	= armature external and stator internal diameter	42
D_c	= commutator diameter	111
D_r	= mean diameter of rotor end connector	372
E	= volts per phase.	
E_{11}	= component of applied e.m.f. equal and opposite to the back generated e.m.f.	331
E_{1b}	= back generated e.m.f. in primary	329
E_f	= volts per field coil	65
E_s	= generated voltage in the short circuited coil	85
E_t	= terminal voltage	169

		Page
F	= commutator face	111
F	= finish of winding	161
I	= amperes per phase.	
I_{11}	= component of primary current with a .m.m.f. equal and opposite to that of the secondary current	331
I_a	= armature current	103
I_c	= current per conductor	55
I_d	= maximum current in induction motors	337
I_e	= exciting current	343, 427
I_f	= field current	42
I_l	= line current	169
I_m	= magnetizing current	428
I_o	= magnetizing current	329
I_{sc}	= short circuit current	344
K	= hysteresis constant	100
K_1	= reactance constant	369
K_2	= reactance constant	369
K_e	= eddy current constant	100
$K.V.A.$	= kilo volt amperes.	
$K.W.$	= kilowatts.	
L	= length of transformer coils	445, 448
L	= coef. of self induction	75
L_b	= length of conductor	102
L_c	= axial length of core	43
L_e	= length of end connections	108
L_f	= radial length of field coil	65
L_g	= gross iron in frame length	43
L_{ip}	= axial length of interpole	94
L_n	= net iron in frame length	43
L_p	= axial length of pole	43
L_s	= axial length of pole shoe	51
M	= section in circular mils	65
M	= coef. of mutual induction	75
MT	= mean turn of coil	65
MT	= mean turn of transformer coils	448
N	= number of slots	43
$P.F.$	= power factor.	
R	= reluctance.	
R	= resistance per phase.	
R	= brush contact resistance	75
R_e	= equivalent secondary resistance	431
R_{eq}	= equivalent primary resistance	431
$R.V.$	= reactance voltage	121
S	= number of commutator segments	18
S	= space between high and low voltage coils	445, 448
S	= start of winding	161
S	= total rotor surface	503

		Page
T	= time constant	376
T	= turns per coil.	
T_c	= time of commutation	75
T_f	= field turns per pole	42
V	= air velocity in ft. per min.	106
V	= peripheral velocity of rotor in 1000s of ft. per min.	396
V_b	= rubbing velocity of bearing	97
V_r	= rubbing velocity of brush	98
W	= weight of iron in lb.	100
W_{ip}	= interpole waist	93
W_p	= pole waist	43
W_s	= width of pole shoe	52
X	= leakage reactance per phase	211
X	= synchronous reactance	214
X_2	= rotor reactance at standstill	330
X_e	= equivalent secondary reactance	431
X_{eq}	= equivalent primary reactance	431
Z	= total face conductors in D-C. armatures	11
Z	= conductors in series per phase	189
Z_c	= total number of conductors in alternators	248
a	= slots per pole	184
b	= conductors per slot	216
c	= slots per phase per pole	216
$\cos\theta$	= power factor	169
d	= slot depth	43
d_1, d_2, d_3, d_4	= slot dimensions	218
d_1, d_2	= dimensions of transformer coils	445, 448
d_a	= depth of armature core	43
d_b	= bearing diameter	97
d_f	= depth of field winding	65
d_s	= shaft diameter	278
$d.c.c$	= double cotton covering on wires.	
e	= volts per conductor	188
f	= frequency	162
f	= constant in Carter formula	43
h_p	= height of pole	52
h_s	= height of pole shoe	51
$h.p.$	= horse-power.	
i	= current in amperes.	
k	= constant in reactance voltage formula	84
k	= distribution factor	189
k	= a whole number, in winding formula	18
k_r	= end ring resistance factor	372
k_t	$= \frac{\text{magnetic loading in transformers}}{\text{electric loading}}$	476
$(-1k)$	= per cent. of copper in a given thickness of winding	473
$k.v.a.$	= kilo volt amperes	188

		Page
$k.w.$	= kilowatts.	
l_1	= length of leakage path	51
l_3	= length of leakage path	52
l_b	= length of bearing	97
l_c	= magnetic length of core	49
l_p	= magnetic length of pole core	48
l_y	= magnetic length of yoke	46
$lf.$	= leakage factor	49
m.m.f.	= magneto-motive force.	
n	= number of phases.	
p	= number of poles.	
p_1	= number of paths through the armature	11
q	= ampere conductors per inch	114, 255
r.p.m.$_1$	= synchronous speed of induction motors	333
r.p.m.$_2$	= rotor speed	330
s	= per cent. slip	328
s	= slot width	43, 219
$sf.$	= space factor	65
t	= thickness.	
t	= time in secs.	
t	= tooth width	43, 221
v_2	= leakage constant in induction motors	334
w	= slot width	218
δ	= air-gap clearance	43
η	= efficiency.	
λ	= slot pitch	43
λ_e	= distance the end connections project	108
τ	= pole-pitch	42
ϕ	= magnetic flux.	
ϕ_a	= useful flux per pole	11, 178
ϕ_a	= flux in transformer core at no-load	439
ϕ_e	= leakage flux between poles	42
ϕ_e	= end connection leakage constant	79, 215, 360
ϕ_m	= total flux per pole	42
ϕ_s	= slot leakage constant	79, 216, 360
ϕ_t	= tooth tip leakage constant	216
ϕ_{ta}	= tooth tip leakage constant	221
ϕ_{tp}	= tooth tip leakage constant	221
ϕ_z	= zig zag leakage constant	360
ψ	= per cent. pole enclosure	42

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